Advanced Windowed Interpolated FFT Algorithms for Harmonic Analysis of Electrical Power System

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***Abstract* - Fast Fourier Transform (FFT) is used to obtain the Fourier spectrum of a periodic signal, but it is well known that special care must be taken to avoid severe distortions introduced by the sampling process. The major errors associated with this algorithm are spectral leakage and picket fence effect. This paper discusses the advancement in FFT algorithms for harmonic analysis in power system by applying windowed interpolation techniques. It is observed that the application of different WIFFT algorithms has different effects on the measuring accuracy. The advanced minimized sidelobe window (MSW) algorithm has better performance characteristics when compared to the conventional hanning and hamming windows adopted for harmonic analysis. By considering the practical harmonic data the hanning, hamming and MSW algorithms are validated using matlab programming. The simulation results demonstrate the effectiveness of hanning, hamming and MSW by considering the magnitude and phase angle error variations with respect to the harmonic order.**

Keywords—Harmonic Analysis; Hamming; Hanning; Minimized sidelobe window (MSW); Picket Fence effect; Spectral Leakage; Windowed Interpolated Fast Fourier Transform (WIFFT)

I. INTRODUCTION

Electrical power systems are scattering to a huge area and ample array of loads are connected to power system. During the planning and management of power systems, precise analysis of load characteristics that are connected to the power system is very important. Thus power system problems that may occur can be pre-determined and precautions must be taken against them. Especially with the emergent semi-conductor technology, harmonics have become one of the most popular power quality issues in power system. Therefore, for avoiding the harmful effects of harmonics and maintaining the secure operation of power system, it is necessary to measure the real-time content of harmonics that are present in the power system network [1].

The harmonic analysis of power system is achieved by the Fast Fourier Transform (FFT). The FFT-based harmonic analysis can be accurate under synchronous sampling. However, it is complicated to execute exact synchronous sampling, even when using the discrete phase-locked loop technique. This is due to the fundamental frequency offset and harmonic interferences that are inevitable in practical application. Without proper synchronization, the direct use of FFT suffers major drawbacks, i.e., the spectral leakage and picket fence effect. Hence to eliminate the spectral leakage and picket fence effect, windowed Interpolated FFT (WIFFT) algorithms have been introduced [2]-[4].

Several papers in the literature presents different windowed interpolation techniques used for reduction of spectral leakage and picket fence effect [6]-[13]. It is also reported that the WIFFT algorithms have the advantages of robustness toward signal model inaccuracies and low computational effort [5], [14], [15]. It has been observed that the performances of the WIFFT algorithms are mainly effected by the data windows. WIFFT can be implemented with an easy calculation formula when the classical windows are used. However, when the window function is complex, the WIFFT algorithms involve solving high-order equations on the harmonic parameter estimation. Different solving methods, presented in the literature suffer from complex computation [5], [13].

The selection of a window with good sidelobe behaviour is of immense significance for the precise harmonic analysis because the width of the major lobe in a window's spectrum relates to the frequency resolution, while the amplitude and decay of the sidelobe determine leakage effect. The minimized sidelobe windows (MSWs), proposed by Wen et.al. have low-peak sidelobe levels along with high sidelobe-decaying rates, to attain desirable performance for the suppression of spectral leakage [5]. Hence, this paper analyzed the characteristic of MSW interpolated FFT algorithm, which can greatly increase the accuracy of FFT to meet the precision and its effectiveness, is verified by comparing with classical windows such as hanning and hamming using matlab simulation.

II. HARMONIC ANALYSIS USING FFT

A. Synchronous sampling

If the signal has to be analysed is an analog signal, it should processed through an anti aliasing filter and then sample it at a rate of $f_s \geq 2B$, where B is bandwidth of the filtered signal. Thus the highest frequency that is contained in the sample signal is $f_s/2$. By assuming that the Nyquist frequency $f_{max} = 1/(2T_s)$ is larger than all of the f_h , the aliasing effects that arise from finite sampling rate can be neglected. Here, we considered the sampled multi frequency signal of length N ($n = 0, 1,..., (N-1)$):

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$$x(nT_s) = x(t)|_{t=nT_s} = \sum_{h=1}^{H} A_h \sin(2\pi f_h nT_s + \varphi_h) \quad (1)$$

Where H is the harmonic order and, f_h is harmonic frequency, A_h is amplitude, and φ_h is phase respectively, the frequency, amplitude and phase of the fundamental component are f_1, A_1 and φ_h, $T_s=1/f_s$ is the sampling interval, and f_s is the sampling frequency.

