

# Matrix Based Univariate And Multivariate Linear Similar Day Approach Towards Short Term Solar Radiation Forecasting

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**Abstract**—Increase in the penetration of renewable generation like solar and wind in power system requires expansion of transmission structure as well as more flexible generation, better cycling of existing generation, accurate spinning reserve planning, optimum power dispatch from renewable plants and security evaluations and stability analysis with high penetrated renewables. Almost all these operations require real time predicted generation from intermittent renewable sources. There are several models present in literature for solar power prediction, but accuracy and real time applications are the major challenges. Artificial neural network (ANN) models, Numerical Weather Prediction (NWP) models and Machine Learning models like Support Vector Regression, Bayesian Approaches and Hybrids models are used for prediction. The processing time of these models for decently accurate prediction is quite high. This paper investigate the similarity of solar radiation of similar days in different years and also implements one univariate and two multivariate similar day based linear models for short term solar power prediction. Similar days inputs are used prediction because, position of sun and earth is same corresponding to previous year similar day, only difference is cloud cover and cloud movement, wind speed and temperature. If we assume dependent factors will remain same with respect to previous years similar day, solar radiation will be same as previous years similar day values. Results obtained from these models shows that these models have strong potential for accurate real time solar power prediction and their accuracy can be enhanced by considering factors affecting solar irradiation.

**Index Terms**—Short Term Solar Power prediction; Linear Time Series Model, Similar Day Approach.

## I. INTRODUCTION

Renewable integration in the grid is progressively increasing over the whole world and some of the countries are planning 20 to 30% integration by 2020. Solar and wind are the most widely used renewables till the date. Secure operation of grid and optimum dispatch of renewable energy demands accurate power prediction models. Countries like India have strong potential for solar energy due to its geographical specialties and it is planning for 175 GW renewables by 2020, out of these large contribution is from solar. To maintain the grid codes utility level solar power plants requires accurate solar power prediction models otherwise they may face economical penalties and technical challenges like frequency stability, voltage stability etc [1], [2], [3], [4] and [5].

High penetration of renewable generation in grid makes spinning reserve planning, optimum power dispatch from renewable plants, security evaluations and stability analysis are more complex. Spinning reserve planning is one of the major operation challenge, which will accommodate load and generation variability. Unavailability of advance generation information demands more spinning reserve in the system. Spinning reserve plants are mostly conventional fossil fuel plants, which emits more carbon. Therefore renewable plants without accurate prediction will not reduce carbon emission as expected. Planning, scheduling and operations of network needs accurate load prediction as well as accurate generation forecast. Generation forecast accuracy have great impact on system security as well reliability [6], [7], [8], [9], [10], [11], [12] and [13].

Solar power prediction is important in distributed generation systems containing renewable resources. Large scale solar power proliferation is only probable with integration of accurate solar power prediction techniques in Energy Management System (EMS) for the better operation of electrical grid. The accurate PV prediction provides better assistance for the operators in grid, which helps them to make better scheduling, regulation of power, dispatching and also help to make efficient market bidding strategies. Model Predictive Control (MPC) strategy is used to manage the real time energy generation of a grid connected solar power plant with a reduced capacity energy storage system which participates in the market. Accurate PV power forecasting is the base of MPC, the saturation of the ES system (ESS) is advanced by the use of a solar power forecasting model. Therefore, based on market deviations better production strategies can develop by the plant operators for reducing economic penalties. Grid connected PV plants requires accurate solar forecasting model for creating better power dispatching plans, but in the case of standalone and hybrid systems accurate solar power prediction required for improving the control strategies of charge controllers [14-19] [22-26].

Real time solar power prediction has two objectives, first one is accuracy and second one is processing time. ANN, Fuzzy Logic based ANN, Support Vector Regression, ARIMA, Grey Model, Wavelet Neural Networks, General Simulation Model and Expert System based models, Sky Imagery based techniques, Numerical weather prediction models Conven-

tional models with modern optimization techniques are used to address short term forecasting problem in literature [1-9], [12-18] and [20-38]. All these non-linear models are complex in nature, data required for accurate prediction is quite high and processing time is high. This paper uses similar day based linear model for short term solar power prediction, which produces comparable results with conventional complex and time consuming techniques. There is further scope for improvement by the consideration of factors affecting solar radiation [39].

