Power Flow Analysis under Fuzzy Environment

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Abstract- **In this paper power flow solutions with realistic loads has been presented. In conventional power flow algorithms the values of real and reactive loads are specified. Unfortunately the solutions obtained based on this approach do not reflect actual conditions because the variables are fuzzy in nature. As a case study the classical algorithms for power flow such as Gauss-Seidel and Newton Raphson have been implemented under fuzzy environment so that they can handle fuzzy variables. The results so obtained have been found to be accurate and satisfactory.**

***Index Terms-* Fuzzy logic and load flow.**

I. INTRODUCTION

Load flow (or power flow) analysis is the determination of current, voltage, active and reactive powers at various points in a power system operating under normal steady-state or static conditions. Load flow studies are performed to calculate the magnitude ($|V_i|$) and phase angles (δ_i) of voltages at the buses, and also the active power (P_i) and reactive volt-amperes (Q_i) for the given terminal or bus conditions. The power flow problem is formulated as a set of nonlinear equations. Many calculation methods have been proposed to solve this problem. Among them, Newton-Raphson method and fast decoupled load flow method are two very successful methods. In general, the decoupled power flow methods are only valid for weakly loaded network with large X/R ratio network. For system conditions with large angles across lines (heavily loaded network) and with special control that strongly influence active and reactive power flows, Newton-Raphson method may be required [1-3]. To make the power flow solutions more realistic, the variables such as real and reactive powers at different buses are considered fuzzy in nature. In present day scenario power flow solutions under fuzzy environment [4-5] has been considered to be of practical use.

II. FUNDAMENTAL OF FUZZY SET THEORY

Fuzzy Set: Let X be a collection of objects (X is the universal set). Then a fuzzy set $\tilde{A}$ in X is defined to be a set of ordered pairs[4]:

$$\tilde{A} = \{(x, \mu_A(x)), \mid x \in X\} \qquad (1)$$

where $\mu_A(x)$ *is called the membership function of x in* $\tilde{A}$. *Note that the membership function* $\mu_A(x)$ *denotes the degree that x belongs to* $\tilde{A}$ *and is normally limited to* values between 0 and 1. A high value of $\mu_A(x)$ implies that it is very likely for x to be in $\tilde{A}$. Elements with a zero degree of membership are normally not listed. If we limit the values of the membership function to be either 0 or 1, then $\tilde{A}$ becomes a crisp (non-fuzzy) set.

The Union of two Fuzzy Sets: Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy sets with membership functions $\mu_A(x)$ and $\mu_B(x)$ respectively. The membership function of the union $\tilde{C} = \tilde{A} \cup \tilde{B}$ is point-wise defined by

$$\mu_C(x) = \max\ (\mu_A(x), \mu_B(x)),\ x \in X \quad (2)$$

The Intersection of Two Fuzzy Sets: Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy sets with membership functions $\mu_A(x)$ and $\mu_B(x)$ respectively. The membership function of the intersection $\tilde{D} = \tilde{A} \cap \tilde{B}$, is defined by

$$\mu_D(x) = \min\ (\mu_A(x), \mu_B(x)), x \in X \quad (3)$$

The Complement of a Fuzzy Set: Let $\tilde{A}$ be the fuzzy set with membership function $\mu_A(x)$. The membership function of complement of set $\tilde{A}$ is defined by

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \qquad (4)$$

III. ARITHMETIC OPERATIONS OF DISCRETE FUZZY NUMBERS

Arithmetic operations of fuzzy numbers are normally carried out using α cut [5] or extension principle method. The detailed procedure of α cut method is explained as follows. Improvement over α cut and extension principle will be suggested in the next section.

