Power System Voltage Security Using Eigen Analysis

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***Abstract*-This paper deals with Monitoring Voltage stability of power systems for voltage security studies using Singular value decomposition and Eigen value properties. The voltage stability of the system is tested using a Standard IEEE 14 bus system and sensitivities of the buses are calculated. The voltage profile of the buses and their sensitivities are declared using a Rule Based module. The software was developed using C++.**

***Key Words*- Eigen value, Rule Base, Singular value Decomposition, Voltage Stability.**

I. Introduction

Problems related to Voltage instability in power systems are today, in many countries, one of the major concerns in power system planning and operation [1,2,3]. One of the causes for this increased interest in voltage stability problems is that a load growth without a corresponding increase of transmission capacity has brought many power systems closer to their voltage stability boundaries. Throughout the world there have been a number of system black – outs caused by voltage instabilities, which in turn are caused by an unexpected raise in the load level, sometimes in combination with unusual conditions in the system, or by a network disturbance such as the loss of an important transmission line, a transformer or a generator, some of the voltage collapses and other similar instabilities can be characterized by a voltage drop, which at first is slow and then becomes rapid. The voltage collapse phenomenon can be related to the action of tap – changers on transformers, current limiters of generators, inadequate reactive power resources (at least locally) and load characteristics at low voltage magnitudes [1].

In order to identify voltage instability problems in large power systems several different methods such as load modeling, power flow equations are used [1,2]. But these methods are analytical solutions only and cannot be incorporated for solving practical real time voltage violations. The application of Rule Based techniques greatly reduces analytical procedures.

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The application of Rule Based techniques for voltage stability was discussed in paper [4,5]. Above papers uses power flows and converting them into knowledge base. The drawback of the above works is not incorporating the system dynamic condition resulting inaccuracy. The previous works [1,2] uses properties of Jacobian matrices and the paper [1] uses only Eigen value monitoring. This paper attempts to combine Eigen values and its sensitivity for identifying system state.

One of the aims of Academics research in this area is to find a voltage stability indicator, available from normal power flow calculations, which is suited for both planning and operational purposes. The purpose of this work is to find a voltage stability index is to, in some respect, quantify how 'close' a particular operating point is to the point of voltage collapse, i.e. to estimate the steady state voltage stability limit of the power systems. This work utilizes the minimum singular value of the power flow Jacobian matrix and Eigen values of its J_R [1,2,3,4]. The use of this indicator is obtained from a (full) singular value decomposition of the power flow Jacobian matrix. The sensitivity factors are found using this singular value decomposition and they are used to form Rule base.

This paper adapts the modal analysis techniques [1,6] to enable very large systems to be studied and Rule Based module is developed by suitable step-by-step algorithm.

II. Power System Voltage Security

Voltage stability [3] is a dynamic phenomenon and can be studied using time domain simulation method with appropriate system modeling. However such simulations do not readily provide information regarding the degree of stability. They are also time consuming in terms of CPU requirement. Therefore the applications of dynamic simulations are limited to the investigation of specific voltage instability incidents. Steady state analysis is much more efficient in terms of computational speed and, if used properly, can provide much insight into the voltage problem of the system.

III. Security Analysis

It is now commonly accepted that for security assessment of a power system, the most efficient and practical strategy is to deal with the problem in two stages. First in contingency selection, those potentially critical contingency cases are ranked by the severity of their impacts on a system. Then in contingency analysis, detailed A.C power flows are applied only to the most dangerous cases appearing on the top of the ranked list. In the past decade, the problem of ranking outage cases in the order of decreasing severity has attracted a lot of intensive studies and many efficient and reliable algorithms were developed. The analysis of the effect of hundreds of outages on line flows and bus voltages is required for the real – time security analysis and contingency enumeration. This increases the speed, accuracy and adaptability of the solution methods. The security level of the network is established to assure the consumers a reasonable availability of the energy supply. In the power industry, it is very important that the operating personnel be able to predict the consequences of disturbances and be able to take measures to prevent deterioration in service quality. Intensive research in the past two decades has resulted in significant advances so that a large number of analytical security monitoring techniques are presently available and used routinely at advanced control centres. Real time security assessment, however, is still a challenging task. The objective of power system security assessment is to investigate the extent to which a computer program with knowledge and inference capabilities can assist a human operator in this task. The rule-based system is independent of the size of the power system, and derives partly from simulation models. The approach should be cost effective and has the potential for application to on – line operation.

