SEISMIC DESIGN CRITERIA FOR REINFORCED CONCRETE BUILDINGS

by

J. Ferry Borges(*)

ABSTRACT

Available information on the non-linear response of oscillators and on the non-linear behaviour of reinforced concrete elements is combined to define design criteria for reinforced concrete buildings.

Special emphasis is laid on the response of simple linear and non-linear oscillators in function of the spectral densities of the earthquakes and on the displacement capacity of reinforced concrete columns.

The design criteria are established by comparing the displacements to be expected during earthquakes with the allowable values. Complementary conditions to limit deformability and to guarantee the safety against collapse are also considered.

The above conditions are turned into design rules that are compared with the usual values based on seismic factors.

1 - INTRODUCTION

To improve seismic design of structures it is of great interest to combine the information obtained during the last years on the response of linear and non-linear oscillators to random noise vibration and on the non-linear behaviour of reinforced concrete structures.

This information is not yet complete and further studies are necessary. Even so it is already possible to derive useful rules for simple cases. The logical way these rules are obtained seems particulary important. Further studies shall allow a refinement of the basic relations or their generalization to other cases but may follow the general procedure here outlined.

The simple expression derived for the mean maximum relative displacements of oscillators in function of their frequency could be improved so as to fit better the results available and to consider several para-

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meters. This was not judged necessary due to the variability of all the factors involved.

In what concerns the behaviour of the reinforced concrete elements, the conclusions are based on results established not considering the shake-down problem. This is not so far from reality as could be thought. For a well designed structure the number of cycles in the non-linear range is small. Even so, it will be possible in the future to introduce corrections to take this problem into account.

In this paper only simple one-degree-of-freedom structures are analysed, but the conclusions can be generalized to other cases.

2 - SEISMIC ACTIONS

For the purpose of the present paper earthquake actions are represented by a random vibration of constant spectral density in the range from 0 to 5 Hz and with 30 s duration.

It was thought unnecessary to replace this representation by a more correct one, in which the spectral density is assumed to vary in function of the frequency (1) with a law analogous to that of a simple oscillator transfer function. In fact it was shown (2) that both for linear and elasto-plastic simple oscillators, the maximum mean values of the displacements do not depend much on the shape of the spectral density curve, but, mainly, on the mean value of the spectral density in the considered range. As the analysis is limited to oscillators with natural frequencies lower than 5 Hz, the hypothesis of a sharp cut off in the spectral density for frequencies higher than this is not of practical importance.

The problem of the probability of occurrence of earthquakes is not dealt with here. Even so, the randomness of this occurrence cannot be forgotten in the choice of convenient statistical magnitudes to represent the structural behaviour and in the discussion of the precision of hypotheses and results.

In fact combining the randomness of seismic loadings with that of structural behaviour it can be shown that the final probability of collapse is the probability of occurrence of earthquakes with a spectral density higher than a given value. This value is that for which collapse is produced, as a mean (3 and 4). So only the mean of the maximum values of the response are of interest. Due to the mentioned random character, with high coefficients of variation, it is also of no interest to search for results with a too high precision.

3 - RESPONSE OF A SIMPLE OSCILLATOR

It is important to relate the maximum relative displacements undergone by a simple oscillator to its natural frequency. For a simple

linear oscillator with a fraction of critical damping 0.05 acted by a random vibration of spectral density of acceleration, S_o=675 $cm^2 s^{-4} Hz^{-1}$ (that represents El Centro N-S 1940 earthquake) the mean maximum values of the displacements are related to the frequencies by curve 1 of fig. 1. For an elasto-plastic oscillator with an yielding factor, λ_y=0.10, and the fraction of critical damping indicated above, this relation is represented by curve 2. These results obtained by J. Pereira (2) confirm the well-known conclusion that the maximum displacements in elastic and elasto-plastic oscillators with different yielding factors are not much different (5, 6 and 7).

In fact, fig. 2 also indicates the mean values of the maximum displacements undergone by oscillators with fractions of critical damping 0.03 and 0.10 and yielding factors 0.03, 0.06, 0.12 and ∞, as computed by Berg and Thomaides (6) for El Centro, N-S, 1940 earthquake. Also represented in the same figure are the maximum displacements computed by the Japanese Committee SERAC (8) for a linear oscillator with a fraction of critical damping 0.05, for the same component of El Centro earthquake.

