Comparison and Allocation of Reactive Power for Practical Utility Systems

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Abstract— **Continuing trend towards deregulation and unbundling of transmission services has resulted in the need to access what the impact of a particular generator to load is on the power system. This paper presents the three different methods namely, Modified Ybus,Virtual flow approach and Power flow tracing to determine or measure the reactive power output from a particular generator to particular load. It also presents the electricity tracing method in detail with an algorithm of apportioning the transmission loss to individual loads or generator. This Method can be useful in providing additional insight into power system operation and can be used to modify existing tariffs of charging for transmission loss, reactive power and transmission services. Simulation and comparisons results are shown by taking Indian utility 62 bus system as Test system.**

Keywords*- Modified Ybus method, Virtual flow approach, Power flow Tracing Method*

I. INTRODUCTION

THE modern power industry is changing from one based on vertically integrated monopolies to a new form based on competition and privatization. This results in the unbundling of the vertically integrated functions of generation, transmission, distribution and retail sales. The transmission and distribution functions remains regulated whereas competition occurs in both the generation and retail electricity services market. Competition is a preferred practice in order to bring fairness and open access to the transmission network. Fairness can only be achieved by adopting a fair and transparent usage allocation methodology acceptable to all parties.

With deregulation of electricity sector each electric power service should be economically valued and the fair rules for evaluation and compensation should be established. Reactive power service is one of the key ancillary services and its trading is becoming a reality for deregulated electricity markets [1-7]. This has resulted in a need to quantify the value and to compensate the service of reactive power support. Some methods for evaluating the reactive power is given in [8-9].

In view of market operation it becomes more important to know the role of individual generators and loads to the networks and power transfer between individual generators to loads.

This is necessary for the competitive market to operate economically, efficiently and for the guarantee of open access to all system users. Several schemes have been developed to solve the allocation problem in the last few years. Methods based on the system Y-bus or Z-bus matrix methods can integrate the network characteristics and circuit theories [10]. Starting from the load flow solution, branch currents are determined as a function of generators' injected currents by using information from the bus impedance matrix. Similarly, contribution to bus voltages is computed as a function of each generator current injection by decomposing the network into different networks [11]. The concept of virtual flows using the principle of superposition is applied to obtain virtual contributions of individual sources to line flows and loads are given in[12].

Tracing of electricity gains importance as its solution could enhance the transparency in the operation of the transmission system. Recently a novel electricity tracing method has been proposed in [13- 14] which, assumes that nodal inflows are shared proportionally between the nodal outflows. This allows one to trace the flow of electricity in a meshed network. A similar approach has been proposed in [15] around same time. Bialek proposed upstream and downstream looking algorithms for tracing reactive power flow [16].In case of reactive power this approach introduces fictitious nodes on transmission line which act as of these fictitious reactive power source or sink [17].

A modified radial equivalent networks for reactive power tracing throughout the power grid is presented in paper [18].Due to the addition of this fictitious node the network size increases, thus requiring more computation memory. To overcome this problem a modify methodology for tracing reactive power is proposed in [19-20].

This paper shows the three different methods to solve the reactive power allocation problem. The effectiveness of Modified Ybus Matrix method is given in section II. In section III virtual flow approach is presented. In section IV, modified reactive power tracing methodology is described. Simulation Results and comparision of this method is shown in section V.Finally a discussion with concluding remarks is given in section VI,while section VII lists the relevant references.

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II. MODIFIED YBUS METHOD

In this method, a new modified nodal equation has been developed for identifying reactive power transfer between generators and load. The purpose is to represent each load current as a function of the generator's currents and load voltages. Starting from the concept of circuit theory, it uses the modified admittance matrix to decompose the load voltage dependent term into components of generator dependent terms. By using these two decompositions of current and voltage terms, the real and reactive power transfer between loads and generators are obtained [10].

The proposed methodology begins with the system node equation. For the convenience of explanation, it is assumed that the power system has a total number of *n* buses,*g* generators, and *l* loads, among which bus no.1~g are generation buses and bus no.g+1~n are load buses. Therefore, the Y bus of n*n dimension can be divided into four sub matrixes as shown in (1)

$$\left[\begin{array}{cc|cc} Y_{1,1} & Y_{1,g} & Y_{1,g+1} & Y_{1,n} \\ \ddots & & \ddots & \\ Y_{g,1} & Y_{g,g} & Y_{g,g+1} & Y_{g,n} \\ \hline Y_{g+1,1} & Y_{g+1,g} & Y_{g+1,g+1} & Y_{g+1,n} \\ \ddots & & \ddots & \\ Y_{n,1} & Y_{n,g} & Y_{n,g+1} & Y_{n,n} \end{array}\right] \times \left[\begin{array}{c} V_1 \\ \vdots \\ V_g \\ \hline V_{g+1} \\ \\ V_n \end{array}\right] = \left[\begin{array}{c} I_1 \\ \vdots \\ I_g \\ \hline I_{g+1} \\ \vdots \\ I_n \end{array}\right] \qquad (1)$$

