Dynamic Modeling and Control of Shunt Active Power Filter

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Abstract—**Overall performance of the shunt active power filter depends on the method of reference current extraction. The conventional extraction methods for harmonic and reactive currents is having inaccuracy, in case of distorted and unbalanced supply; non-linear and unbalanced load. Based on adaptive interference cancelling theory, a method is presented to estimate the harmonic, fundamental frequency unbalanced and reactive current components. A simulation is performed in MATLAB/SIMULINK environment which to verify the feasibility of the proposed method. This method can be useful for the dynamic compensation equipment such as static Var compensator, unified power quality conditioner and so on.**

Keywords: ***Adaptive cancellation theory, Adaptive detection algorithm, Instantaneous power theory, Instantaneous symmetrical components, Shunt active power filter, Voltage source inverter.***

I. INTRODUCTION

Recently, there has been a rapid development of nonlinear loads due to the intensive use of power electronic control in industry as well as by the general consumers. This results many undesirable phenomena in the operation of power system. The most important among these are harmonic contamination, increased reactive power demand and power system voltage fluctuations. Harmonic current components increase power system losses, cause excessive heating and vibration in rotating machinery. Also, some precision instruments and communication equipment will be interfered by the EMI. Therefore, utility power quality has become an important issue in the recent past. Often, end user must maintain nearly sinusoidal line currents at a high power factor to comply with the specified in the IEEE-519 and limits proposed by the IEC-555 [1]. However, the most significant limit on the total harmonic distortion (THD) is specified by the IEEE-519 standard and is 5% for a load connected to a utility system.

Conventionally, a passive LC filter is used to compensate the harmonics; capacitors being used to compensate the lagging power factor. However, they have many disadvantages such as, large size, resonance and fixed compensation characteristics. These difficulties bring the alternative solution as shunt active power filter (SAPF) which use voltage source inverter to nullify the harmonic. In addition, shunt active power filter can be optimized for power factor correction, power flow control, load balancing, voltage regulation.

In this paper, a simplified dynamic model of the shunt active power filter is proposed with an PI controller for dc-link voltage regulation. Using the derived dynamic model, analysis of DC-link voltage response and current tracking capability for the active power filter will be easier. Applying the proposed control strategy, the current harmonics of a nonlinear and unbalanced load can be compensated quickly. Also, fluctuations of DC-link voltage during transient and steady states are effectively suppressed. A detailed simulation program of the scheme is developed to predict its performance for different operating conditions.

The salient features of this paper are summarized as follows:

1) The current injected by the SAPF, passes through a filter inductor. The behavior of filter inductors is frequency dependent [2]. The influence of these parameter uncertainties on stability and performance of the current controller can be avoided by using the dynamic model of SAPF.
2) The delay times of both current response of SAPF and DC-link voltage feedback are considered. This results in decreasing the settling time of the DC-link voltage and reducing the high frequency current components of the power system.
3) The control scheme is suitable for both distorted and unbalanced supply; non-linear and unbalanced loads.

II. SAPF DYNAMIC MODELLING

A. System Configuration

The configuration model of shunt active power filter using a voltage source converter (VSC) is shown in Fig. 1 [4]. In this model, the resistance R_f in series with the voltage source inverter represents the sum of the coupling inductor resistance losses and the inverter conduction losses. The inductance L_f represents the leakage inductance of the coupling inductor. The sum of the switching losses of the inverter and the power losses in the capacitor is represented by R_{dc} which is in shunt with the DC-link capacitor C_{dc}. In Fig. 1, v_{fa} , v_{fb} , and v_{fc} are the three-phase SAPF output voltages; v_{la} , v_{lb} , and v_{lc} are the three phase bus voltages at load-side; i_{fa} , i_{fb} , and i_{fc} are the three-phase SAPF output currents.