IV. Rule Base System

1. During a real time environment, if a problem is detected, a Rule base system can be applied to suggest a solution to the operators in time, based on the incorporated expert knowledge. An example is the Emergency control problem of power system[5].

2. The knowledge required to perform a task is expressed in terms of production rules (IF – THEN Structures), which are very close to natural language and therefore easy to understand to users.

3. Each production rule represents the knowledge relevant to the task. Hence it is very convenient to add or remove a rule when more experience is gained during operation or testing.

4. Each production can be provided with an explanation of why an action is taken under certain situation. This is desirable from the training point of view, since the trainee can learn from the previous experience in a systematic way.

This paper is aimed to develop a Rule Based system to assist the decision making of the power systems in the presence of a voltage problem.

V. Components Of Rule Based System

An Rule Based system consists of three fundamental components:

A global database, knowledge base which is usually a set of IF – THEN structures, which represent the knowledge required to solve a problem and an inference engine utilized to chain a set of rules to form a line of reasoning. The knowledge used by human experts in solving a problem may include facts derived from physical laws and heuristics based on engineering judgments.

VI. Database And Knowledge Base

A Rule Based Algorithm for power system uses the following data:

1. Upper and lower limits of voltage at each bus
2. Upper and lower limits of each controller
3. Sensitivity factor for each load bus and controller

Note the sensitivity factors vary with different system operating conditions:

When a bus voltage exceeds its specified limits. Either high or low, the usual actions to be taken by a system operator are either to switch a capacitor bank, adjust the tap positions of transformer tap changers or vary the generator bus voltage in order to restore a normal voltage profile. When a low voltage occurs on a load bus, a capacitor bank or synchronous condenser can provide an additional VAR to the power system thus raising bus voltages. The tap changer can adjust the turns ratio of a transformer and hence increase the secondary voltage magnitude. However, as a result of the higher secondary voltage, the total reactive power demand may also increase. The inferencing process involves the use of the sensitivity tree technique.

VII. Problem Formulation

A. Jacobian Matrix

The Eigen value analysis on load flow Jacobian matrix will indicate the closeness of the system to steady state bifurication point and voltage collapse. The Jacobian matrix relates the linear variation in load bus voltage magnitude and angle to the linear variation in real and reactive power injection in to the load bus. Here the load itself is considered as part of the systems and jacobian matrix is formed [2].

B. Reduced Jacobian Matrix

J_R is called the reduced jacobian matrix, which is a scalar. The Eigen value analysis of the reduced Jacobian matrix is very important for voltage stability analysis of large systems; it provides a unified criterion for assessing system voltage stability [7,8].

C. Singular Value Decomposition

The Singular value decomposition is an important and practically useful orthogonal decomposition method used for making computations. The minimum value of the power flow jacobian matrix has been used as a static voltage stability index, indicating the distance between the studied operating point and the steady state voltage stability limit. Here a fast method to calculate the minimum singular value and the corresponding (Left and right) singular vectors is proposed [1,2].

D. Algorithm

Step 1: Input values of **n, P_{sp}, Q_{sp}, e, f, G, B.**

Where

n is the total no. of buses,

$\mathbf{P_{sp}}$ is the specified value of real power of the bus,

$\mathbf{Q_{sp}}$ is the specified value of reactive power of the bus,

e is the real part of voltage (in rectangular co ordinates) of the bus,

f is the imaginary part of voltage (in rectangular co ordinates) of the bus,

G is the conductance of the transmission line,

B is the Susceptance of the transmission line,

Step 2: Calculate the values of $\mathbf{P_{cal}, Q_{cal}}$.

