Lecture – 3

Optimal Guidance using Model Predictive Static Programming (MPSP)

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Outline

- Philosophical overview of guidance
- Applications in Missile Systems
  - Strategic Missile Guidance
  - Tactical Missile Guidance
- Applications in Space Missions
  - Reentry Guidance of a Reusable Launch Vehicle
  - Formation Flying of Satellites
  - Soft-landing on Moon
- Summary
Guidance of Aerospace Vehicles: A Philosophical Overview

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Achievement in Aerospace: A Remarkable Journey

- Early attempts: Lot of failures
- "Heavier-than-air flying machines are impossible" - Lord Kelvin, President, Royal Society, 1895.
- Right Brothers’s first flight: 1903
- History of Flight: Approx. 100 years
- Currently: 1,00,000 flights carrying more than 15 million passengers daily!
- 37.6 million commercial aircraft flights globally transported more than 3.5 billion passengers in 2015 with no accident (IATA)
- Quickest & Safest mode of transport
- Other aerospace vehicles: Military aircrafts, missiles, launch vehicles, spacecrafts, UAVs, MAVs etc.
- Key Enabling Technology: Guidance and Control
Navigation, Guidance and Control

• **Navigation**
  • Determination of position and velocity (both translational and rotational) from sensors

• **Guidance**
  • Online determination of the path to be followed for mission success

• **Control**
  • Achieving the commanded guidance by appropriate actuator action accounting for the flight dynamics
**Guidance? What is it??**

**Question:** What is $R(s)$? How to design it??

Unfortunately, books remain completely silent on this!

![Guidance Diagram](image)

**Fundamental Problem of Strategic Missile Guidance**

- Guidance phase
- LH: Local Horizontal
- GCL: Guidance Loop Closure
- BO: Burn Out Point
- T: Target Point

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**Fundamental Problem of Tactical Missile Guidance**

PN Guidance: \( a_M = NV_M \dot{\lambda} \)

**Missile Guidance Laws**

- Many classical guidance laws are inspired from “observing nature” (e.g. Proportional Navigation (PN) guidance law is based on ensuring “collision triangle”)
- Control theoretic based guidance laws are usually based on “kinematics” and/or “linearized dynamics”. Hence, they are usually not very effective!
- Nonlinear optimal control theory is a “natural tool” to obtain effective missile guidance laws
MPSP for Strategic Missile Guidance
(with solid motors)

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Fundamental Problem of Strategic Missile Guidance

[Diagram showing Guidance phase with terms like LH, BO, T, T₀, and specific angles and coordinates]
Introduction: Liquid Engines vs. Solid Motors

**Liquid Engines**
- Quick firing is not possible
- Sloshing and TWD effect
- Higher cost
- Thrust cut-off facility
- Burnout time is certain
- Manipulative T-t curve
- Guidance is easier

**Solid Motors**
- Quick firing is possible!
- No sloshing and TWD effects
- Lower cost
- No thrust cut-off facility
- Burnout time is uncertain
- Non-manipulative T-t curve
- Guidance is difficult!

System Dynamics

\[
\begin{align*}
\dot{r} &= V \sin \gamma \\
\dot{V} &= \frac{T}{M} \cos \delta - g \sin \gamma \\
\dot{\gamma} &= -\frac{T}{MV} \sin \delta + (\frac{V}{r} - \frac{g}{V}) \cos \gamma \\
\dot{\phi} &= -\frac{V \cos \gamma}{r}
\end{align*}
\]

- \( r \) : Local radius
- \( V \) : Velocity
- \( \gamma \) : Flt. path angle
- \( \phi \) : Range angle
- \( \delta \) : Shear angle (guidance parameter)
Free-Flight and Hit Equation

**FF Equation**

\[ \frac{r_{bo}}{r} = \frac{1 - \cos \theta}{\lambda_{bo} \cos^2 \gamma_{bo}} + \frac{\cos(\theta + \gamma_{bo})}{\cos \gamma_{bo}} \]