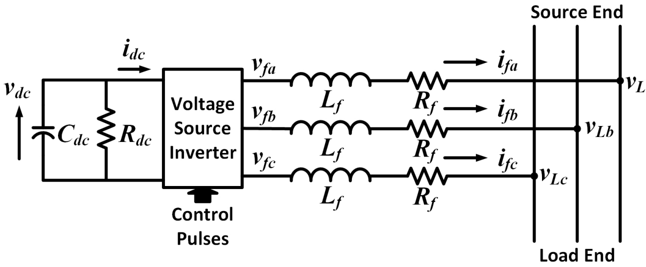

Fig. 1. Equivalent circuit of SAPF

B. Modeling

In order to analyze the balanced three-phase system more conveniently, the three-phase voltages and currents are converted to synchronous rotating frame by $abc/dq0$ transformation. The dq-frame rotate with an angle $\theta = \omega t$ from the reference axis of the abc-frame. By this transformation, the control problem is greatly simplified since the system variables become DC values under the balanced condition. The transformation from phase variables to d and q coordinates is given as follows:

$$\begin{bmatrix} v_d \\ v_q \\ v_0 \end{bmatrix} = T \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{1}$$

where

$$T = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin\theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

A linear mathematical model for each phase of the SAPF shown in Fig. 1 can be written as:

$$\begin{aligned} L_f \frac{di_{fa}}{dt} &= -R_f i_{fa} + v_{fa} - v_{La} \\ L_f \frac{di_{fb}}{dt} &= -R_f i_{fb} + v_{fb} - v_{Lb} \\ L_f \frac{di_{fc}}{dt} &= -R_f i_{fc} + v_{fc} - v_{Lc} \end{aligned} \tag{2}$$

Equations (2) can be written in following form:

$$L_f \frac{d}{dt} \begin{bmatrix} i_{fa} \\ i_{fb} \\ i_{fc} \end{bmatrix} = -R_f \begin{bmatrix} i_{fa} \\ i_{fb} \\ i_{fc} \end{bmatrix} + \begin{bmatrix} v_{fa} \\ v_{fb} \\ v_{fc} \end{bmatrix} - \begin{bmatrix} v_{la} \\ v_{lb} \\ v_{lc} \end{bmatrix} \tag{3}$$

With dq-transformation from equation (1),

$$L_f \frac{d}{dt} \left(T^{-1} \begin{bmatrix} i_{fd} \\ i_{fq} \\ i_{fo} \end{bmatrix} \right) = -R_f T^{-1} \begin{bmatrix} i_{fd} \\ i_{fq} \\ i_{fo} \end{bmatrix} + T^{-1} \begin{bmatrix} v_{fd} \\ v_{fq} \\ v_{fo} \end{bmatrix} - T^{-1} \begin{bmatrix} v_{ld} \\ v_{lq} \\ v_{lo} \end{bmatrix}$$

The above equation can be simplified as:

$$L_f T \left(\frac{d}{dt} \left(T^{-1} \right) . \begin{bmatrix} i_{fd} \\ i_{fq} \\ i_{fo} \end{bmatrix} + T^{-1} . \frac{d}{dt} \begin{bmatrix} i_{fd} \\ i_{fq} \\ i_{fo} \end{bmatrix} \right) = -R_f \begin{bmatrix} i_{fd} \\ i_{fq} \\ i_{fo} \end{bmatrix} + \begin{bmatrix} v_{fd} \\ v_{fq} \\ v_{fo} \end{bmatrix} - \begin{bmatrix} v_{ld} \\ v_{lq} \\ v_{lo} \end{bmatrix} \tag{4}$$

where

$$\frac{d}{dt} \left(T^{-1} \right) = \omega \sqrt{\frac{2}{3}} \begin{bmatrix} -\sin\theta & \cos\theta & 0 \\ -\sin(\theta - \frac{2\pi}{3}) & \cos(\theta - \frac{2\pi}{3}) & 0 \\ -\sin(\theta + \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) & 0 \end{bmatrix}$$

$$T . \frac{d}{dt} \left(T^{-1} \right) = \omega \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{d\theta}{dt} = \omega$$