If the synchronous sampling condition is satisfied, i.e., $Nf1/fs$ is an integer, the complex coefficients of the FFT relevant to the sampled signal (1) on N is

$$X(\lambda\Delta f) = \sum_{k=1}^{N-1} x(nT_s) e^{-j2\pi k\lambda/N} \quad (2)$$

Where $\Delta f = 1/(NT_s)$ is the frequency resolution and $\lambda = 0, 1 \ldots N-1$.

From (2), the harmonic components of the signal $x(nT_s)$ fall at frequencies, which are integer multiples of Δf in the case of a synchronous sampling. It is thus possible to determine amplitude and phase of the signal [5].

B. Non synchronous sampling

It is evident that a synchronized sampling is purely theoretical because of either the practical requirement of processing or unfamiliar fluctuations of the input signal frequency. As an end result, the spectral leakage is inevitable, and the samplings are weighted by the time window to suppress the leakage. The FFT of the windowed samples $x(n)$ and $w(n)$ on N sampled points can be written as follows:

$$X(k) = \sum_{h=1}^{H} \left[e^{j\varphi_h} W_f(k-k_h) - e^{-j\varphi_h} W_f(k+k_h) \right] \quad (3)$$

Where $k=0,1,\ldots,N-1$,

W_f is the FFT of the adopted window,

k_h is the division coefficient of signal frequency and the frequency resolution Δf of the window $w(n)$ [5].

The FFT exhibits correct results only when the input data sequence contain energy precisely at the analysis frequency that is an integral multiple of fundamental frequency. The k^{th} array element in the frequency domain is referred to as a frequency bucket. Thus we can relate the frequency of bucket k to the frequency of the input signal using following equation at a sampling interval of Δt :

$$f_k = \frac{k}{N\Delta t} \quad (4)$$

Where N=Number of samples

$$k = \frac{Nf_k}{f_s} \quad (5)$$

From (5) there are two basic observations about trade-offs in the FFT:

1. The frequency response is inversely proportional to sample rate, for a given number of samples.
2. For a given sample rate, frequency response is proportional to the number of samples.

According to the Heisenberg uncertainty principle certain pairs of physical properties cannot be known to random precision. In concern with quantum physics, the properties are position and momentum. In concern with FFT, the properties are frequency and time. Thus, it requires more samples to get precise frequency response. Nevertheless, this causes our capability to focus the event precision in time to suffer. The contrary is also true. If we isolated the event time, the number of samples has declined and so has the accuracy in frequency. Actually, from (4) we can observe that the frequency precision is proportional to the number of samples. Hence, for any bucket, we can compute the frequency resolution [16].

III. CHARACTERISTICS OF MSW

The time-domain discrete MSW of length N is defined as [5]

$$w_{MS}(n) = \sum_{m=0}^{M-1} (-1)^m a_m \cos\left(\frac{2\pi}{N} mn\right) \quad (6)$$

Where M is the item number, $n=1,2........N-1$ is the sequence number, a_m is the window coefficient, which follow the restrict condition as

$$\sum_{m=0}^{M-1} (-1)^m a_m = 0. \quad (7)$$

The spectral window corresponding to the MSW is

$$w_{MS}(k) = \sum_{m=0}^{M-1} \frac{a_m}{2} \begin{bmatrix} e^{-\frac{j\pi(k-m)(N-1)}{N}} \dfrac{\sin((k-m)\pi}{\sin((k-m)\pi/N} \\ +e^{-\frac{j\pi(k+m)(N-1)}{N}} \dfrac{\sin((k+m)\pi}{\sin((k+m)\pi/N} \end{bmatrix} \quad (8)$$