**Hit Equation**

\[ \frac{r_{bo}}{r_T} = \frac{1 - \cos \phi_{bo}}{\lambda_{bo} \cos^2 \gamma_{bo}} + \frac{\cos(\phi_{bo} + \gamma_{bo})}{\cos \gamma_{bo}} \]

\[ \lambda_{bo} = \frac{r_{bo} v_{bo}^2}{GM} \]

Reference:

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Numerical Result:
With Nominal Thrust

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**Numerical Result:**

**With Uncertainties in Motor Performance**

![Graph showing thrust vs time with different performance levels](image1)

- Nominal performance
- Over performance
- Under performance

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**Numerical Result:**

**With online re-adjustment of Thrust-time curve**

![Graphs showing trajectory and speed regulation over range and time](image2)

- Nominal performance
- Over performance
- Under performance

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Comparison Between MPSP and Nonlinear Programming

Cost function computation

\[ J_{NLP} = 1.6843 \]
\[ J_{MPSP} = 1.6689 \]

A Hybrid Design For Energy-Insensitive Guidance

- Step 1: Assume that the motor is guaranteed to burn up to a certain duration of predicted burnout time (say 90%). Design the MPSP Guidance
- Step 2: Switch over to Dynamic inversion guidance, which assures that the free flight equation is satisfied for the remaining time continuously.
- Motivation: To eliminate VTP requirement
Numerical Results:
MPSP + Dynamic Inversion

MPSP Guidance of Anti-Ballistic Tactical Missiles under Partial IGC Framework

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**Missile for missile (air) defense: A typical engagement scenario**

Challenges

- Very high speed targets
  - Very less engagement time
  - Very high line-of-sight rate
- Zero/Near-zero miss distance is desired
- Impact angle constraint (mission requirement)
- Directional warhead
- Latax saturation (due to less dynamic pressure) should be avoided
Philosophy of Partial Integrated Guidance & Control (PIGC)

- Exploits the inherent time scale separation property between faster rotational and slower translational dynamics
- Operates in a Two Loop Structure:
  - Commanded body rates generated in outer loop – MPSP
  - Commanded deflections generated in inner loop – Dyn. Inv.

MPSP for Tactical Missile Guidance with Impact Angle Constraint

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**Why Impact Angle Constrained Guidance?**

- Enhancement of warhead lethality (e.g. front-attack)
- Terminal trajectory shaping for attacking weak locations of targets (e.g. top attacks for tanks)
- Mission demand (e.g. bunker buster mission, terrorist hideouts in urban areas, water reservoir attack etc.)
- Coordinated attack by multiple munitions
- Countermeasure by enhanced stealthiness
- Range enhancement (indirectly)
- Increase in observability of the target

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**Motivations**

- Classical (PN) guidance laws are usually adequate to assure small miss distance, but are silent on impact angle.
- Optimal control theory based techniques are available, but they rely on “linearized kinematics”...need to do better than that!
- The aim here is to develop a nonlinear optimal guidance law with 3-D impact angle constraints, using nonlinear point-mass dynamic models.
Challenges

- Strong (nonlinear) coupling between elevation angle and azimuth angle dynamics should be accounted for.
- Zero/Near-zero miss distance requirement cannot be compromised.
- Impact angle constraints in 3D (i.e. two angle constraints at the same time) must be ensured.
- Latex demand has to be as minimum as possible.
### Assumptions about target

- Speed $V_t$ is constant
- Target moves with a lateral acceleration command $a_{y_t}$ normal to its velocity $V_t$, which is known.