Applying all the above relations in equation (4)

$$\frac{d}{dt} \begin{bmatrix} i_{fd} \\ i_{fq} \\ i_{fo} \end{bmatrix} = \begin{bmatrix} \frac{-R_f}{L_f} & \omega & 0 \\ -\omega & \frac{-R_f}{L_f} & 0 \\ 0 & 0 & \frac{-R_f}{L_f} \end{bmatrix} \begin{bmatrix} i_{fd} \\ i_{fq} \\ i_{fo} \end{bmatrix} + \frac{1}{L_f} \begin{bmatrix} v_{fd} \\ v_{fq} \\ v_{fo} \end{bmatrix} - \frac{1}{L_f} \begin{bmatrix} v_{ld} \\ v_{lq} \\ v_{lo} \end{bmatrix} \tag{5}$$

Suppose, the output voltage of the SAPF can be expressed as:

$$v_{fd} = K v_{dc} \cos\alpha \tag{6}$$

$$v_{fq} = K v_{dc} \sin\alpha \tag{7}$$

where K is a factor that relates the DC voltage to the peak phase-to-neutral voltage on the AC side; v_{dc} is the DC-link voltage; α is the phase angle which the SAPF output voltage leads the bus voltage.

Using the relation (6) and (7), the equation (5) can be modified as

$$\frac{d}{dt} \begin{bmatrix} i_{fd} \\ i_{fq} \end{bmatrix} = \begin{bmatrix} \frac{-R_f}{L_f} & \omega & \frac{K\cos\alpha}{L_f} \\ -\omega & \frac{-R_f}{L_f} & \frac{K\sin\alpha}{L_f} \end{bmatrix} \begin{bmatrix} i_{fd} \\ i_{fq} \\ v_{dc} \end{bmatrix} - \frac{1}{L_f} \begin{bmatrix} v_{ld} \\ v_{lq} \\ v_{lo} \end{bmatrix} \tag{8}$$

From the SAPF input-output power balance equation, it can be written as:

$$\begin{aligned} & p_{dc} = p_f \\ \text{or,} \quad & v_{dc} i_C + v_{dc} i_R = v_{fa} i_{fa} + v_{fb} i_{fb} + v_{fc} i_{fc} \\ \text{or,} \quad & v_{dc} C_{dc} \frac{dv_{dc}}{dt} + \frac{v_{dc}^2}{R_{dc}} = v_{fd} i_{fd} + v_{fq} i_{fq} \\ \text{or,} \quad & \frac{dv_{dc}}{dt} = \frac{K \cos\alpha}{C_{dc}} i_{fd} + \frac{K \sin\alpha}{C_{dc}} i_{fq} - \frac{v_{dc}}{R_{dc} C_{dc}} \end{aligned} \tag{9}$$

From the equations (8) and (9), the relation for the dynamic model of the SAPF can be derived and is given below:

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} i_{fd} \\ i_{fq} \\ v_{dc} \end{bmatrix} = & \begin{bmatrix} \frac{-R_f}{L_f} & \omega & \frac{K}{L_f} \cos\alpha \\ -\omega & \frac{-R_f}{L_f} & \frac{K}{L_f} \sin\alpha \\ \frac{K}{C_{dc}} \cos\alpha & \frac{K}{C_{dc}} \sin\alpha & \frac{1}{R_{dc} C_{dc}} \end{bmatrix} \begin{bmatrix} i_{fd} \\ i_{fq} \\ v_{dc} \end{bmatrix} \\ & - \frac{1}{L_f} \begin{bmatrix} v_{ld} \\ v_{lq} \\ v_{lo} \end{bmatrix} \end{aligned} \tag{10}$$

III. PROPOSED REFERENCE CURRENT EXTRACTOR

The source could be balanced or unbalanced, spectrally pure or distorted. At the same instant, the load could be balanced or unbalanced, linear or nonlinear. Irrespective of the nature of the load, the proposed algorithm is reliable to estimate the reference current which is shown in Fig. 2.