$$for\ k = 0,1,\ldots\ldots\ldots,N-1.$$

It is clear that the major lobe width of a window is defined as the distance between zero-crossings in the window spectrum (in angular frequency) [6]. Hence, from (8), the following conditions must be satisfied to make $|W_{MS}(k)|$ zeros:

$$\begin{cases} k \pm m = d, \\ k \pm m \neq dN, \end{cases} \quad (9)$$

$$for\ d = 0, \pm 1, \pm 2, \ldots\ldots, +\infty$$

In practical harmonic analysis, the k_h th spectral line, which represents the h^{th} harmonic component, should be frequently well separated. Therefore, the minimum allowed difference between two adjacent frequency components $|fh-fh+1|$ should satisfy the following inequality:

$$\psi_{MLW} < \frac{4\pi |f_h - f_{h+1}|}{f_s} \qquad (10)$$

Where ψ_{MLW} is the major lobe width of the adopted window. According to the definition of the major lobe width of a window, d= ±1, the spectrum of the MSW attain the nearest zeros point to the origin at the first time. The major lobe width of the MSW is thus given by

$$\psi_{MLW} = 4M\pi / N. \qquad (11)$$

The MSW with item number of 4(I) & 4(II) have the fastest sidelobe roll off rate as well as a narrow major lobe width.

IV. INTERPOLATED FFT ALGORITHM

A. Formulas of the Interpolated FFT Algorithm [5]

The complex spectrum of the l_h^{th} spectral line representing the *h*th harmonic can be written as [5]

$$X(\xi_h) = \frac{A_h}{2j} e^{j\varphi_h} W_{MS}(\xi_h - k_h) \qquad (12)$$

$$for\ \xi_h = 0, 1, \ldots\ldots, N-1.$$

As mentioned above, suppose the two maximum amplitude spectral lines corresponding to the h^{th} harmonic are obtained at l_{h1} and l_{h2} ($l_{h2} = l_{h1} + 1$, $l_{h1} < k_h < l_{h2}$).

Hence $y_1 = |X(l_{h1})|$ and $y_2 = |X(l_{h2})|$ can be written as

$$y_1 = |X(l_{h1})| = |A_h| \cdot |W_{MS}(2\pi(l_{h1} - k_h)/N)| \quad (13)$$

$$y_2 = |X(l_{h2})| = |A_h| \cdot |W_{MS}(2\pi(l_{h2} - k_h)/N)| \quad (14)$$

The ratio of the two magnitudes y_1 and y_2 can be used for interpolated algorithm and the fractional part ζ_h caused by the non synchronous sampling can thus be solved. However, the solution of the fractional part ζ_h may include division of polynomials. Here a polynomial-fitting procedure based on the least-square method is used for easy calculation.
Since $0 \le k_h - l_{h1} \le 1$, here a symmetrical coefficient α defined as

$$\alpha = k_h - l_{h1} - 0.5, for\ -0.5 \le \alpha \le 0.5 \qquad (15)$$

Then y_1 and y_2 are both an even and symmetrical on α, which can be expressed as

$$y_1 = |X(l_{h1})| = |A_h| \cdot |W_{MS}(2\pi(-\alpha + 0.5)/N)| \quad (16)$$

$$y_2 = |X(l_{h2})| = |A_h| \cdot |W_{MS}(2\pi(-\alpha - 0.5)/N)| \quad (17)$$

An odd and symmetrical coefficient β as a function of α can then the defined as

$$\beta = (y_2 - y_1)/(y_2 + y_1). \qquad (18)$$

Substituting y_1 and y_2 into (18), β can be rewritten as

$$\beta = g(\alpha) = \frac{|W_{MS}(2\pi(-\alpha+0.5)/N)| - |W_{MS}(2\pi(-\alpha-0.5)/N)|}{|W_{MS}(2\pi(-\alpha-0.5)/N)| + |W_{MS}(2\pi(-\alpha-0.5)/N)|} (19)$$