- $a_{y_t}$ can be:
  - Zero (straight line movement)
  - Constant (constant $g$ maneuvers)
  - Sinusoidal (periodic maneuvers)
Stationary Targets
MPSP Vs. APN: A Comparison

\[ \gamma_{mf} = -90^\circ \]
\[ \gamma_{mb} = 10^\circ \]
\[ \psi_{mb} = 20^\circ \]

Constraint in single angle

Stationary Targets
MPSP Vs. APN: A Comparison

Guidance commands

Constraint in single angle
Stationary Targets:
Same initial conditions & Different Terminal Constraints

Various constraints in both angles

Stationary Targets:
Different initial conditions for Same Terminal Constraints

Initial condition perturbation with same terminal constraint
Stationary Targets: 
Perturbation in initial conditions 
(Angle Histories)

MPSP for Re-entry Guidance of a 
Re-usable Launch Vehicle (RLV)

Co-workers: Omkar Halbe, Scientist, Airbus, Germany and S. Mathavaraj, Scientist, ISAC/ISRO, Bangalore

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Objective and Challenges

- **Objective**
  - To develop advanced nonlinear and optimal guidance for a reusable launch vehicles (RLV) in the descent phase, with special emphasis on the critical re-entry segment.

- **Challenges**
  - **Path constraints**: Structural load, Thermal load, Angle of attack boundary
  - **Terminal constraints**: Final position and velocity
  - Optimal online trajectory generation
  - Robustness wrt. uncertainties in parameters
  - Real-time computability, Smoothness in guidance command etc.

Kinematic Equations over Spherical Rotating Earth

\[
\begin{align*}
\dot{r} &= V \sin \gamma \\
\dot{\phi} &= \frac{V \cos \gamma \sin \psi}{r} \\
\dot{\theta} &= \frac{V \cos \gamma \cos \psi}{r \cos \phi}
\end{align*}
\]

Dynamic Equations over Spherical Rotating Earth

\[
\begin{align*}
\ddot{V} &= - \frac{D}{m} - g \sin \gamma + \Omega_e^2 r \cos \phi (\sin \gamma \cos \phi - \cos \gamma \sin \phi \sin \psi) \\
\dot{\gamma} &= \frac{L \cos \sigma}{mV} - \frac{g \cos \gamma}{V} + \frac{V \cos \gamma}{r} + 2\Omega_e \cos \phi \cos \psi \\
&\quad + \frac{\Omega_e^2 r}{V} \cos \phi (\cos \gamma \cos \phi + \sin \gamma \sin \phi \sin \psi) \\
\dot{\psi} &= \frac{L \sin \sigma}{mV \cos \gamma} - \frac{V \cos \gamma}{r} \cos \gamma \cos \psi \tan \phi + 2\Omega_e (\tan \gamma \cos \phi \sin \psi - \sin \phi) \\
&\quad - \frac{\Omega_e^2 r}{V \cos \gamma} \sin \phi \cos \phi \cos \psi
\end{align*}
\]
**RLV Guidance using MPSP: Problem Formulation**

- **Normalized State Vector**
  \[ Z = \begin{bmatrix} V \\ \gamma \\ \phi \\ \theta \end{bmatrix}^T \]

- **Control Vector**
  \[ U = \begin{bmatrix} \alpha \\ \psi \end{bmatrix}^T \]

- **Output Vector**
  \[ Y = \begin{bmatrix} V \\ \gamma \\ \phi \\ \theta \end{bmatrix}^T \]

- **Goals**
  1. \( \Delta Y = (Y_n - Y^*_n) \rightarrow 0 \)
  2. Normal Load Minimization
  3. Control Deviation Minimization
  4. Control Smoothness

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**Normalization of State Variables**

- Absolute values of state variables are not of the same order of magnitude
- May create disparity during the control and state update process
- State variables are normalized using the desired final values

\[
\begin{align*}
V_n &= \frac{V}{V^*} \\
\gamma_n &= \frac{\gamma}{\gamma^*} \\
\phi_n &= \frac{\phi}{\phi^*} \\
\theta_n &= \frac{\theta}{\theta^*}
\end{align*}
\]
**RLV Guidance using MPSP: Performance Index Selection**

**“Soft” Path Constraints**

\[ J_1 = \frac{1}{2} \sum_{k=1}^{N} A e^{B N_k} N_k \]

- Minimize normal load only if value close to bound
- Exponential weight of nominal NL profile to achieve this

\[ J_2 = \frac{1}{2} \sum_{k=1}^{N-1} d U_k R_d d U_k^T \]