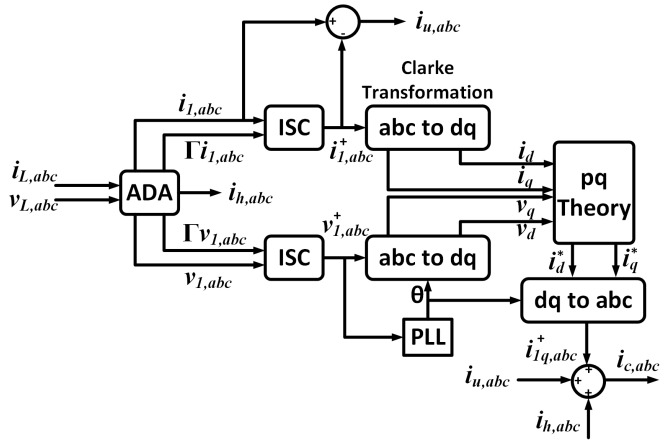

Fig. 2. Proposed reference current extractor

The current drawn by the load is measured through the current sensor at the point of common coupling. The measured current is then decomposed into following components:

- Harmonic component: $i_{h,abc}$
- Unbalanced component of fundamental frequency: $i_{1u,abc}$
- Active component of the fundamental frequency: $i^+_{1p,abc}$
- Reactive component of the fundamental frequency: $i^+_{1q,abc}$

The role of SAPF is to supply the desirable current components such that the utility source supply only the active current component required by the load. The total compensation current for the SAPF is given by

$$i_{c,abc} = i_{h,abc} + i_{1u,abc} + i^+_{1q,abc} \tag{11}$$

The voltage across DC-link capacitor decays due to switching losses in the voltage source inverter. In order to maintain this voltage constant, SAPF will draw the necessary amount of current from the utility source. So, the reference current should be the combination of the compensation current $i_{c,abc}$ and the switching loss component $i_{loss,abc}$ and is given by

$$i^*_{f,abc} = i_{c,abc} + i_{loss,abc} \tag{12}$$

A. Deriving $i_{h,abc}$

Based on the principle of adaptive noise cancelling theory [5], adaptive detecting algorithm (ADA) extracts harmonic current from the load current which is shown in Fig.3. The system is composed of an adaptive filter, a band pass filter (BPF) with 50Hz cut-off frequency and 90^0 phase-shifter. The primary input is the load current: $i_{L,abc} = i_{1,abc} + i_{h,abc}$, where $i_{1,abc}$ is the fundamental load current, $i_{h,abc}$ is the sum of all harmonic components.In the Fig. 3 $v_{L,abc}$ and $v_{1,abc}$ are the load voltage and its fundamental component, respectively.

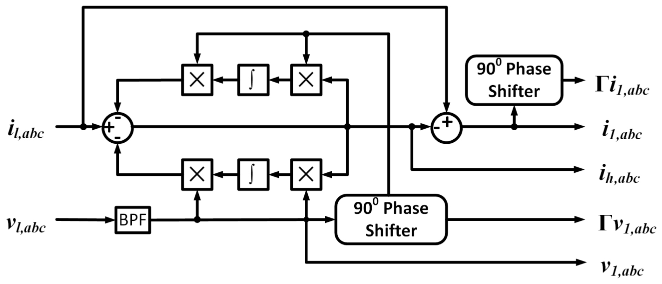

Fig. 3. Adaptive detection algorithm

B. Deriving $i_{1u,abc}$

Instantaneous symmetrical component (ISC) method is a time-domain approach which is mainly deployed in power system protection applications [3]. ISC can only be applicable to the system with single frequency component. With ADA, the signals (load current and voltage) and its 90^0-shifted forms are readily available. The load current at fundamental frequency is given by

$$i_{1,abc} = i_{L,abc} - i_{h,abc} \tag{13}$$

By using this method, load current i_{La}, i_{Lb}, i_{Lc} could be mapped onto positive sequence components as,