As a result, the coefficient α can be obtained by the least square method of the inverse of (19). According to the method of least squares, the best-fitting polynomial has the property that

$$min\|\gamma\|_2^2 = min\sum_{i=0}^{L-1}[S(\beta_i) - \alpha_i]^2 \ (20)$$

Then, the polynomial $s_J(\beta) = \sum_{j=0}^{J-1} v_j \beta^j$ with the order *J* should satisfy

$$I_p = min\sum_{i=0}^{L-1}\left(\sum_{J=0}^{J-1} v_j \beta_i^j - \alpha_i\right)^2 \quad (21)$$

Taking the derivative of I_p with respect to α and β setting them to zero gives the following equation:

$$\frac{\partial I_p}{\partial v_j} = 2\sum_{i=0}^{L-1}\left(\sum_{j=0}^{J-1} v_j \beta_i^j - \alpha_i\right)\beta_i^j \qquad (22)$$

and then the coefficients $v_0, v_1, \ldots, v_{J-1}$ can be obtained by solving (22). The order *J* of the polynomial $\alpha = S(\beta)$ should be determined according to the accuracy requirement. Once α is calculated, the parameters f_h , A_h , and □$_h$ of the signal can be obtained by using the following formulas

$$k_h = \alpha + l_{h1} + 0.5 \qquad (23)$$

$$f_h = k_h f_s / N \qquad (24)$$

$$A_h = \frac{2y_1}{|w_{MS}(2\Pi(l_{h1} - k_h)/N|} \qquad (25)$$

$$\varphi_h = arg[X(l_{h1})] + \frac{\pi}{2} - arg[W_{MS}(l_{h1} - k_h)] \qquad (26)$$

V. VALIDATION OF WIFFT ALGORITHMS USING MATLAB SIMULTION

To authenticate the effectiveness of the Advanced MSW Interpolated FFT algorithm, simulations are done in MATLAB programming. The mathematical representation of the simulated signal is as follows:

$$x(n) = \sum_{h=1}^{9} A_h \sin\left(2\pi h \frac{f_0}{f_s} n + \varphi_h\right) \qquad (27)$$

Where the fundamental frequency f_1 is set as 50.5Hz, the sampling frequency f_s is 1500 Hz, window length N is 512, and the value of the amplitude and phase of the fundamental and harmonic component are given in Table I. The fundamental frequency can also set to 49.5Hz as there is +0.5 or -0.5 variations in 50Hz practically. Simulations are done for different frequencies to observe the magnitude and phase angle errors variation with respect to the harmonic order.

TABLE I

PRACTICAL SAMPLED SIGNAL VOLTAGE MAGNITUDE AND PHASE ANGLE

Parameters	**Harmonic orders**				
	1	**2**	**3**	**4**	**5**
Amplitude/V	1	0.02	0.1	0.05	0.05
Phase angle/°	-23	115.6	59.3	52.4	123.8
Parameters	**Harmonic orders**				
	6	**7**	**8**	**9**	
Amplitude/V	0	0.02	0	0.01	
Phase angle/°	-	-31.8	-	-63.7	

The interpolated algorithm with MSW of item 4(I) & 4(II) are adopted for the practical signal. Here 4(I) & 4(II) represent the modified forms of MSWs for different window coefficients. As a result, the MSW with different coefficients exhibit different performance in frequency domain, the percentage relative error of the fundamental frequency is $2.385\mathrm{x}10^{-8}$ % for MSW 4(II) window function. The percentage relative errors of the amplitude and frequency of the harmonics by using different windows and algorithms are listed in Table II.

