Abstract

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Traffic flow is not merely a logistical problem but a reflection of human behavior in motion, an emergent phenomenon shaped by infrastructure, culture, and individual decision-making. In its ideal form, traffic is often imagined as an orderly system of lanes, signals, and compliance. Nonetheless, the situation in numerous regions globally, especially in developing nations like India, presents a stark contrast. In this context, traffic systems often function under minimally regulated, lane-free conditions, where drivers navigate the road space fluidly, relying on instinct, experience, and opportunity rather than formal regulations. This lateral freedom, although seemingly chaotic, follows its own implicit logic and social norms, rendering it both complex and fascinating for scholarly examination.

Traditional traffic models, especially those based on the Lighthill–Whitham–Richards (LWR) framework, were developed in the context of well-disciplined, lane-based traffic. These models reduce traffic to a one-dimensional flow, positing that vehicle interactions are predominantly longitudinal. Nevertheless, these assumptions inadequately represent the actual dynamics of disordered traffic, in which lateral interactions, such as position searching, edge effects, and density fluctuations, are fundamental to driver behavior. The identified shortcomings present a significant research gap, particularly within the domains of intelligent transportation systems and real-time traffic management in developing regions.

This thesis seeks to bridge that gap by proposing a two-dimensional extension of the classical LWR model, specifically tailored to represent lane-free traffic environments. The model is grounded in the fundamental conservation of vehicles but expands the spatial framework to accommodate both longitudinal and lateral vehicle movements. At its core, it embodies the philosophy that traffic is not strictly longitudinal; rather, it represents a two-dimensional collective motion influenced by local interactions and boundary constraints. The longitudinal behavior is characterized as anisotropic, acknowledging that drivers primarily respond to vehicles in front of them rather than those behind. Lateral motion is characterized as a combination of convective drift, which denotes intentional sideward movement such as inner-edge bias, and diffusion, which accounts for random displacements influenced by surrounding traffic or road conditions.

This modeling framework is based on several assumptions: a no-flux boundary condition at either side of the road to maintain mass conservation within the road width; a lateral velocity bias to reflect real-world observations of position preference; and a speed-density relationship derived from fundamental traffic principles. These considerations are grounded in empirical observations and a comprehensive understanding of driver psychology within disordered traffic conditions.

The model exhibits intrinsic nonlinearity and is analytically intractable in dynamic scenarios, necessitating a robust numerical solution method. A finite volume technique has been formulated to ensure conservation and stability by adhering to the Courant–Friedrichs–Lewy (CFL) condition. The numerical approach enables the simulation of various intricate traffic scenarios, allowing for the analysis of essential wave phenomena, such as shockwaves, rarefaction waves, and symmetric (both convex and concave) and asymmetric (half-concave) lateral distribution profiles. The simulations demonstrate the transmission of localized disruptions within the traffic flow, accurately reflecting features seen in actual traffic that are often neglected in one-dimensional assessments.

Furthermore, the model is validated by its ability to replicate the empirical observation of a bicycle traffic distribution evolution study conducted on a circular track. This validation exercise demonstrates the model's effectiveness in accurately representing the lateral distribution of vehicles and their tendency to be inclined towards specific road edges, thereby ensuring that simulated outcomes align with observed human behavior.

This thesis presents a mathematical model that represents a conceptual shift, transitioning from the perception of traffic as a one-dimensional queue to understanding it as a dynamic, two-dimensional system of interacting entities. This framework presents various practical applications, including its potential as a foundation for traffic simulation tools, congestion management systems, emission prediction models, and real-time adaptive traffic control strategies in heterogeneous and infrastructure-limited environments.

This model provides a foundation for future extensions, such as the integration of real-world field data for validation, the adaptation of the framework to accommodate on- and off-ramps through space-time-dependent source terms, and the consideration of traffic heterogeneity through multi-class modeling. To address the limitation of infinite accelerations in first-order LWR models, the model can be extended to higher-order models that incorporate momentum equations.

This thesis bridges conceptual understanding and practical aspects of traffic in developing regions, contributing to the development of traffic models that are both mathematically robust and socially relevant.