$$\begin{aligned} i^+_{1a}(t) &= \frac{1}{3}[i_{La}(t) - \frac{1}{2}(i_{Lb}(t) + i_{Lc}(t)) - \frac{\sqrt{3}}{2}(\Gamma i_{Lb}(t) - \Gamma i_{Lc}(t))] \\ i^+_{1b}(t) &= \frac{1}{3}[i_{Lb}(t) - \frac{1}{2}(i_{Lc}(t) + i_{La}(t)) - \frac{\sqrt{3}}{2}(\Gamma i_{Lc}(t) - \Gamma i_{La}(t))] \\ i^+_{1c}(t) &= \frac{1}{3}[i_{Lc}(t) - \frac{1}{2}(i_{La}(t) + i_{Lb}(t)) - \frac{\sqrt{3}}{2}(\Gamma i_{La}(t) - \Gamma i_{Lb}(t))] \end{aligned}$$

where $\Gamma i_{La}(t),\ \Gamma i_{Lb}(t),\ \Gamma i_{Lc}(t)$ are 90^0 shifted form of i_{La}, i_{Lb}, i_{Lc} respectively. Similarly, load voltage v_{La}, v_{Lb}, v_{Lc} could be mapped onto positive sequence components (v_{1a}^{+}, v_{1b}^{+}, v_{1c}^{+}) using ISC method.

The unbalance component corresponding to the fundamental frequency can be derived by subtracting the positive sequence current from the load current due to the fundamental frequency.

C. Deriving $i_{1q,abc}^{+}$

The instantaneous power (pq) theory can be defined for three phase systems, with or without neutral conductor. Zero-sequence components are absent in three phase three-wire systems, i.e. $i_0 = 0$. The instantaneous power only exists on the dq-axes , as the product $v_0 i_0$ is zero. Hence,the instantaneous power p represents total energy flow per unit time, in terms of dq-components. The instantaneous imaginary power q represents the quantity of the of energy that is being exchanged between the phases of the system [6].

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} v_d & v_q \\ v_q & -v_d \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} \tag{14}$$

The above powers can also be expressed in terms of the real and reactive powers corresponding to fundamental ($\bar{p}$ and $\bar{q}$) and harmonic frequencies ($\tilde{p}$ and $\tilde{q}$). That is,

$$p = \bar{p} + \tilde{p} \tag{15}$$

$$q = \bar{q} + \tilde{q} \tag{16}$$

As the active and reactive current component due to harmonics is eliminated, they are excluded from the calculation of the reference dq-frame current. Hence, the pq-theory is supposed to supply the reactive power component corresponding to fundamental frequency ($\bar{q}$). So, the reference current in dq-frame is given by:

$$\begin{aligned} \begin{bmatrix} i_d^* \\ i_q^* \end{bmatrix} &= \begin{bmatrix} v_d & v_q \\ v_q & -v_d \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \bar{q} \end{bmatrix} \\ &= \frac{1}{v_d^2 + v_q^2} \begin{bmatrix} v_d & v_q \\ v_q & -v_d \end{bmatrix} \begin{bmatrix} 0 \\ \bar{q} \end{bmatrix} \end{aligned} \tag{17}$$

The pq-theory followed by inverse Clarke transformation (dq to abc) gives the reactive current component ($i_{1q,abc}^{+}$) corresponding to fundamental frequency.

D. Deriving $i_{loss,abc}$

After a few moment of SAPF operation, DC-link voltage will decay and become zero because of switching losses in VSI. In order to maintain DC-link voltage constant, the compensating current should flow along with the loss current drawn from the utility source. Initially, the DC-link voltage is sensed through a voltage sensor and then is compared with constant reference voltage. To nullify the comparator error, a PI regulator is used to get the corresponding loss current. Further, this current multiplied to three phase unit vector gives three-phase loss current.