TABLE II

HARMONIC ANALYSIS RESULTS BY USING DIFFERENT ALGORITHMS

Harmonic Order (h)	**Relative Errors of Amplitude at f_l=50.5**			
	Hanning WIFFT	**Hamming WIFFT**	**MSW 4(I)**	**MSW 4(II)**
1	-6.50E-6	-8.30E-2	3.18E-5	-2.38E-8
2	8.58E-5	-6.29E-3	-1.63E-3	-2.24E-5
3	-5.79E-6	-6.94E-3	1.38E-4	3.65E-8
4	1.78E-6	-3.24E-3	-1.65E-4	-2.26E-6
5	-9.39E-8	-4.53E-3	-1.65E-5	1.41E-7
7	-1.35E-7	-2.24E-3	1.2 E-6	-2.81E-9
9	-5.57E-8	-8.87E-4	-1.35E-5	-1.6E-8
Harmonic Order (h)	**Relative Errors of Phase at f_l=50.5**			
	Hanning WIFFT	**Hamming WIFFT**	**MSW 4(I)**	**MSW 4(II)**
1	-3.5E-001	2.8E+001	-2.3E-006	-2.1E-8
2	4.7E-001	-1.9E+001	-6.5E-004	-5.9E-6
3	-1.6E-001	1.1E+001	-1.7E+000	-1.7E00
4	-2.7E-002	3.1E+000	-3.4E-001	-3.4E-1
5	5.3E-002	-3.9E+000	-9.9E-007	1.17E-8
7	-1.1E+003	-1.1E+003	-7.4E+000	-7.48E0
9	-7.6E-002	9.4E+000	4.6E-006	1.89E-9

Fig.1. to Fig.12. demonstrate the pictorial representation of relative Magnitude and phase angle errors variation with respect to frequency variation at fixed sampling frequency f_s =1500 for hanning, hamming and MSW.

From Fig.1. to Fig.8. shows the response of magnitude and phase angle error variation for frequency 50.5Hz and 49.5Hz where Table II shows only for 50.5Hz. These results shows that based on the frequency variation there will be considerable amount of variation in the magnitude and phase angle errors.

In this simulation study hanning and hamming windows, which are common window techniques adopted for harmonic analysis in electrical power system are considered to compare the performance of MSW.