Equation (1) can be briefly represented as the following (2):

$$\left[\begin{array}{c|c} YGG & YGL \\ \hline YLG & YLL \end{array}\right] \left[\begin{array}{c} VG \\ \hline VL \end{array}\right] = \left[\begin{array}{c} IG \\ \hline IL \end{array}\right] \qquad (2)$$

Calculate the equivalent admittance of each load bus with equation (3):

$$YL_j = \frac{1}{VL_j}\left(\frac{SL_j}{VL_j}\right)^* \qquad (3)$$

where

SL_j is the apparent power of load on bus j,

YL_j is the equivalent admittance of load on bus j,

VL_j is the resultant voltage of bus j of power flow analysis.

We use (3) to calculate the equivalent admittance of every load and then modify the sub matrix [YLL] in the original Y bus matrix. The modification is executed by adding the corresponding, YL_j to the diagonal elements in the [YLL] matrix, so the original matrix [YLL] is replaced by matrix[YLL'] . With the equivalent admittances of loads being represented, the load buses will have no injection current, thus reducing the sub-matrix [IL] to [0]. Hence, (3) is changed as shown:

$$\left[\begin{array}{c|c} YGG & YGL \\ \hline YLG & YLL \end{array}\right] \left[\begin{array}{c} VG \\ \hline VL \end{array}\right] = \left[\begin{array}{c} IG \\ \hline 0 \end{array}\right] \qquad (4)$$

Equation (4) on the lower half part of the matrix is used to arrive at:

$$[YLG][VG] + [YLL'][VL] = 0 \qquad (5)$$

And then the relationship functions can be obtained as follows:

$$[YLL'][VL] = -[YLG][VG] \qquad (6)$$

$$[VL] = -[YLL']^{-1}[YLG][VG] \qquad (7)$$

In (7), it is assumed that

$$[YA] = -[YLL']^{-1}[YLG] \qquad (8)$$

and (7) can be rewritten as

$$[VL] = [YA]\,[VG] \qquad (9)$$

The voltage of each load bus consisting of the voltages contributed by individual generators is expanded as shown in the following equation:

$$VL_j = \sum_{i=1}^{g} YA_{j,i} * VG_i \qquad (10)$$

and it is assumed that

$$\Delta VL_{i,j} = YA_{j,i} * VG_i \qquad (11)$$

Where ΔVL_j is the voltage contribution that load acquires from generator . Equation (10) may also be expressed as

$$VL_j = \sum_{i=1}^{g} \Delta VL_{i,j} \qquad (12)$$

With (12), it can be recognized that the voltage contribution of each load bus received from individual generators is ΔVL . The reactive power contributions that load acquire from generator *i* is as follows:

$$QL_{i,j} = \mathrm{Im}\{\Delta VL_{i,j} * IL_j^*\} \qquad (13)$$

where IL_j is the load current which is to divide the power of the load by known load bus voltage and take the conjugate of the complex number on load bus *j.* Reactive Power Contribution that load j acquires from generator *i* can be determined from (13). The calculation results might bring about some differences from those based on other methods. But the contribution of reactive power to the transmission line cannot be calculated [11]. If any static capacitor is added to load bus then the power flows and voltages of this system have been changed. The bus voltage contributions from each generator also changed, reflecting in a change that can be seen as a reduced share on each load bus of the reactive power from existing six generators.

III. VIRTUAL FLOW APPROACH

This approach presents the concept of virtual flows using the principle of superposition. The concept is applied to obtain virtual contributions of individual sources to line flows and loads. It is established that the virtual contribution to loads is by each source of the network in some proportion and the actual contribution is the superposition of the all the respective virtual contribution. The procedure of this method to find the contribution of an each generator to the line flow, loads and losses are given below.