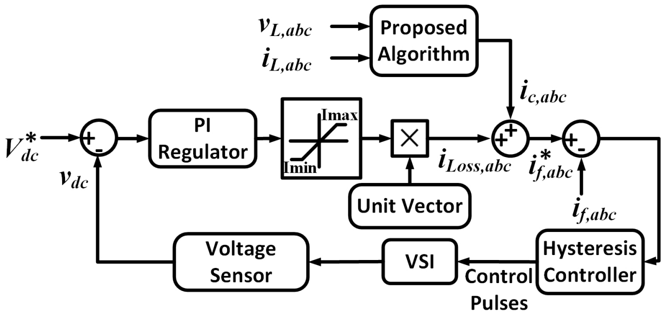

Fig. 4. Voltage controller using PI regulator

IV. RESULTS AND DISCUSSION

Distortion and unbalanceness of both load and source is considered for simulation study. Initially, a balanced load of $R = 100\,\Omega$ and $L = 75\,mH$ and balanced supply with phase-a voltage, $V_{sa} = 312\,sin\,(100\pi t)$ is taken for all the cases. The reference current would be equal to the fundamental reactive component. The waveforms corresponding to all the cases are shown in Fig. 5 between 0 and 0.4 s. The effectiveness of the proposed algorithm can be explicitly studied by considering the following cases.

A. Non-linear Load

The nonlinear load is formed by using three-phase uncontrolled rectifier module with load of $R = 100\,\Omega$ and $L = 250\,mH$. The output current of this load has 33% of harmonic content. The waveforms corresponding to above case are shown in Fig. 5 between 0 and 0.1 s. This harmonic current is generated by the SAPF and is injected in anti-phase with the load current at the point of common coupling. After compensating the current harmonic, it is observed that the total harmonic distortion is 1.96% within IEEE-519 limits.

B. Non-linear and Unbalanced Load

Functioning of the SAPF during a non-linear and unbalanced load condition is shown in Figure 5. A three-phase unbalance load ($R_a = 100\,\Omega$, $L_a = 75\,mH$, $R_b = 50\,\Omega$, $L_b = 90\,mH$, $R_c = 150\,\Omega$, $L_c = 65\,mH$) and a non-linear load of previous case is taken into consideration. As load current contains an unbalance current along with the harmonic one, the compensating current must contain these components. The waveforms corresponding to above case are shown in Fig. 5 between 0.1 and 0.2 s. After supplying the compensating filter current, the source current becomes balanced, sinusoidal and in phase with the source voltage.

C. Linear Load and Distorted Supply

A sample case of distorted supply with 20% fifth and 13% seventh harmonic is considered for the simulation study. The waveforms corresponding to above case are shown in Fig. 5 between 0.2 and 0.3 s. These harmonic content ($i_{h,abc}$) would obviously be reflected on the load current which is being extracted by ADA. The fundamental component can be derived by subtracting the harmonic one from the original load current (Fig. 5(e)). If the load is having some lagging power factor, it will definitely require reactive power. Due to balanced load and supply, the harmonic current component will be zero. As a result, The compensating current is the algebraic sum of harmonic and fundamental reactive current components (Fig. 5(d)).

D. Linear Load, Distorted and Unbalanced Supply

The performance of the proposed technique can be evaluated precisely by considering the case of both distorted and unbalanced supply (20% of fifth and 13% harmonics, 20% of fundamental negative sequence voltage). The waveforms corresponding to above case are shown in Fig. 5 between 0.3 and 0.4 s. The results represent the robustness of the proposed algorithm for estimating compensating current even when the supply is distorted and unbalanced. It is observed from [8] that three cycles is the minimum time required to estimate the reference current. But the proposed algorithm is fast enough to give the response in less than one cycle which is shown in fig. 5.

V. CONCLUSION

In this paper, a novel reference current extraction method using adaptive detection algorithm is proposed which is able to extract the harmonic content from the load current. Along with instantaneous symmetrical component method and pq theory, this algorithm is able to estimate the reference current even with distorted and unbalanced supply, as well as the non-linear and unbalanced load. This closed loop system is independent of parameter variation and behave as a notch filter. The proposed method is studied analytically and verified by MATLAB/Simulink simulation environment. Finally, simulation result is given to conform the feasibility of the hardware realization.