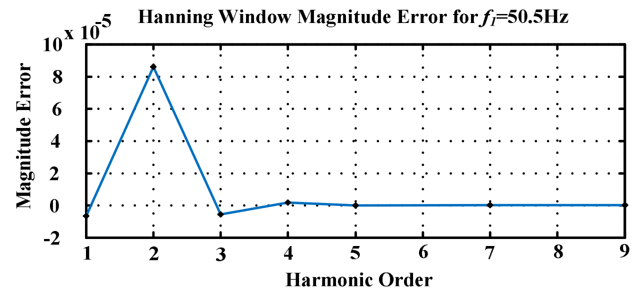

Fig.1. Magnitude Error plot for Hanning Window for f_l=50.5Hz

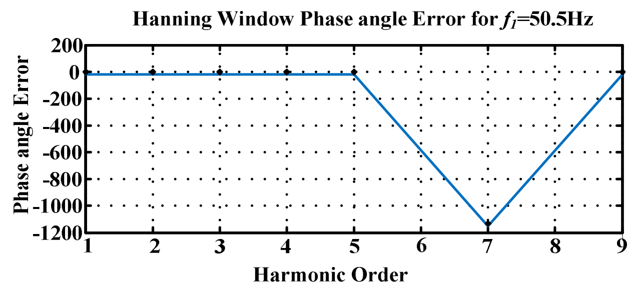

Fig.2. Phase angle Error plot for Hanning Window for f_l=50.5Hz

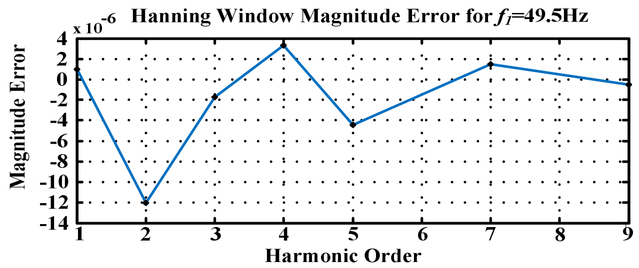

Fig.3. Magnitude Error plot for Hanning Window for f_l=49.5Hz

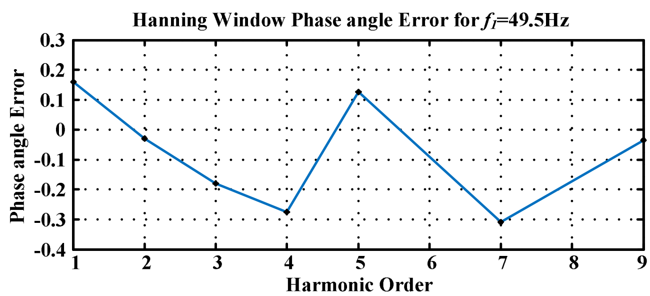

Fig.4. Phase angle Error plot for Hanning Window for f_l=49.5Hz

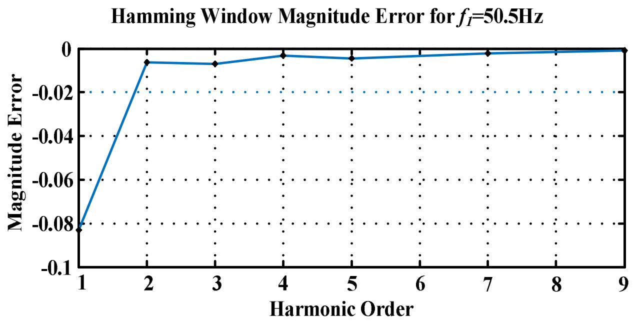

Fig.5. Magnitude Error plot for Hamming Window for f_l=50.5Hz

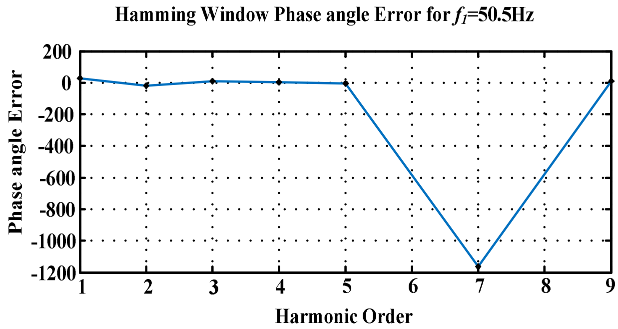

Fig.6. Phase angle Error plot for Hamming Window for f_l=50.5Hz

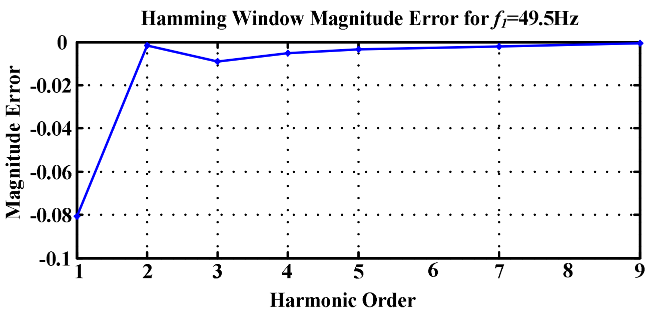

Fig.7. Magnitude Error plot for Hamming Window for f_l=49.5Hz

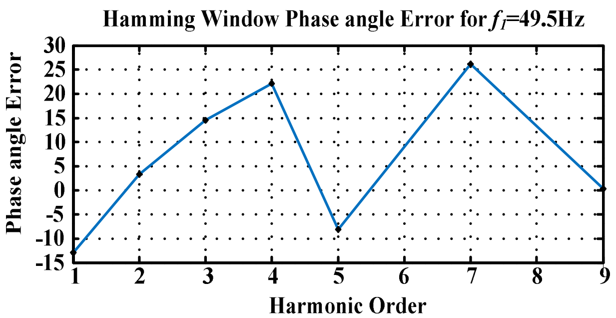

Fig.8. Phase angle Error plot for Hamming Window for f_l=49.5Hz

The conventional hanning and hamming windowed FFT algorithms are classical window techniques, but these are not advisable to adopt for harmonic analysis in modern power system networks. Hence the MSW is one of the options for accurate harmonic analysis.