Using given formula,

$$P_{cal\ p} = \sum_{q=1}^{n} \{ e_p(e_pG_{pq} + f_q B_{pq}) + f_p(f_qG_{pq} - e_qB_{pq})\} \quad (1)$$

$$Q_{cal\ p} = \sum_{q=1}^{n} \{ f_p(e_pG_{pq} + f_q B_{pq}) - e_p(f_qG_{pq} - e_qB_{pq})\} \quad (2)$$

Step 3: Formation of Jacobian matrix **J.**

Step 4: Compute the Reduced Jacobian matrix $\mathbf{J_R}$.

The reduced system steady state Jacobian Matrix represents the linearized relationship between incremental change in bus voltage magnitude and the bus reactive power injecting at a given steady state operating condition [9].

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J1 & J2 \\ J3 & J4 \end{bmatrix} \begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix} \quad (3)$$

$$J_R = J_4 - J_3 J_1^{-1} J_2 \quad (4)$$

Step 5: Determination of Eigen values of the reduced jacobian matrix.

Let A be the given real symmetric matrix. The Eigen values of A are real and there exists a real orthogonal matrix **S** such that $\mathbf{S^{-1}AS}$ is a diagonal matrix D. The Diagonalization is done by applying a series of orthogonal transformations S_1, S_2, S_3 …….S_n as follows:

Among the off – diagonal elements, left $|a_{ik}|$ be the numerically largest element. Then the elements a_{ii}, a_{ik}, a_{ki}, a_{kk} form a 2*2 sub matrix A_1 which can be transformed to a diagonal form [9].

We choose

$$S_1^* = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} \quad (5)$$

and find θ such that 2^*2 sub matrix A_1 is diagonalized.

$$\mathbf{S_1{*}A_1S{*}} = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} \begin{vmatrix} a_{ii} & a_{ik} \\ a_{ki} & a_{kk} \end{vmatrix} \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$$

$$= \begin{vmatrix} a_{ii}\cos^2\theta + a_{ik}\sin 2\theta + a_{kk}\sin^2\theta & (a_{kk} - a_{ii})\sin\theta\cos\theta + a_{ik}\cos 2\theta \\ (a_{kk} - a_{ii})\sin\theta\cos\theta + a_{ik}\cos 2\theta & a_{ii}\sin^2\theta - \sin 2\theta + a_{kk}\cos^2\theta \end{vmatrix}$$

We now choose θ such that this matrix reduces to a diagonal form. That is, we equate

$$(1/2)\ (a_{kk} - a_{ii})\sin 2\theta + a_{ik}\cos 2\theta = 0 \qquad (6)$$

or

$$\tan 2\theta = \frac{2a_{ik}}{a_{ii} - a_{kk}} \qquad (7)$$

This equation produces four values of θ and in order to get the smallest rotation we require $-\Pi/4 \le \theta \le \Pi/4$.

From eqn.(7) we get,

$$\theta = 1/2 \tan^{-1} 2a_{ik}/ a_{ii} - a_{kk} \qquad \text{if } a_{ii} \neq a_{kk} \qquad \ldots (8)$$

if $a_{ii} = a_{kk}$, then

$$\theta = \begin{vmatrix} \Pi/4, & a_{ik} > 0 \\ -\Pi/4, & a_{ik} < 0 \end{vmatrix}$$

With the value of θ given in equation (8) the off – diagonal elements in equation (6) vanish and the diagonal elements are simplified.

The first step is now completed by performing the rotation $S_1^{-1}AS_1$. In the next step the largest off – diagonal element in magnitude in the new rotated matrix is found and the procedure is repeated. We now perform a series of such two dimensional rotations. After finding θ at each step, the rotation is performed with the corresponding orthogonal matrix. For example if $|a_{ik}|$ is the largest off – diagonal element then we write S_1 as:

$$\mathbf{S_1} = \begin{vmatrix} 1 & & & & \\ & 1 & & & \\ & & \cos\theta & -\sin\theta & \\ & & \sin\theta & \cos\theta & \\ & & & & 1 \end{vmatrix} \qquad (9)$$

where $\cos\theta, -\sin\theta, \sin\theta, \cos\theta$ are located in (i,i), (i,k), (k,i) and (k,k) positions respectively. After making r transformations, we get

$$\begin{aligned} B_r &= S_r^{-1} S_{r-1}^{-1} \ldots S_1^{-1} A\ S_1 \ldots S_{r-1} S_r \\ &= (S_1 S_2 \ldots S_r)^{-1} A\ (S_1 \ldots S_{r-1} S_r) \\ &= S_1^{-1} A\ S \end{aligned} \qquad (10)$$

Where $S=S_1S_2\ldots S_r$

As $r \to \infty$, By approaches a diagonal matrix with the Eigen values on the leading diagonal. We then have the Eigen vectors as the corresponding columns of S. The minimum no.of rotations required to bring A into a diagonal form may be (n-1)n/2.