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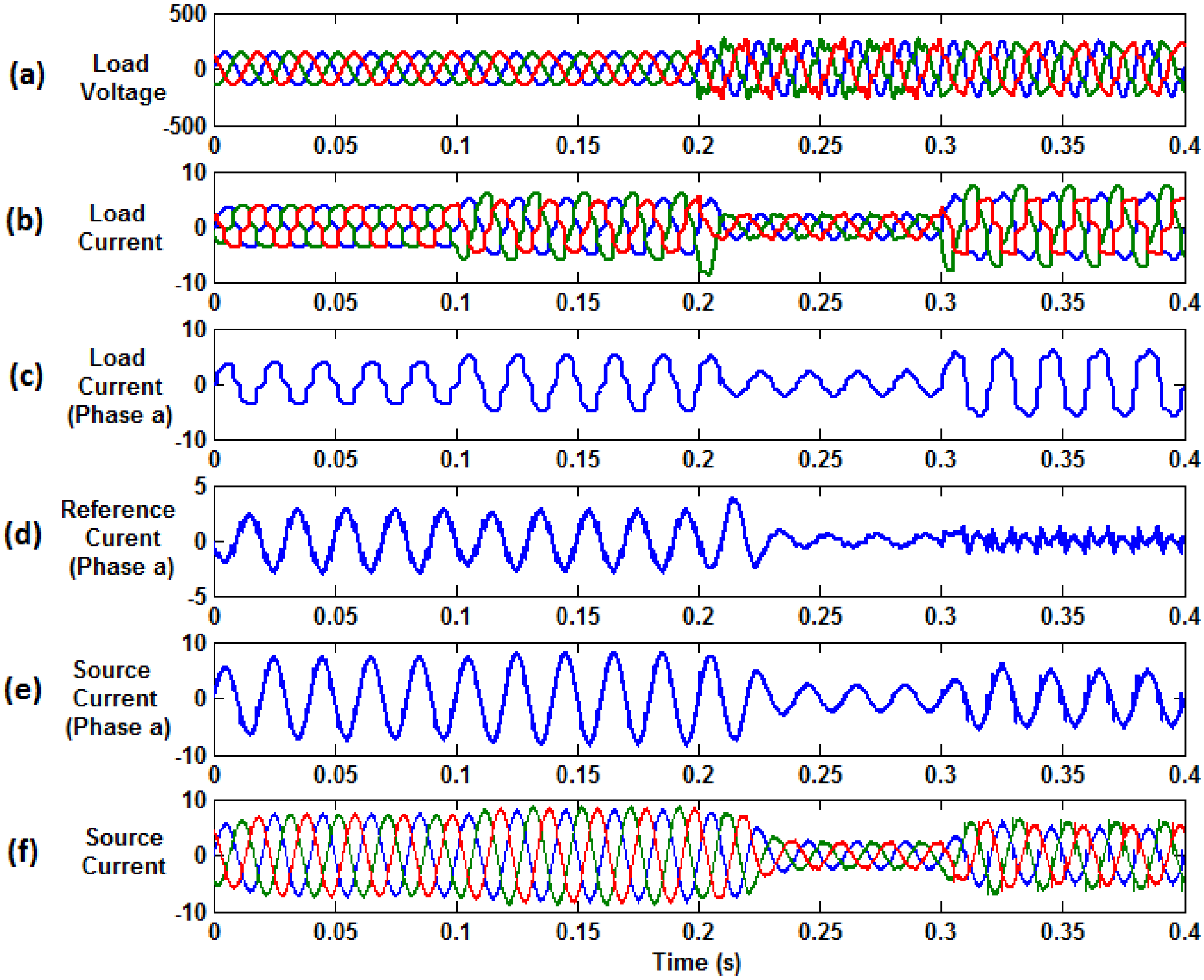

Fig. 5. Simulation results of SAPF by proposed method