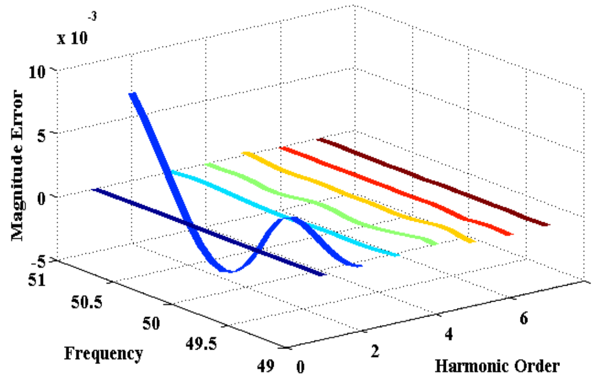

Fig.9. Magnitude Error plot for MSW 4(I)

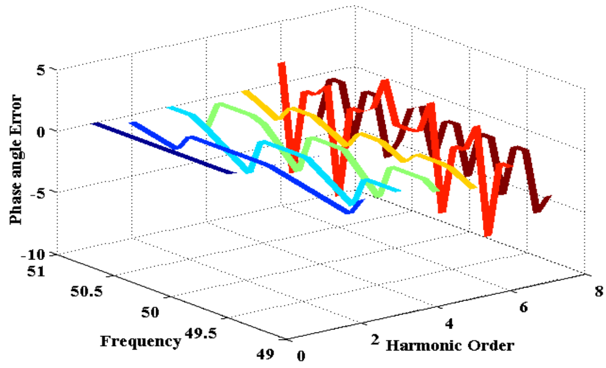

Fig.10. Phase angle error plot for MSW 4(I)

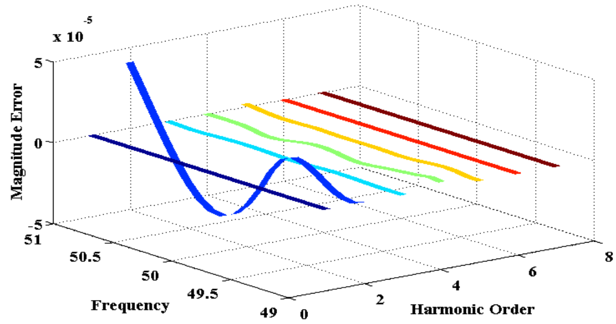

Fig.11. Magnitude Error plot for MSW 4(II)

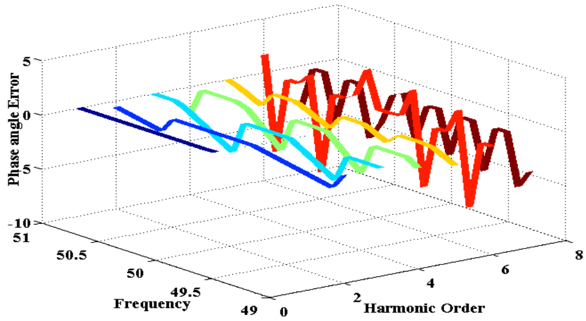

Fig.12. Phase angle Error plot for MSW 4(II)

From Fig.9. to Fig.12. shows the simulated results of MSW interpolated FFT algorithm. The simulation results presented in Tables II ensures the improved accuracy of MSW interpolated algorithm, as the relative error variation of fundamental is brought down to 4E-04 using MSW when compared to the classical windows. Table II also gives the detailed snapshot of MSW and other classical windows such as hanning and hamming window performance on measuring the ratio and phase angle error.

From the simulation results, the percentage relative error of the fundamental frequency is 2.385×10^{-8} % for improved MSW 4(II) window function presented in this paper, whereas the percentage relative error of the fundamental frequency of MSW proposed by Wen et.al. has 1.3×10^{-6}%. Hence the presented MSW algorithm is more accurate than the previously cited MSW interpolated algorithm.

VI. CONCLUSION

Harmonics analysis of electrical power system based on the FFT algorithm suffers from spectral leakage and picket fence effect. To overcome these effects this paper demonstrated the MSW interpolated FFT algorithm for accurate harmonic analysis in electrical power systems. Its effectiveness is verified by the existing classical hanning and hamming WIFFT algorithms. The MSW Interpolated FFT algorithm analyse harmonic parameters by the polynomial approximation theory without solving high-order equations. By observing the estimation results for the simulated signals, it is observed that the MSW based interpolated FFT algorithm is more accurate and needs less computation than the existing WIFFT algorithms.

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