1) Perform load flow / state estimation of the network
2) Read bus voltage phasors, real and reactive power injections at generator buses, loads and network parameters.
3) Convert all the loads to equivalent admittances at the operating point by the relation,

$$y_i^{load} = \frac{\left(-P_i^{(o)} + jQ_i^{(o)}\right)}{\left|V_i^{(0)}\right|^2} \qquad i=g+1,g+2\ldots n$$

4) Modify the network Y bus matrix to include loads as admittances.
5) For *i = 1 to g* (number of sources) Inject equivalent current from one source at a time to respective bus and obtain corresponding bus voltage profile.

$$I_i^{(o)} = \frac{S_i^{(0)^*}}{V_i^{(0)^*}} \quad \text{where } S_i^{(0)} = P_i^{(0)} + jQ_i^{(0)}$$

6) Determine all the resulting branch currents for the voltage profile obtained from this source.
7)The total complex power flow in the line i-j is given by,

$$S_{i-j}^{(0)} = \left\{ (V_i^{(0)} - V_j^{(0)}) y_a + V_i^{(0)} y_b \right\}^* V_i^{(0)}$$
$$= \left| V_i^{(0)} \right|^2 (y_a^* + y_b^*) - V_i^{(0)} V_j^{(0)*} y_a^*$$
$$= \Delta S_{i-j}^{(1)} + \Delta S_{i-j}^{(2)} .$$

Where

y_a is the series admittance and y_b is the half line charging susceptance.

$\Delta S_{i-j}^{(1)}, \Delta S_{i-j}^{(2)}$ are called as virtual flows due to source at node1 and node2.

8)The total contributions to given load from all the sources is obtained by the summation of partial contribution by all individual sources and it agrees with load power as in base case. It can be ascertained that the load power.

$$S_i^{(0)} = \sum_{k=1}^{g} \Delta S_i^{(k)}$$

Where $\Delta S_i^{(k)} = \sum_j \Delta S_{j-i}^{(k)}$ j-i line incident on load bus i.

IV. POWER FLOW TRACING METHOD

The electricity tracing methodology is topological in nature. It is based on actual flows in the network and proportionality share principle. It deals with a general problem of how to distribute flows in a meshed network [12-14]. The proportional sharing principle basically applies Kirchhoff 's current law at nodal power. The node and applies proportionality principle to find the relationship between incoming and outgoing flows. Thus this method is equally applicable to real and reactive power flows and direct currents. The only assumption that is made in this methodology is that the system is lossless [15, 16]. This is achieved by averaging the sending and receiving end line flows and by - adding half of the line loss to the power injections at each terminal node of the line.

The main objective of reactive power tracing method is to calculate reactive power loss allocated to each line for particular load by using the following equation

$$Q_{Dij,k} = QD_{ij,k} Q_{Dij} \qquad (14)$$

where, $$QD_{ij,k} = \frac{\left(\dfrac{Q_{ij,k}}{\sin\phi_k} \right)^2}{\sum_{k=1}^{l} \left(\dfrac{Q_{ij,k}}{\sin\phi_k} \right)^2}$$

QD_{ij} = total reactive power loss in line i-j;

$QD_{ij,k}$ = reactive power loss distribution factor

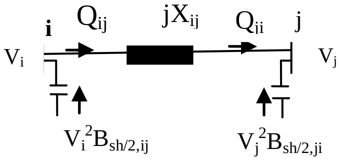
$Q_{ij,k}$ = reactive power flowing in line i-j due to kth load.

The Procedure to obtain this objective is given below

1.Obtain the Power Flow solution for given system.

2.Calculate new reactive power in each line due to the reactive power generated by shunt admittance Q $_{\text{shunt}}$ connected to each bus, by assuming that voltage of shunt admittance is equal to the nearby nodal voltage. The nodal voltage can be obtained from power flow using the formula:

$$Q_{shunt,i} = V_i^2 B_{sh/2,ij};\ Q_{shunt,j} = V_j^2 B_{sh/2,ij};$$
$$Q_{ij,New} = Q_{ij} + Q_{shunt,i} \qquad Q_{ji,New} = Q_{ji} - Q_{shunt,j}$$



3. Form the Lossless Network by dividing the line loss by

a)Calculate the Reactive Power injection at each bus i.e. equal to Total generated power + (Σ half of the Transmission line loss connected to that bus)

b) Calculate the average value of sending and receiving end reactive power of each transmission line.

c)Calculate the reactive power at each bus i.e. equal to sum of outflows of that bus.