This paper is further organized as follows: Section II defines linear time series model and it also describes one day model, two day model and three day model. Section III gives data selection for forecasting. Section IV compares the different forecasted outputs with actual values for all the above mentioned models. Section V concludes that similar day based univariate and multivariate models have great potential towards real time solar power forecasting and the accuracy can be improved further by the addition of exogenous factors affecting radiations.

## II. MATRIX BASED LINEAR REGRESSION MODEL

### A. Matrix based Univariate Model (MUM)

One day model is a univariate model which uses data from 8 am to 5 pm of previous year similar day  $H_P$  and predicts the next year similar day data  $H_{P+1}$ , for the same time interval. The one day univariate model can be represented by the formula [40], which is a modified version of [41].

$$H_{P+1}(i) = a + b * H_P(i) \quad (1)$$

$$A = \begin{bmatrix} a \\ b \end{bmatrix} = [B^T * B]^{-1} . B^T . H_P \quad (2)$$

$$B = \begin{bmatrix} 1 & H_{P-1}(1) \\ 1 & H_{P-1}(2) \\ 1 & H_{P-1}(3) \\ \vdots & \vdots \\ 1 & H_{P-1}(n) \end{bmatrix} \quad H_P = \begin{bmatrix} H_P(1) \\ H_P(2) \\ H_P(3) \\ \vdots \\ H_P(n) \end{bmatrix} \quad (3)$$

Where,  $H_P$  represents similar day data from year P and  $H_{P+1}$  represents similar day data from year P+1, n represents hours (8 am to 5 pm). B is matrix of  $n \times 2$  and i varies from 0 to n. a and b represent model parameters.

### B. Matrix based Bi-variate Model (MBM)

Two day model is multivariate model which uses data from 8 am to 5 pm of previous two years similar days  $H_P$  and  $H_{P-1}$  and predicts next year similar day data,  $H_{P+1}$ , for the same time interval. Two day multivariate model can be represented by the formula [40], which is a modified version of [41].

$$H_{P+1}(i) = a + b * H_P(i) + c * H_{P-1}(i) \quad (4)$$

$$A = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = [B^T * B]^{-1} . B^T . H_P \quad (5)$$

$$B = \begin{bmatrix} 1 & H_{P-1}(1) & H_{P-2}(1) \\ 1 & H_{P-1}(2) & H_{P-2}(2) \\ 1 & H_{P-1}(3) & H_{P-2}(3) \\ \vdots & \vdots & \vdots \\ 1 & H_{P-1}(n) & H_{P-2}(n) \end{bmatrix} \quad H_P = \begin{bmatrix} H_P(1) \\ H_P(2) \\ H_P(3) \\ \vdots \\ H_P(n) \end{bmatrix} \quad (6)$$

Where,  $H_P, H_{P-1}$  represents similar day data from year P and P-1 respectively,  $H_{P+1}$  represents similar day data from year P+1, n represents hours (8 am to 5 pm). B is matrix of  $n \times 3$  and i varies from 0 to n. a, b and c represent model parameters.

### C. Matrix based Tri-variate Model (MTM)

Three day model is a multivariate model which uses data from 8 am to 5 pm of previous three years similar days  $H_P, H_{P-1}$  and  $H_{P-2}$ , then predicts next year similar day data,  $H_{P+1}$  for the same time interval. Three day multivariate model can be represented by the formula [40], which is a modified version of [41].

$$H_{P+1}(i) = a + b * H_P(i) + c * H_{P-1}(i) + d * H_{P-2}(i) \quad (7)$$

$$A = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = [B^T * B]^{-1} . B^T . H_P \quad (8)$$

$$B = \begin{bmatrix} 1 & H_{P-1}(1) & H_{P-2}(1) & H_{P-3}(1) \\ 1 & H_{P-1}(2) & H_{P-2}(2) & H_{P-3}(2) \\ 1 & H_{P-1}(3) & H_{P-2}(3) & H_{P-3}(3) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & H_{P-1}(n) & H_{P-2}(n) & H_{P-3}(n) \end{bmatrix} \quad (9)$$

Where,  $H_P, H_{P-1}$  and  $H_{P-2}$  represents similar day data from year P, P-1 and P-2 respectively,  $H_{P+1}$  represents similar day data from year P+1, n represents hours (8 am to 5 pm). B is matrix of  $n \times 4$  and i varies from 0 to n. a, b, c and d represent model parameters.