Addition

When adding two fuzzy numbers A and B we seek to compute a new fuzzy number C = A+B. The new number C is uniquely described when we obtain its membership function, $\mu_C(z) \equiv \mu_{A+B}(z)$, with z being the crisp sum of x and y, the elements of the universe of discourse of A and B. The addition of A and B may be defined in terms of addition of the α=cuts of the two numbers as follows:

$$A + B = [a_1^{(\alpha)}, a_2^{(\alpha)}] + [b_1^{(\alpha)}, b_2^{(\alpha)}] \qquad (5)$$

where $[a_1^{(a)}, a_2^{(a)}]$ is the collection of intervals representing the fuzzy number A, and $[b_1^{(a)}, b_2^{(a)}]$ is the collection of intervals representing the fuzzy number B. Intervals are added by adding their corresponding left and right endpoints.

$$A + B = [a_1^{(\alpha)} + b_1^{(\alpha)}, a_2^{(\alpha)} + b_2^{(\alpha)}] \qquad (6)$$

Subtraction

The difference C of two fuzzy numbers A, B may be defined utilizing the α-cut representation and it has been subtracted as follows:

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$$A - B = [a_1^{(\alpha)}, a_2^{(\alpha)}] - [b_1^{(\alpha)}, b_2^{(\alpha)}] \quad (7)$$

Where $[a_1^{(a)}, a_2^{(a)}]$ is the collection of closed intervals representing A, and $[b_1^{(a)}, b_2^{(a)}]$ is the collection of closed intervals representing b. Two intervals are subtracted by subtracting their left and right endpoints.

$$A - B = [a_1^{(\alpha)} - b_2^{(\alpha)}, a_2^{(\alpha)} - b_1^{(\alpha)}] \quad (8)$$

Multiplication

Using the α-cut representation of two numbers A, b, their product is defined as

$$A.B \equiv [a_1^{(\alpha)}, a_2^{(\alpha)}] . [b_1^{(\alpha)}, b_1^{(\alpha)}] \quad (9)$$

In general, the product of two intervals is a new interval whose left endpoint is the product of the left endpoints of the two intervals and the right endpoint is the product of the right endpoints of the two intervals.

So, $A.B = [a_1^{(\alpha)} .b_1^{(\alpha)}, \; a_2^{(\alpha)} .b_2^{(\alpha)}]$ (10)

Division

We can find the division of two fuzzy numbers A and B in terms of their α-cut representation, we write the division of the two numbers as,

$$A \div B = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right] \div \left[b_1^{(\alpha)}, b_2^{(\alpha)}\right] \quad (11)$$

Hence, provided that $b_2^{(\alpha)} \neq 0$ and $b_1^{(\alpha)} \neq 0$, the quotient of A,B is

$$A \div B = \left[\frac{a_1^{(\alpha)}}{b_2^{(\alpha)}}, \frac{a_2^{(\alpha)}}{b_1^{(\alpha)}}\right] \quad (12)$$

IV. FUZZY APPROACH TO LOAD FLOW

All values of Pk and Qk are fuzzy numbers in the proposed algorithm and their intervals/fuzzy values for example are calculated by using their respective membership functions. A typical membership function has been shown in Fig.1. The deviation in values for example has been fixed for all membership functions with +- 0.2 p.u. only. Also an appropriate value of α may be selected (example 0.8).

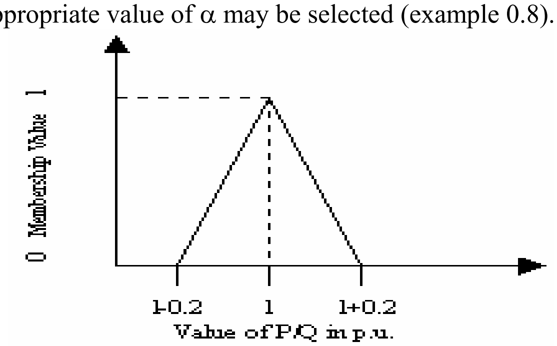

Figure 1. Triangular membership function

We have applied the method to Gauss-Seidel and Newton-Raphson method of load flow analysis.