This procedure, described to reduce the symmetric matrix A to a diagonal matrix D is called **Jacobi method [9].**

Step 6: Determination of sensitivity factor "**S**" using the obtained Eigen values of the reduced Jacobi matrix.

$$\mathbf{S} = \sum \frac{\mathbf{U_i\ V_i}}{\boldsymbol{\lambda_i}} \qquad (11)$$

Where

$\mathbf{U_i}$ is the left row Eigen vector of the i[th] bus,
$\mathbf{V_i}$ is the right column Eigen vector of the i[th] bus
$\boldsymbol{\lambda_i}$ is the Eigen value of the i[th] bus.

Determination of $\mathbf{U_i}$ **and** $\mathbf{V_i}$:

$$\mathbf{U = J_R\ J_R^T} \qquad (12)$$

$$\mathbf{V = J_R^T\ J_R} \qquad (13)$$

Sensitivity technique is widely used to analyze linear systems. It represents the major relationship between the control action and their effects. As the power system is actually a non – linear system, the sensitivity factor between a reactive control measure and the bus voltages cannot be a constant value. The first order sensitivity function is commonly used for simplicity especially when system non – linearity is not large.

Test results show only a small difference over a wide range of system operating conditions. This implies that the sensitivity technique can be used successfully to analyze this reactive power and voltage control problem.

Step 7: Declaration of the stability of the buses using the obtained sensitivity factors and the Eigen values.

- ❖ If all the Eigen values are positive then the V – Q sensitivities are all positive indicating that the system is voltage stable.
- ❖ If at least one Eigen value is equal to zero then the V – Q sensitivity for some buses become infinite indicating that the system is on the verge of voltage instability.
- ❖ If any one of the Eigen values is negative the system has passed the voltage stability critical point therefore the system is voltage unstable.

VIII. Test And Results

The above formulation is applied to a standard IEEE 14 bus system and the results shows Eigen value variation, sensitivities of buses and clearly indicates the system state.

The above Algorithm is tested and the values are tabulated.

Bus	Eigen Values	Sensitivity Factors	System State
6	0.464360	Positive	Stable
7	-0.000072	Negative	Unstable
8	5.000362	Positive	Stable
9	-2781.6967	Negative	Unstable
10	-15.947819	Negative	Unstable
11	91.028946	Negative	Unstable
12	0.285195	Positive	Stable
13	0.0	Positive	Presently Stable and going to be Unstable
14	-0.158789	Negative	Unstable

IX. Conclusion

This paper has presented a Rule based solution for solving power system voltage security problems.

This paper also uses the concept of Eigen value and sensitivities with respect to the reduced Jacobian matrix. In small signal voltage security studies using Eigen value monitoring is the useful tool. The knowledge base consists of no. of production rules, which are formed using these Eigen value.

The main advantage of this work is incredibly fast, since it uses SVD and sparsity of Jacobian Matrix. By using an interesting technique called Jacobi Method, J_R [9] is found and this method takes less computation time and the results are highly accurate.

The work also shows Eigen Values of the multi machine system tends to have large sensitivities. Since this Eigen value and sensitivity factors are obtained directly from a base Load flow results, and can be applied to On line monitoring of voltage security at control centres. The developed methods are applied to a standard IEEE 14 bus system and the results are found satisfactorily.

The authors hope that by applying a Rule based Voltage security monitoring will reduce the time for decision-making during voltage stability problems. This developed Rule Based system based ranking algorithm will stimulate further research of AI applications like Neuro, Neuro – Expert, Neuro – Fuzzy for power system security studies. Further development of this work will include the Large signal voltage stability studies by incorporating Load-shedding control into the knowledge base.

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