### D. Performance Parameters

Error is calculated by taking the difference between actual value and forecasted value for each data point. The principal statistics used to evaluate the performance of the proposed model are Mean Absolute Percentage Error (MAPE) and Mean Absolute Error (MAE). Error E is calculated by,

$$E = Y - Q \quad (10)$$

Where Y is actual output, Q is predicted value. Mean average error is calculated by taking the mean of absolute error.

$$MAE = E_A / N \quad (11)$$

$$E_A = E_1 + E_2 + E_3 \dots E_N \quad (12)$$

Where  $E_1, E_2, E_3, \dots, E_N$  are absolute values of individual errors. Percentage error is calculated by dividing absolute value of error by corresponding actual value.

Mean absolute percentage error is the mean of absolute percentage errors

$$MAPE = (1/N) \sum |PE| \quad (13)$$

### III. DATA SELECTION

For this work data has been selected for Chicago, USA [42]. The hourly solar radiation data (8 am to 5 pm) from the similar days of 2010, 2011, 2012 are used as input and 2013 similar day data is used for validation. Global horizontal irradiation data in  $W/m^2$  is used. Data selection table for March 15 is shown in TABLE I, rest days selection is similar.

TABLE I  
DATA SELECTION

Model	Data used as input	Forecasted data
MUM	15/03/2012	15/03/2013
MBM	15/03/(2012,2011)	15/03/2013
MTM	15/03/(2012,2011,2010)	15/03/2013

### IV. RESULTS AND DISCUSSION

The global horizontal irradiation forecasted by the models were compared to the actual global solar irradiation data and the error was calculated using Matlab2015a. Performance parameters of randomly selected days from alternative months are tabulated in Table II and over all performance parameters are tabulated in Table III.

TABLE II  
PERFORMANCE PARAMETERS FOR DIFFERENT DAYS

Days	MAE ( $W/m^2$ )			MAPE (%)		
	MUM	MBM	MTM	MUM	MBM	MTM
Jan 15	34.5	24.91	8.06	16.89	10.34	4.67
Mar 15	67.04	60.83	50.47	15.82	15.82	14.39
May 15	4.18	4.28	3.74	0.55	0.56	0.46
Jul 15	7.34	3.21	2.76	0.95	0.48	0.42
Sep 15	9.43	5.49	3.37	1.55	0.87	0.63
Nov 15	9.71	9.39	7.39	4.11	3.53	2.65

TABLE III  
COMPARISON OF PERFORMANCE PARAMETERS FOR DIFFERENT MODELS

Model	MAE( $W/m^2$ )	MAPE( %)
MUM	22.03	6.64
MBM	18.01	5.26
MTM	12.63	3.87

Temperature is an indicator of sun position, average monthly temperature from US climatic data impacts the results obtained from LTS models. Average monthly temperature of Chicago is shown in Fig. 7. From the temperature data it is clear that the months January and March has an average monthly temperature value less than 5 degree celsius. These months have an average values of MAPE are 16.35%, 13.08% and 9.05% for LTS one day, LTS two day and LTS three day respectively and average values of MAE are  $50.77W/m^2$ ,  $42.87W/m^2$  and  $29.26W/m^2$  for LTS one day, LTS two day and LTS three day respectively. Rest months have an average values of MAPE 1.79%, 1.36% and 1.04% for LTS one day, LTS two day and LTS three day respectively and average values of MAE  $7.665W/m^2$ ,  $5.59W/m^2$  and  $4.31W/m^2$  for LTS one day, LTS two day and LTS three day respectively. The months having low average temperature shows large error and months having high average temperature shows lower error.