- Minimize deviation (error) of updated profile from nominal one
- Maintain control profile in the vicinity of nominal profile

\[ J_3 = \frac{1}{2} \sum_{k=2}^{N-1} (U_k - U_{k-1}^p) R_{im} (U_k - U_{k-1}^p)^T \]

- For smoothness, minimize distance CD
- Equivalent to minimizing distances AB, BD and AD
- AB is nominal profile
- Min (BD) = min (J2)
- Min (AD) = min (J3)
RLV Guidance using MPSP: Nominal Trajectory with 8 Initial Conditions

- Smooth Profile
- Trim Bounds not Violated
- Stable Lateral Profile

Guidance Simulation Results: 8 Cases of Initial Condition Perturbations

- Smooth Trajectories, Terminal Constraints Met
Guidance Simulation Results: 8 Cases of Initial Condition Perturbations

Path Constraints Met
1. Normal Load < 3g
2. Heat Flux < 60 W/cm²
3. Dynamic Pressure < 25 kPa

Summary of Results

- MPSP is a potential technique to implement for Re-entry guidance as it leads to:
  - Very good terminal accuracy
  - Path-constraints are met
  - Maximum normal load is below 3g at all time
  - Bank-angle reversals are in the middle
  - Shows good amount of robustness
  - Computational time is small
Satellite Formation Flying (SFF) through MPSP

Co-worker: Girish Joshi, Scientist, ISAC/ISRO, Bangalore

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Satellite Formation Flying

(Image Source: en.wikipedia.org, newsscientist.com)

Two or more satellites flying in prescribed orbits at a fixed separation distance for a given period of time forming a large virtual spacecraft

Trailing Formation
Constellation
Cluster Formation
**Satellite Formation Flying: Advantages & Objectives**

- **Advantages**
  - Higher redundancy: Improved fault tolerance
  - On orbit configuration: Coordinated multi missions
  - Lower individual launch mass: Reduced launch cost and increased mission flexibility
  - Minimal financial loss in case of failure

- **Objectives**
  - Reconfigure the formation of satellites to new desired relative orbit: Minimize terminal errors
  - Minimize the control effort required

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**System Dynamics and Problem Formulation**

**Hill’s reference Frame:**

- Chief Satellite Centered non inertial reference frame

\[
\hat{e}_x = \frac{\dot{r}_c}{|\dot{r}_c|} \\
\hat{e}_z = \frac{\hat{h}}{|\hat{h}|} \\
\hat{e}_y = -\hat{e}_x \times \hat{e}_z
\]

- $h$: Angular momentum Vector

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System Dynamics

- Complete nonlinear SFF system dynamics (Hill’s Equations)

\[
\begin{align*}
\ddot{x} - 2\dot{v}\dot{y} - \dot{v}\ddot{y} - \dot{v}^2 x + \frac{\mu}{\gamma} x + \frac{\mu}{\gamma^2} r_e - \frac{\mu}{r_e^3} \\
\dot{y} + 2\dot{v}\dot{x} + \dot{v}x - \dot{v}^2 y + \frac{\mu}{\gamma} y \\
\ddot{z} + \frac{\mu}{\gamma^2} z
\end{align*}
\]

Where \( x, y, z \) and \( \dot{x}, \dot{y}, \dot{z} \) are position and velocities in Hill’s frame. \( \mu = GM = 398601 \frac{km^3}{s^2} \) & \( \gamma = \lvert \vec{r} + \vec{\rho} \rvert^2 \)

Formation flying of satellite using MPSP:

- State Vector

\[ X = [x \ \dot{x} \ y \ \dot{y} \ z \ \dot{z}]^T = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T \]

- Control Vector

\[ U = [a_x \ a_y \ a_z]^T = [u_1 \ u_2 \ u_3]^T \]

- Output Vector

\[ Y = X \]

- Final time \( t_f = 2000 \text{ sec} \)

Selected from LQR method (time at which tracking error becomes small)
### Circular Chief Satellite Orbit: Results and Discussion

Position Error units: \( km \); Velocity Error units: \( km/sec \)