A. Step-by-step algorithm for Gauss-Seidel power flow algorithm under fuzzy environment :

1. Read the load flow data and form the Y_{bus} matrix.
2. Represent Pi and Qi as fuzzy numbers $^{\alpha}$(Pi) and $^{\alpha}$(Qi) where μ is fuzzy membership function as shown above (Fig. 1) assuming a ∝ cut of 0.8.
3. Assume initial bus voltages $V_i(0)$ for i=m+1,m+2,….n (PQ bus)and δ_i for I=2,3,…m(PV bus)
4. Calculate C_i for i=m+1,m+2,…n and D_i for i=2,3,….n k=1,2,….n and i≠k using $C_i=\{^{\alpha}(P_i)-j^{\alpha}(Q_i)\}/Y_{ii}$ and $D_{ik}=Y_{ik}/Y_{ii}$.
5. Set iteration count=0.
6. Set bus count i=2 and maximum voltage change ΔVmax=0.
7. If the bus is a PQ(i>m) bus then jump to (11) else continue(i<=m).
8. Calculate $^{\alpha}Q_k(r+1)$ using $^{\alpha}Q_k(r+1)=-\text{Im}[(^{\alpha}V_k(r))^* \Sigma Y_{ki} {}^{\alpha}V_i(r+1) + (^{\alpha}V_k(r))^* \Sigma Y_{ki} {}^{\alpha}V_i(r)]$
9. Defuzzify Q_{k+1} and Q_k. .If $Q_k(r+1)>Q_k(\max)$ put $^{\alpha}(Q_k(r+1))=^{\alpha}(Q_k(\max))$ else if $Q_k(r+1)<Q_k(\min)$ put $^{\alpha}(Q_k(r+1))=^{\alpha}(Q_k(\min))$ else go to (10).
10. Find new C_k.
11. Find $^{\alpha}(V_k(r+1))=(C_k/(Vnr)^*) - \Sigma D_{ki}.^{\alpha}(Vi(r+1)) - \Sigma Dki.^{\alpha}(V_i(r))$.Where the value of Vnr is defuzzified value of $^{\alpha}(V_n(r))$. Go to (14).
12. Set $V_k r = |V_k|$sp treat δk(r) as phase angle of $|V_k|$sp
13. Find new Ck and from it determine $^{\alpha}(V_k(r+1))$. Put $V_k(r+1)=|V_k|$sp. Treat $\delta_k(r+1)$ as the phase angle of $|V_k|$sp go to (15).
14. Calculate $|\Delta V_k(r+1)|=|V_k(r+1)-V_k r|$.Use defuzzified values of $V_k(r+1)$ and V(r).
15. If $|\Delta V_k(r+1)|<\Delta V\max$ go to (17) else continue.
16. Set $\Delta V\max=|\Delta V_k(r+1)|$
17. Replace $^{\alpha}(V_k(r))$by $^{\alpha}(V_k(r+1))$and increment bus count i=i+1.
18. Loop to (7) if i<=n else continue
19. Test Convergence. Is max$|V_k(r+1) - V_k r|$<∈? If no then advance iteration from r to r+1 and loop to (6); else proceed.
20. Calculate losses, line flows and Slack Bus Power.
21. STOP

* Variables which do not have a α in superscript are assumed to be crisp or have been defuzzified.

B.

Step-by-step algorithm for Newton-Raphson power flow algorithm under fuzzy environment :

1.Form or take as input the Y_{BUS} matrix.

2 Assume bus voltages $E_p^{(o)}$ p=1,2,3…..n, $p \neq s$.

3 Set iteration count k=0

4 Calculate real and reactive bus powers using necessary equations →

$$^{\alpha}P_p^k = \sum_{q=1}^{n} \{ {}^{\alpha}e_p^k ({}^{\alpha}e_q^k G_{pq} + {}^{\alpha}f_q^k B_{pq}) + {}^{\alpha}f_p^k ({}^{\alpha}f_q^k G_{pq} - {}^{\alpha}e_q^k B_{pq}) \}$$

$$^{\alpha}Q_p^k = \sum_{q=1}^{n} \{ {}^{\alpha}f_p^k ({}^{\alpha}e_q^k G_{pq} + {}^{\alpha}f_q^k B_{pq}) - {}^{\alpha}e_p^k ({}^{\alpha}f_q^k G_{pq} - {}^{\alpha}e_q^k B_{pq}) \}$$

p=1,2…n, $p \neq s$

(Also $E_p = e_p + jf_p$ and $Y_{pq} = G_{pq} - jB_{pq}$)

5 Calculate difference between scheduled and calculated powers →

$$^{\alpha}\Delta P_p^k = {}^{\alpha}P_{p(scheduled)} - {}^{\alpha}P_p^k$$

and $^{\alpha}\Delta Q_p^k = {}^{\alpha}Q_{p(scheduled)} - {}^{\alpha}Q_p^k$

p=1,2,3….n, $p \neq s$

6. Determine maximum change in power max ΔP^k and max ΔQ^k .

7.Test for convergence $|\max \Delta P^k| \le \in$; $|\max \Delta Q^k| \le \in$ if equal or less the calculate line flows and power at the slack bus and exit.