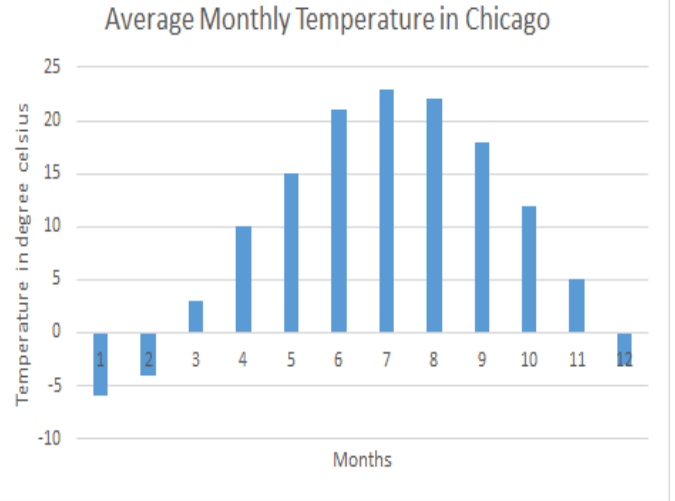


Fig. 1. Average Monthly temperature of Chicago

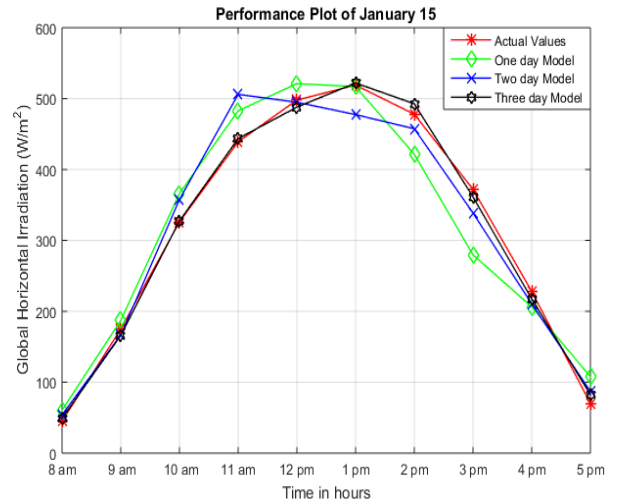


Fig. 2. Performance plot of January 15

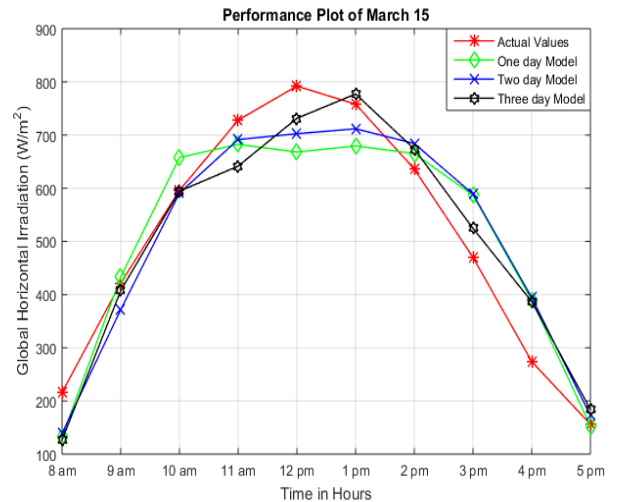


Fig. 3. Performance plot of March 15

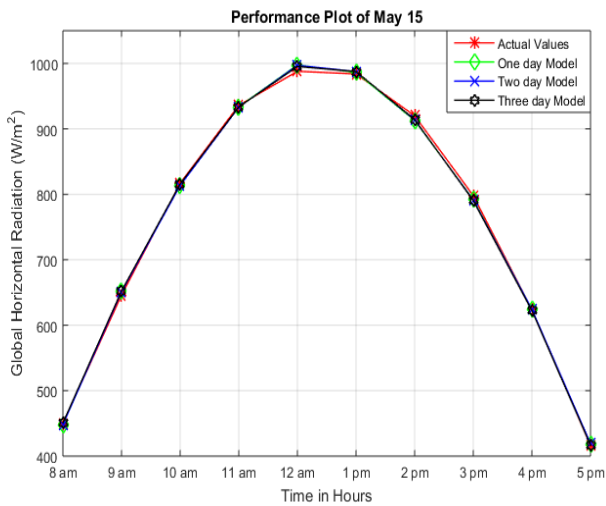


Fig. 4. Performance plot of May 15

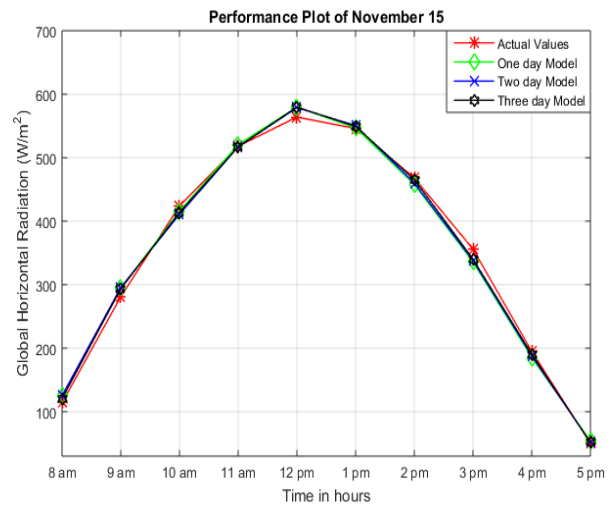


Fig. 7. Performance plot of November 15

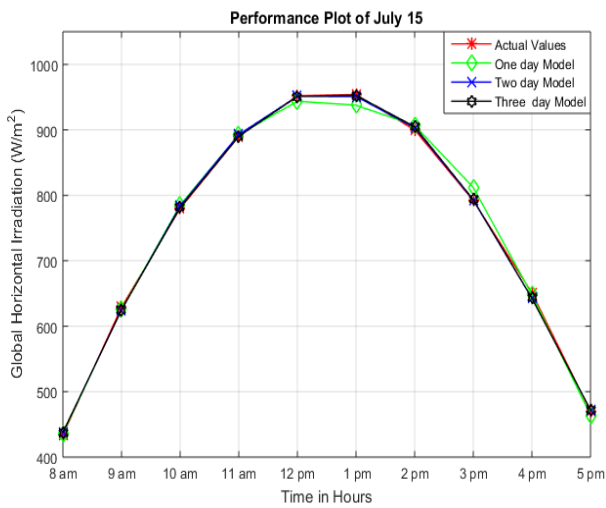


Fig. 5. Performance plot of July 15

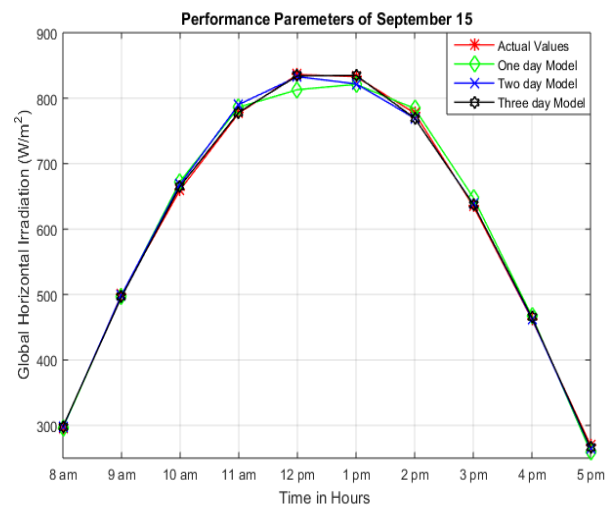


Fig. 6. Performance plot of September 15

There is 26.23 % reduction in MAPE from LTS one day to LTS two day and 71.57 % reduction in MAPE from LTS one day to LTS three day model. Solar radiation of months having lower temperature shows high irregularity in previous year similar days, this results high forecasting error in these months. The performance plots of January 15, March 15, May 15, July 15, September 15 and November 15 are shown in Fig 1, Fig 2, Fig 3, Fig 4, Fig 5 and Fig 6 respectively.

## V. CONCLUSION

Increased penetration of renewables in the grid made load generation balancing is more complex than the past years. In modern power system, both Load and generation are uncertain in nature due to the presence of intermittent sources like solar and wind. Real time load and generation forecasting is a vital for in modern grid operation planning. The increase in the number of utility level solar power plants again increases the demand of solar radiation forecasting. This paper introduces one univariate and two multivariate similar day based linear regression models for short term solar radiation forecasting. These models uses simple matrix based mathematical equations for prediction and produces accurate forecasts with negligible time. Results are comparable with conventional complex artificial and machine learning based models and have strong potential in real time application due to the negligible processing time. The forecast error for months with low average temperature is quite high as compared to months with high average monthly temperature, because these months showing irregular radiation pattern in previous year similar days. The accuracy of these models can be increased by considering the factors affecting solar radiation and it can be do as future work.

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