<table>
<thead>
<tr>
<th>State error</th>
<th>LQR</th>
<th>MPSP (Iter No. 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>0.1683</td>
<td>-0.4578 \times 10^{-3}</td>
</tr>
<tr>
<td>( \dot{x} )</td>
<td>0.00012</td>
<td>-0.0007 \times 10^{-3}</td>
</tr>
<tr>
<td>( y )</td>
<td>-0.1042</td>
<td>-0.4184 \times 10^{-3}</td>
</tr>
<tr>
<td>( \dot{y} )</td>
<td>-0.0002</td>
<td>-0.0003 \times 10^{-3}</td>
</tr>
<tr>
<td>( z )</td>
<td>0.2003</td>
<td>0.754 \times 10^{-4}</td>
</tr>
<tr>
<td>( \dot{z} )</td>
<td>0.00008</td>
<td>0.1071 \times 10^{-10}</td>
</tr>
</tbody>
</table>

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### Results and Discussion

Position States Error History LQR, MPSP

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Results and Discussion
Velocity States Error History LQR, MPSP

LQR  MPSP

Results and Discussion
Control History LQR, MPSP

LQR Control  MPSP Control
Numerical Results: Output Error Convergence

<table>
<thead>
<tr>
<th>Error in states</th>
<th>Initial Error</th>
<th>Iteration 1</th>
<th>Iteration 2</th>
<th>Iteration 3</th>
<th>Iteration 4</th>
<th>Iteration 5</th>
<th>Iteration 6</th>
<th>Iteration 7</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1.0e-003</td>
<td>1.0e-004</td>
<td>1.0e-005</td>
<td>1.0e-003</td>
<td>1.0e-004</td>
<td>1.0e-005</td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td>0.0275</td>
<td>0.0374</td>
<td>0.0214</td>
<td>0.0024</td>
<td>-0.0388</td>
<td>-0.7360</td>
<td>-0.1512</td>
<td>-0.1345</td>
</tr>
<tr>
<td>$\dot{x}$</td>
<td>-0.0223</td>
<td>0.0832</td>
<td>0.0315</td>
<td>0.0047</td>
<td>0.1959</td>
<td>-0.8402</td>
<td>-0.2370</td>
<td>-0.2949</td>
</tr>
<tr>
<td>$y$</td>
<td>-0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0002</td>
<td>-0.0014</td>
<td>-0.0004</td>
<td>-0.0004</td>
</tr>
<tr>
<td>$\dot{y}$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0012</td>
<td>-0.0003</td>
<td>-0.0004</td>
<td>-0.0007</td>
</tr>
<tr>
<td>$z$</td>
<td>0.0456</td>
<td>-0.1314</td>
<td>-0.0000</td>
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<td>-0.0000</td>
<td>-0.0000</td>
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<td>-0.0000</td>
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<tr>
<td>$\dot{z}$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.0000</td>
<td>-0.0000</td>
<td>-0.0000</td>
</tr>
</tbody>
</table>

Results and Discussion
Composite Plot for MPSP solution for different initial conditions:

- $\rho_{\text{initial}} = 0.5km$
- $\rho_{\text{final}} = 1.5km$
- Circular chief satellite orbit
Rendezvous (Docking)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Desired Final States</th>
<th>Error in States (LQR)</th>
<th>Error in States (MPSP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>-1.1086 km</td>
<td>-56.1m</td>
<td>4 mm</td>
</tr>
<tr>
<td>y</td>
<td>2.0209 km</td>
<td>39.1m</td>
<td>9 mm</td>
</tr>
<tr>
<td>z</td>
<td>0.3579 km</td>
<td>-57.1m</td>
<td>0 mm</td>
</tr>
<tr>
<td>x_dot</td>
<td>0.0006 m/s</td>
<td>0 m/s</td>
<td>1.3 mm/s</td>
</tr>
<tr>
<td>y_dot</td>
<td>1.4 m/s</td>
<td>0.1m/s</td>
<td>2.2 mm/s</td>
</tr>
<tr>
<td>z_dot</td>
<td>2.4 km/s</td>
<td>0 m/s</td>
<td>0.0 m/s</td>
</tr>
</tbody>
</table>