8.Calculate bus currents

$^{\alpha}I_p^k = ({}^{\alpha}P_p^k - j\,{}^{\alpha}Q_p^k)/(E_p^k)^*$ p=1,2,3,….n , $p \neq s$.

9.Calculate the elements of the jacobian.

10.Solve for voltage corrections using the inverse of the given basic power voltage equation using the jacobian J.

11.Calculate new bus voltages →

$$^{\alpha}e_p^{k+1} = {}^{\alpha}e_p^k + {}^{\alpha}\Delta e_p^k \quad \text{and}$$

$$^{\alpha}f_p^{k+1} = {}^{\alpha}f_p^k + {}^{\alpha}\Delta f_p^k$$

p=1,2,3…n, $p \neq s$.

12. Replace $^{\alpha}e_p^k$ with $^{\alpha}e_p^{k+1}$ and $^{\alpha}f_p^k$ with $^{\alpha}f_p^{k+1}$ p=1,2,3…. n, $p \neq s$.

13. Advance iteration k+1 → k and go to step 4.

* Variables which do not have a α in superscript are assumed to be crisp or have been defuzzified.

III. RESULTS

To validate the proposed model the IEEE standard 6-bus test system shown in Fig. 2 has been considered, and the results so obtained are shown in Table 1.

Both the conventional and fuzzy version of the Gauss-Seidel and Newton-Raphson algorithm were carried out for 5 and 4 iterations respectively. The self-explanatory results have been shown in Table 1 and 2. The coding for both the algorithms has been carried out in MATLAB®.

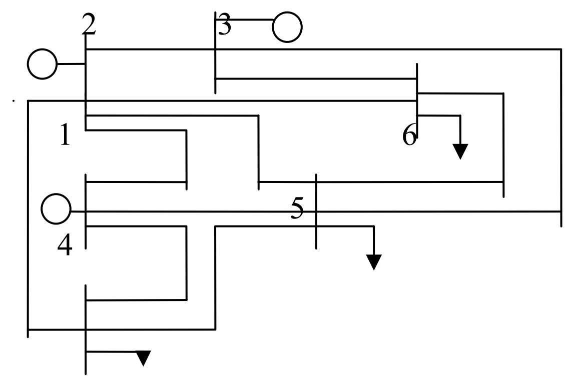

Figure 2. IEEE 6 Bus System

IV. CONCLUSIONS

The results prove that the fuzzy environment has led to results, which are very much close to the classical results. Hence it can be concluded that fuzzy modeling gives a true picture of the actual situation faced in practical power systems.

Further work to model using exact membership function is being conducted which will give more exact solutions to large systems consisting of many buses.

VII. REFERENCES

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TABLE 1

Comparison of conventional and fuzzy environment based Gauss-Seidel method.

Bus No.	Conventional Solution	Fuzzy Solution (range)	
1	1.0500	1.0300	1.0700
2	1.0191 - 0.0726i	1.0122-0.0724i	1.0191-0.0860i
3	1.0280 - 0.0860i	1.0215-0.0849i	1.0280-0.0757i
4	0.9602 - 0.0757i	0.9559 - 0.0743i	0.9602-0.0757i
5	0.9448 - 0.0936i	0.9368 - 0.0952i	0.9448-0.0936i
6	0.9534 - 0.1078i	0.9480 - 0.1061i	0.9534-0.1078i

TABLE 2

Comparison of conventional and fuzzy environment based Newton-Raphson method.

Bus No.	Conventional Solution	Fuzzy Solution (range)	
1	1.0500	1.0300	1.0700
2	1.0295 - 0.0600i	0.9944 - 0.0654i	1.0585 - 0.0575i
3	1.0433 - 0.0707i	1.0039 - 0.0783i	1.0762 - 0.0668i
4	0.9700 - 0.0678i	0.9349 - 0.0733i	0.9954 - 0.0652i
5	0.9586 - 0.0839i	0.9193 - 0.0903i	0.9854 - 0.0807i
6	0.9740 - 0.0960i	0.9315 - 0.1038i	1.0039 - 0.0924i