4.Calculate the Upstream Distribution Matrix(**A$_{\mathbf{u}}$**):

These can be calculated using Upstream Looking Algorithm, it states that total flows (inflows and outflows) in bus 'i' i.e. P$_i$ can be expressed as

$$P_i = \sum_{j=\alpha_i^{(u)}} \left| P_{i-j} \right| + P_{Gi} \qquad \text{Let } C_{ij} = \left| P_{j-i} \right| / P_j$$

Therefore,

$$P_i - \sum_{j=\alpha_i^{(u)}} c_{ij} \left| P_{i-j} \right| = P_{Gi} \text{ (or) } A_u P = P_{Gi} \qquad (15)$$

The Elements of upstream distribution matrix can be calculated by

$$[A_u]_{ij} = \begin{cases} 1 & for \quad i = j \\ -C_{ji} = -\left| P_{j-i} \right| \big/ P_j & for \quad l \in \alpha_i^{(u)} \\ 0 & otherwise \end{cases}$$

(16)

5.Find the inverse of upstream distribution matrix (A$_u$)

6.The contribution of kth generator to ith load is found out using

$$P_{Li} = \frac{P_{Li}}{P_i} P_i = \frac{P_{Li}}{P_i} \sum_{k=1}^{n} \left[A_u^{-1} \right]_{ik} P_{GK} \text{ for i=1,2,...n.} \qquad (17)$$

7.The contribution of kth generator to i-l line is found out using

$$\left| P_{i-l} \right| = \frac{\left| P_{i-l} \right|}{P_i} P_i = \sum_{k=1}^{n} D_{i-i,k}^{G} P_{GK} \text{ for all } l \in \alpha_i^{(d)} \qquad (18)$$

Where

$D_{i-i,k}^{G} = \left| P_{i-l} \right| \left[A_u^{-1} \right]_{ik} \big/ P_i$ generation distribution factor

8. Calculate the Downstream Distribution Matrix (A$_d$):

These can be calculated using Downstream Looking Algorithm, it states that, total flows (inflows and outflows) in bus 'i' i.e. P$_i$ can be expressed as

$$P_i = \sum_{l \in \alpha_i^{(d)}} \left| P_{i-l} \right| + P_{Li} = \sum_{l \in \alpha_i^{(d)}} C_{li} P_i + P_{Li} \qquad (19)$$

Let $C_{li} = |P_{l-i}| / P_l$

Therefore,

$$P_i - \sum_{l \in \alpha_i^{(d)}} C_{li} P_i = P_{Li} \text{ (or) } A_d P = P_L$$

The Elements of Down stream distribution matrix can be calculated by

$$[A_d]_{il} = \begin{cases} 1 & \text{for} \quad i = l \\ -C_{li} = -\left|P_{l-i}\right| \big/ P_l & \text{for} \quad l \in \alpha_i^{(d)} \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

9. Find the inverse of downstream distribution matrix (A_d) .
10. Calculate reactive power loss allocated to each line for particular load by using

$$Q_{Dij,k} = QD_{ij,k} Q_{Dij}$$

where

$$QD_{ij,k} = \frac{\left(\dfrac{Q_{ij,k}}{\sin\phi_k}\right)^2}{\sum_{k=1}^{l}\left(\dfrac{Q_{ij,k}}{\sin\phi_k}\right)^2}$$

QD_{ij} = total reactive power loss in line i-l;

$QD_{ij,k}$ = reactive power loss distribution factor

$Q_{ij,k}$ = reactive power flowing in line i-l due to kth load.

The general algorithm for tracing power flows is summarised in Figure 1.