Computational Time (in Matlab): Approx. 20 sec (for one iteration)

Summary of Results

- MPSP is a potential technique to implement for Satellite formation flying as it leads to:
  - Very good terminal accuracy
  - Less number of iterations
  - Computational time per iteration is small

- Current/Future work
  - Evaluation in a Real-time platform
  - Possible implementation in hardware
Fuel-Optimal Guidance of a Lunarcraft for Soft Landing on Moon

Co-workers: Avijit Banerjee & Kapil Sachan, Students, IISc, Bangalore

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A Typical Lunar Mission
Soft-Landing Mission

De-orbit maneuver

Transfer Orbit Phase

100km

Main Breaking

Altitude

18 km

100 km

Attitude Hold Phase

Rough Braking Phase

Precision Braking Phase

50°

E

D

C

B

A

v = 0 m/s

V_H = 1690 m/s

4 x 800

10 km

4.2 km

0

Pitch-In Phase

Hovering Phase

Final Descent Phase

LIRAP

LIRAP, Altimeter

LIRAP, Altimeter, Pattern Matching Camera, Velocity Sensors

LIRAP, Altimeter, Velocity Sensors, HDA Camera

68

7 km

2 km

Down Range

TRAJECTORY PROFILE OF A LUNAR LANDER

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Problem to be Explored:
Optimal Soft-landing of Lunar Module

Objective

- To develop an optimal and robust Guidance algorithm for soft landing of the lander
- The algorithm should ensure:
  - Soft landing at the designated site
  - Minimum fuel consumption
  - Non-violation of mission and hardware constraints
  - Robustness for uncertainties (due to engine misalignment, CG shift, fuel slosh etc.)
  - Real-time computability for onboard implementation
System Dynamics in 2D

\[ \begin{align*}
\dot{r} &= v \\
\dot{\theta} &= \frac{u}{r} \\
\dot{u} &= -\frac{uv}{r} + \frac{T}{M} \cos \beta \\
\dot{v} &= \frac{u^2}{r} - \frac{\mu}{r^2} + \frac{T}{M} \sin \beta \\
\dot{M} &= -\frac{T}{I_{sp}}
\end{align*} \]

Variables
- \( r \) : altitude
- \( \theta \) : range angle
- \( u \) : transversal velocity
- \( v \) : radial velocity
- \( M \) : module mass
- \( T \) : thrust magnitude
- \( \beta \) : thrust direction

Objective: Fuel Minimization

- Minimum fuel consumption is ensured by maximizing the landed mass, i.e. by minimizing \( J_o = \left( \frac{M_0}{M_f} \right) \)
- However, minimization of \( J_o \) is equivalent minimizing \( J_b \), where

\[ J_b = \ln \left( \frac{M_0}{M_f} \right) = - \int_{M_0}^{M_f} \left( \frac{dM}{M} \right) = - \int_{0}^{t_f} \left( \frac{dM}{M} \right) \, dt \]

\[ = - \int_{0}^{t_f} \left( \frac{dM}{M} \right) \, dt = \frac{1}{I_{sp}} \int_{0}^{t_f} \left( \frac{T}{M} \right) \, dt = \frac{1}{I_{sp}} \int_{0}^{t_f} \sqrt{U_x^2 + U_y^2} \, dt \]

- So, the cost function selected is:

\[ J = \frac{1}{2} \int_{0}^{t_f} \left( U_x^2 + U_y^2 \right) \, dt \]
Terminal Vertical Orientation

\[ J = \frac{1}{2} \int_0^{t_f} U^T RU \, dt \quad \text{where} \ U = \begin{bmatrix} U_x \\ U_y \end{bmatrix} = \begin{bmatrix} \frac{T}{M} \cos \beta \\ \frac{T}{M} \sin \beta \end{bmatrix} \]

and \( R = \begin{bmatrix} r_1(t) & 0 \\ 0 & r_2(t) \end{bmatrix} \)

