Synopsis

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Traffic in India does not follow lanes; consequently traffic flow in India is disorderly. In such streams, unlike in streams with lane discipline, longitudinal (i.e., in the primary direction of flow) as well as lateral interactions among vehicles take place continuously. Further, road geometry, especially width, on many Indian roads varies frequently (often owing to poor quality control during construction, unauthorized incursions, etc.). Hence, traffic stream models for India must also incorporate the effect of geometric variations. The effects of road geometry on streams also happen through lateral interactions. Thus, a traffic flow model for India must be able to incorporate both lateral and longitudinal interactions.

In order to incorporate both longitudinal and lateral interactions in driver behavior, a two dimensional model is required. In this thesis, therefore, an attempt is made to develop a two dimensional continuum model of traffic flow. To the best of the knowledge of the author this is the first such attempt. It may be mentioned that even if one wants to understand the impact of road geometry variation on stream behavior of orderly traffic streams (i.e., those that follow lane discipline) then also one would require a two dimensional model capable of incorporating both longitudinal and lateral interactions.

The proposed two dimensional model aims to describe the traffic stream behavior through three partial differential equations (PDEs). One is the continuity equation that describes the conservation of number of vehicles in a section of a road without any entry/exit ramps. The other two equations describe driver behavior in longitudinal and lateral directions. In order to simulate traffic flow governed by the proposed model, the three PDEs need to be solved simultaneously. Analytical solution of these PDEs is not possible and one has to rely on a numerical solution. Further, the nature of the PDEs suggests that the solution might develop shocks (or discontinuities). Therefore, an integral based solution methodology (rather than a difference based method) is considered more suitable for solving these PDEs. Hence, a stabilized Finite element method (FEM) is used here. This thesis also makes a small innovation in the cross term stabilization of numerical oscillations in the solution.

Various numerical experiments are carried out to evaluate the performance of the proposed two dimensional model under different flow and geometric conditions. From these numerical experiments various inferences are drawn. The important ones are mentioned in the next few paragraphs. Numerical experiments show that streams governed by the proposed model can develop oblique shocks, normal shocks, expansion waves and slip lines or a combination of them. Vehicles in streams where these structures (especially oblique shocks) are present are expected to go through frequent and sudden changes in speed. Field experiments using a probe vehicle was conducted on two different roads of Kanpur city. In this experiment speed of the probe vehicle was collected as the vehicle moves along a path parallel to the road edges. It is observed from the field experiments that there are frequent and sudden changes in the speed of the vehicle. Although, these verifications are primitive they reveal the fact that like in the simulated streams vehicles in real streams also go through frequent and sudden speed changes possibly due to the presence of stationary, oblique shock fronts.

Streams governed by the proposed model also show that at steady state at a reasonable distance away from the inlet the stream shows indications of moving towards an equilibrium. Equilibrium here means that there is no variation in traffic density and speed over the width and there is hardly any movement of vehicles in the lateral direction. Such an equilibration is expected since the primary reason for lateral movement is local increments in speed; as streams evolve such lateral movements bring about homogeneity in the lateral direction and at equilibrium no vehicle can increase his/her speed by unilaterally moving in the lateral direction.

Numerical experiments with width variations along a road stretch show how oblique stationary shock waves develop upstream of a narrowing and how expansion waves are formed upstream of the widening. The experiments also indicate that the oblique stationary shocks evolve into normal shocks as the inlet speed falls. The features observed here as well as in the experiments mentioned earlier indicate that traffic streams (as modelled here) behave like compressible fluids (for example, air) at supersonic speed.

These numerical experiments also show that the proposed two dimensional model can explain well the impact of geometric variations (especially, width) on stream behavior. Experiments indicate that roads must have a capacity which depends on width. Then experiments also indicate that traffic operates in two distinct regimes (a free flow regime and a congested regime). Further, it is seen that at high inlet flows backward moving congestion develops upstream of a bottleneck; downstream of the bottleneck the traffic again speeds up through an expansion wave (or fan). In short, traffic streams governed by the proposed model exhibits characteristics of a real world stream.

Even though the primary purpose of this thesis was to develop a two dimensional continuum model of traffic flow and study its properties, this thesis also presents an analysis of the degenerate one dimensional model implied by the proposed two dimensional model. The reason for developing and studying the degenerate model is to establish that the implied one dimensional model behaves in a manner consistent with expectations (it may be pointed out here that over the last five decades a lot of work on one dimensional continuum model has been carried out).

In conclusion, it can be stated that this thesis, in all likelihood for the first time, presents a two dimensional continuum model of traffic flow. Such models, unlike the available one dimensional models, can be used to study disorderly traffic streams like that of India, as well as understand how geometric variations impact traffic flow. The proposed two dimensional model also indicates that in reality traffic streams posses various features like oblique shock waves which often get reflected from edges, normal shock waves, slip lines and expansion waves. It is pertinent to mention that one dimensional continuum models can only predict the presence of normal shocks and expansion waves. The proposed model also shows, like in many other systems where humans interact to achieve their own goals, an equilibrium process is at play in traffic streams. Finally, it is felt that the modeling framework developed here can be used to study other two dimensional flow systems like that of pedestrians.