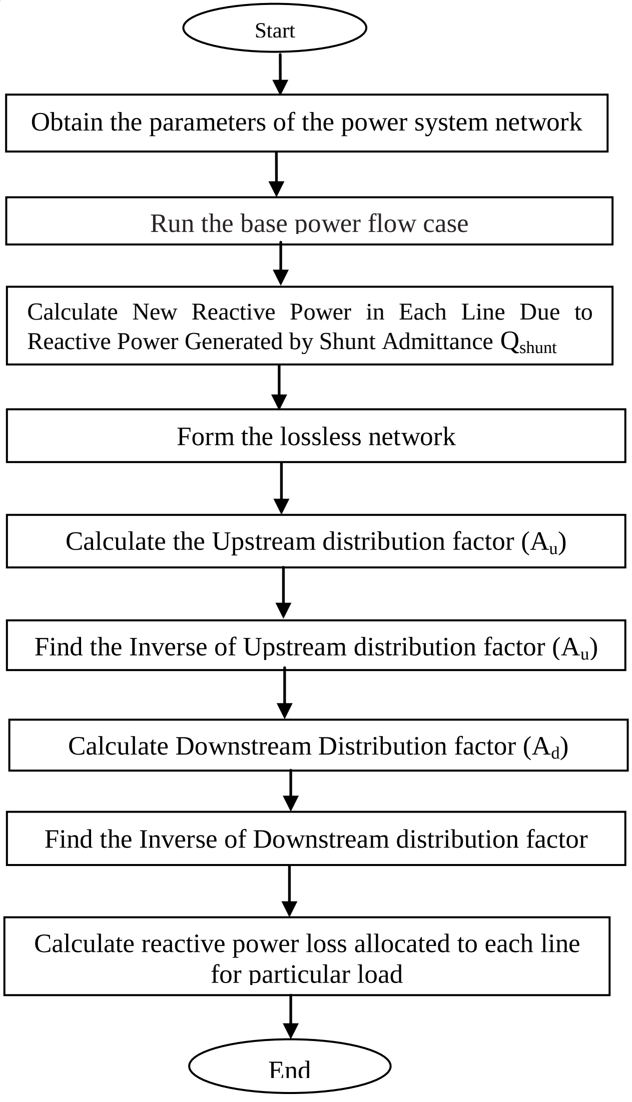


Figure-1 POWER FLOW TRACING ALGORITHM

V. SIMULATION RESULTS

The Indian Utility 62 bus system is taken to demonstrate the reactive power pricing. This system consists of 19generators and 43 load buses with 89(220kv) transmission lines.

The contribution of reactive power from sources to the sinks are analyzed by the four methods (Modified Ybus ,Virtual Flow Approach and Power flow Tracing) through following three case studies.

1.Base case :- From the power flow result, reactive power generated by each generators from Modified Ybus method,Virtual flow approach and Power flow tracing methods are calculated at the system base load condition.

2.Contingency case:-In order to analyze the security and stability single line outage condition is carried out.

3.Increasing load condition:- When the load is increased to 120% generator's contributions are analyzed .

A.Modified Ybus method

From the power flow analysis results the voltages, real power and reactive powers were obtained. The Ybus of n*n dimensions can be divided into four submatrices [Y_{GG},Y_{GL},Y_{LG},Y_{LL}].Then the equivalent admittance YL_j of every load was calculated.The submatrix [Y_{LL}] is modified by adding YL_j to the diagonal elements. The voltages of each load buses (VL_j) were calculated by solving the equations(6)to (9). The reactive power contribution that load acquired from generator is calculated using the equation (13). The total reactive power contribution generated by six generators for three different cases is given by the Table1.

Table 1. Contribution of reactive power in MVAR from Modified Ybus Method

	baseload	**Contingency**	**IncreasingLoad**
GEN1	34.64453567	35.9059067	34.61002
GEN2	26.02412397	26.9650871	25.5865
GEN5	108.0251201	113.900642	112.0412
GEN9	0	0	0
GEN14	168.4417939	170.006435	167.5041
GEN17	-63.4819093	-62.4208241	-76.6116
GEN23	164.0627294	165.38133	150.3066
GEN25	107.6906844	107.779065	104.7654
GEN32	78.21315586	78.2415932	71.69717
GEN33	0	0	0
GEN34	9.111573922	9.11157358	9.109359
GEN37	4.184050643	4.18612285	3.822491
GEN49	77.77687459	77.8371905	56.60387
GEN50	-6.07392869	-6.07049396	-6.66023
GEN51	39.0977259	38.9187915	36.90907
GEN52	36.90169404	36.7824543	34.29005
GEN54	-0.32150979	-0.3183544	-0.34941
GEN57	0	0	0
GEN58	108.9119888	110.914053	106.5988

It is inferred that in all the three cases the generator G_{14} only delivers higher amount of reactive power. But the computational procedure of modified Ybus is more ambiguity.

B. Virtual flow approach

The virtual flow tracing technique approach can identify counter flows and also handles loop flows equally well. Here all the loads are converted into respective complex admittances at the operating point. The diagonal element of the Y_{bus} matrix corresponding to load buses is then modified by including these loads respectively.Partial contribution to line flows from a given source is calculated using the base case voltage profile and the partial current in the network branch due to that source.

Though the flows computed by the proposed approach are virtual, the line flows and counter flows gives information regarding extend of line usage by each sources.