- To ensure the terminal vertical orientation \( \beta \to 90^\circ \)

\[ \tan \beta = \frac{U_x}{U_y}, \quad U_z \to 0 \quad \text{as} \quad t \to t_f \]

Control Weight Profile

Computation of guess control

- Considering the altitude as a function of time

\[ r = -\frac{1}{(a + bt + ct^2 + dt^3)} \quad \dot{r} = v \]

- Considering initial and terminal conditions

\[ r_0 = \frac{1}{a} \quad \text{(at} \ t = 0) \quad r_y = -\frac{1}{(a + bt + ct^2 + dt^3)} \quad \text{(at} \ t = t_f) \]

\[ v_0 = -\frac{b}{a^2} \quad \text{(at} \ t = 0) \quad v_y = -\frac{b + 2ct + 3dt^2}{(a + bt + ct^2 + dt^3)^2} \quad \text{(at} \ t = t_f) \]
Computation of guess control (cont..)

- Considering the linear approximation of transversal velocity profile
  \[ u = u_a + u_b t \quad \Rightarrow \quad u_a = u_0 \quad \Rightarrow \quad u_b = \frac{(u_f - u_0)}{t_f} \]

- The control guess can be computed from following equations
  \[
  u_x = \dot{u} + \frac{uv}{r} \\
  u_y = \dot{v} - \frac{u^2}{r} + \frac{\mu}{r^2}
  \]

Terminal Mass & Cost Function Variation with Final Time

Variation of the mission cost w.r.t. final flight time

- Maximum terminal mass
- Minimum cost
Optimal Flight Time:

- MPSP computes the fuel optimal guidance command based on the selected final time \( (t_f) \).
- To select the optimal \( t_f \): a gradient based off-line computational method is formulated.
- To make the \( t_f \) selection online: a neural net based function learning is incorporated to account for the variation of initial state of the lunar module.

\[
u_0 = \frac{\sqrt{r_0 \mu(1 + e)}}{r_0} \]

Prof. Radhakant Padhi, AE Dept., IISc-Bangalore
Numerical Results: MPSP Guidance

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Indian Institute of Science – Bangalore

Simulation Results with Free Terminal Attitude

Altitude Profile

Mass Profile

Mass consumption 45% of initial mass

20 September 2016
Soft-landing of Lunar Lander
Simulation Results:

Numerical Convergence of MPSP Guidance:

<table>
<thead>
<tr>
<th>Normalized Terminal Error ($\Delta X_n$)</th>
<th>Iteration I (normalized)</th>
<th>Iteration II (normalized)</th>
<th>Iteration III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude</td>
<td>3.63</td>
<td>-45.05 x 10^{-3}</td>
<td>1.72 x 10^{-3}</td>
</tr>
<tr>
<td>Range angle</td>
<td>-1.16</td>
<td>-2.43 x 10^{-3}</td>
<td>0.37 x 10^{-3}</td>
</tr>
<tr>
<td>Transverse velocity</td>
<td>-1.76</td>
<td>74.59 x 10^{-3}</td>
<td>3.23 x 10^{-3}</td>
</tr>
<tr>
<td>Radial velocity</td>
<td>127.91</td>
<td>-32.37 x 10^{-3}</td>
<td>3.66 x 10^{-3}</td>
</tr>
</tbody>
</table>

Algorithm takes only 3 iterations to converge
High Terminal accuracy is ensured

Average computational time for the complete algorithm is 0.576728 second on “Matlab 2013a” platform
Simulation Results:

Thrust magnitude history

![Thrust magnitude history graph]

The terminal body orientation constraints is not satisfied

Thrust direction history

Terminal Vertical Orientation

\[ J = \frac{1}{2} \int_0^T U^T R U \, dt \quad \text{where} \quad U \]

\[ = \begin{pmatrix} \alpha_x \\ \alpha_z \end{pmatrix} = \begin{pmatrix} \frac{T}{M} \cos \beta \\ \frac{T}{M} \sin \beta \end{pmatrix} \]

and \( R = \begin{bmatrix} r_1(t) & 0 \\ 0 & r_2(t) \end{bmatrix} \)

- To ensure the terminal vertical orientation \( \beta \to 90^\circ \)

\[ \tan \beta = \frac{a_y}{a_z} \quad \alpha_z \to 0 \quad \text{as} \quad t \to t_f \]

Control Weight Profile

![Control Weight Profile graph]
Simulation Results:

Thrust direction history

Altitude Profile

• Terminal Mass of Module: 485kg (high mass consumption)
• Thrust limit is violated

Thrust magnitude history

Mass Profile
Remarks:

- Orientation of module, with high velocity is difficult and demands more fuel.

- Segment-I ensures the desired low velocity and altitude

- Terminal vertical orientation considered during the followed by segment.

Simulation results:

Altitude Profile

Mass profile

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Simulation results:

Transversal velocity history

Radial velocity Profile

Thrust magnitude history

Thrust direction history
Remarks:

- Orientation of module, with high velocity is difficult and demands more fuel.

- Segment-I ensures the desired low velocity and altitude

- Terminal vertical orientation considered during the followed by segment.

\[ J = \frac{1}{2} \int_0^t U^T R_1 U \, dt + \frac{1}{2} \int_0^t (U - U_0)^T R_2 (U - U_0) \, dt \]

Simulation Results:

Altitude Profile  
Mass Profile
Simulation Results:

Transversal velocity history

Radial velocity profile

Simulation results:

Thrust direction history

Thrust magnitude history

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### Segmentation:

<table>
<thead>
<tr>
<th>Segment I</th>
<th>Segment II</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial</strong></td>
<td><strong>Terminal</strong></td>
</tr>
<tr>
<td>Altitude</td>
<td>18 km</td>
</tr>
<tr>
<td>Range angle</td>
<td>0°</td>
</tr>
<tr>
<td>Transverse velocity</td>
<td>1692 m/s</td>
</tr>
<tr>
<td>Radial velocity</td>
<td>0 m/s</td>
</tr>
<tr>
<td>Mass</td>
<td>934 kg</td>
</tr>
</tbody>
</table>

Without terminal angle constraint

With terminal angle constraint

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### PIL Simulation: G-MPSP Guidance

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Processor Details

Texas Instruments’ TMS320C6748

Advantages of C6748:
- Low power applications
- Easy memory management
- Scalable

Features of LCDK and C6748:
- 456 MHz C674x Floating point core
- 2 level cache based architecture
- XDS100v2 JTAG debug probe

Details:

Integrated Development Environment (IDE)

Texas Instruments’ Code Composer Studio (CCStudio V5)
- Code Composer Studio used for interfacing between processor and the PC.
- It comprises a C/C++ compiler, source code editor, project build environment, debugger and other features.
PIL Simulation Procedure

- Code the algorithm in C.
- Compile the program and test for errors.
- Configure CCS to connect to the LCKD.
- Launch Target Configuration and connect the target processor to the host PC.
- Build, load and run the program onto the target processor.
- Determine CPU cycles and execution time using the inbuilt clock tool.

Quantitative results

\[
\text{Execution Time} = \frac{\text{CPU Cycles}}{\text{Clock Rate}} = 456 \text{ MHz}
\]

Segment 1
- Total CPUCycles = 38614470
- Total Execution Time = 84.68 ms
- Number of Iterations = 5
- Execution Time per Iteration = 16.93 ms

Segment 2
- Total CPUCycles = 13749110
- Total Execution Time = 30.15 ms
- Number of Iterations = 4
- Execution Time per Iteration = 7.53 ms
Concluding Remarks

MPSP is a promising technique for optimal guidance of aerospace vehicles.

Trajectory optimization philosophy is brought into guidance design (with no hard constraints).

It is not quite sensitive to the initial guess history of control history.

Various challenging aerospace guidance problems have been (and are being) solved.

There are interests in DRDO and ISRO to implement it in various upcoming missions.
Thank you

questions ... ??