Table 2. Contribution of reactive power in MVAR from virtual flow approach method

	Baseload	**Contingency**	**IncreasingLoad**
GEN1	392.9	56.7952	350.152
GEN2	89.71781	241.2842	100.0951287
GEN5	30.94047	27.8804	30.42291738
GEN9	40.96872	66.68587	41.28769579
GEN14	397.294	291.5944	521.6210949
GEN17	26.98254	35.51453	35.68869694
GEN23	89.11916	127.9072	445.753379
GEN25	456.6019	481.4652	521.7141748
GEN32	4.07375	4.068865	4.171460301
GEN33	9.508744	9.508741	9.519251437
GEN34	26.59379	26.69627	25.38810763
GEN37	251.9191	264.4166	256.9818224
GEN49	51.62045	51.06478	51.90982095
GEN50	-0.84042	-0.75619	-0.724881255
GEN51	203.7151	205.5172	196.3430078
GEN52	381.9127	380.8607	353.82909
GEN54	11.4349	10.52002	10.77284423
GEN57	123.1788	447.3538	447.3611379
GEN58	11.11408	60.7066	207.7160671

C.Power Flow Tracing Method

The objective of this method is to determine the amount of reactive power consumed by the corresponding loads. This method gives an exact idea about the contribution of reactive power delivered by the generator, reactive VAR sources and fictitious nodes connected in the system. Hence in this method the fictitious generators are connected to all the nodes.

Then the lossless network was obtained by adding half of the line loss to the power injections at each terminal of the line. Then the above lossless network is obtained by constructing upstream and downstream matrices. This includes the losses along with the nodal loads. In order to include the effect of Power factor in the corresponding load currents Loss distribution factor is computed using the equation (14). It identifies the loads responsible for reactive power loss in a specific transmission line and indicates their responsibility share.

Total amount of reactive power delivered to the load from the sources for three case studies by power flow tracing method is shown in Table 3.In this table, the generator G_{25} delivers the maximum amount of reactive power in all the three cases. In large scale powers system power flow tracing method gives additional information about reactive power generated by var sources, shunt admittance of transmission line and given in Table3.

Table 3. Contribution of Reactive Power(**in MVAR)** due to shunt admittance using power flow tracing method

	Base Load		**Contingency**		**Increasing Load**	
	Generator	**shunt admittance**	**Generator**	**shunt admittance**	**Generator**	**shunt admittance**
GEN1	143.113	28.7811	179.1492	27.1636	154.924	28.7811
GEN2	59.2006	4.0749	32.0798	2.4452	66.6002	4.0749
GEN5	-1.9679	4.5027	-3.3306	4.5027	8.7664	4.5027
GEN9	33.2607	1.218	33.2607	1.218	34.5278	1.218
GEN14	119.5508	31.8501	120.1153	31.8501	265.1553	31.8501
GEN17	196.2974	25.4487	196.2014	25.4487	317.6218	25.4487
GEN23	82.8269	21.2985	82.7887	21.2985	166.3082	21.2985
GEN25	268.7195	24.9698	268.7281	24.9698	291.2281	24.9698
GEN32	29.9932	52.1777	29.9949	52.1777	36.4347	52.1777
GEN33	6.413	18.3134	6.4132	18.3134	7.2124	18.3134
GEN34	95.362	37.6532	95.3651	37.6532	120.4983	37.6532
GEN37	-12.7007	31.2747	-12.693	31.2747	-11.6185	31.2747
GEN49	41.1471	11.3471	41.1477	11.3471	44.2578	11.3471
GEN50	0	5.0438	0	5.0438	0.0315	5.0438
GEN51	0	15.9107	0	15.9107	0	15.6105
GEN52	0	11.0409	0	11.0409	0	10.8244
GEN54	26.7953	9.1889	26.7954	9.1889	10.7956	9.0195
GEN57	124.0077	1.7798	124.0077	1.7798	286.6468	1.7798
GEN58	50.3462	21.2719	50.5705	21.2719	0	20.8682

CONCLUSION

The comparison of three different methods of reactive power valuation is reported in this paper. Different methods have different results. Virtual flow approach is used to evaluate real and reactive power flow in the network due to individual sources and its contribution to each load using the principle of superposition.The modified Ybus Method can identify the source and can calculate the amount of consumed reactive power on each load. The power flow tracing method could have wide applications in the deregulated electricity supply industry. Apart from giving additional insight into how power flows in the network, it can be used to set tariffs or transmission services based on the shared, as opposed to marginal costs. This includes charging for the transmission loss and for the actual usage of the system by a particular generator or the load. This method can also be used to assess the contribution of individual sources of reactive power in satisfying individual reactive power demands and therefore be used as a tool for reactive power pricing.

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