Theoretical and experimental results indicated in fig.2 can be represented with sufficient accuracy for the purpose of the present paper by the simple ratio

$$\delta_s = \frac{6}{f} \qquad (1)$$

where δ_s, expressed in cm, is the mean of the maximum displacements, due to the earthquake considered and f, expressed in Hz, is the natural frequency of the oscillator. For an earthquake of spectral density of acceleration S, other than the one considered, a reduction of the mean maximum displacement proportional to the ratio of the square roots of the corresponding spectral densities could be assumed.

4 - MAXIMUM RELATIVE DISPLACEMENT DUE TO EARTHQUAKE ACTION

The behaviour of a reinforced concrete building at a given floor is assimilated to that of a simple oscillator. The mass considered is that of the upper floors and the stiffness that of the reinforced concrete columns, fig. 2.

If all the columns are similar, with the same axial load, N, and are supposed to be perfectly built-in in the floors, the natural frequency of the system is given by

$$f = \frac{\sqrt{g}}{2\pi} \sqrt{\frac{12\,E\,I}{N\,l^3}} \qquad (2)$$

where E is the modulus of elasticity of the material and I the moment of inertia of the columns.

For rectangular columns, taking h_t = 1.1h, E = 21 000 $\sqrt{\sigma_{cu}}$ (both

E and the ultimate concrete stress, σ_{cu} , expressed in kg cm^{-2}) and $g = 980$ cm s^{-2}

$$f = 830 \frac{\sigma_{cu}^{1/4} h}{\sigma_c^{1/2} l^{3/2}} \qquad (3)$$

where $\sigma_c = \frac{N}{b\ h}$.

The natural frequency thus computed corresponds to the behaviour prior to craking. It is well-known that cracking reduces elastic stiffness. This reduction depends on several factors, namely on the percentage of reinforcement and, even, on the value of the acting moments.It is possible, starting from simple hypotheses (9), to have an idea of this reduction of stiffness and to show that in usual cases it corresponds to multiplying the elastic stiffness of the total concrete cross section by a factor ranging between 0.4 and 0.6. The natural frequency is thus reduced to about 0.63 to 0.77 of the value indicated by expression 3. Recent tests on reinforced concrete columns (10) confirmed the above values of this reduction and so the natural frequency shall be calculated by the following expression

$$f = 600 \frac{\sigma_{cu}^{1/4} h}{\sigma_c^{1/2} l^{3/2}} \qquad (4)$$

According to expression 1 the mean maximum value of the relative displacement due to an earthquake of the intensity considered is then given by

$$\delta_s = 0.01 \frac{\sigma_c^{1/2} l^{3/2}}{\sigma_{cu}^{1/4} h} \qquad (5)$$

5 - ULTIMATE DEFORMABILITY OF COLUMNS

For an elastic rectangular built-in column it is easy to show that the horizontal relative displacement of the ends in function of the difference of maximum strains at the extreme fibers, $\varepsilon = \varepsilon_1 - \varepsilon_2$, is given by

$$\delta^* = \frac{\varepsilon}{h} \frac{l^2}{6} \qquad (6)$$

If the behaviour is not elastic, a correction, k, has to be introduced depending on the non-linear diagram that relates the bending moment, M, with the difference of strains at the extreme fibers. This difference can be considered as a reduced curvature, and so the correcting factor is a function of the moment-curvature diagram.

Fig. 3 shows the reduced moment-curvature diagrams for rectangular cross sections symmetrically reinforced with steel 40 (conventional limit of proportionality at 0.2%, 40 kg mm^{-2}) and for concrete σ_{cu} = = 300 kg cm^{-2} (11). The total percentage of reinforcement varies between 0,5 and 1,5%. For other qualities of steel and concrete similar diagrams can be obtained.

By integrating these diagrams it is easy to obtain the relation, presented in fig.4, between the horizontal force, F, and the horizontal displacement, δ . In this figure the diagram corresponding to elastic behaviour having the same ultimate curvature is also presented.

For usual cases, as shown in fig.4, k can be taken as approximately equal to 0.4 and the maximum allowable transverse displacement of the column in the non-linear range can be expressed in function of the ultimate difference of the extreme strains ε_u by

$$\delta_u = 0.067 \frac{l^2}{h} \varepsilon_u \qquad (7)$$

From the condition that the strains in concrete must not exceed 3.5 °/oo, fig, 3, a relation between ε_u and n can be obtained. this relation is almost independent of the quality and percentage of reinforcement and, for the values of interest, can be expressed with sufficient accuracy by

$$\varepsilon_u = \frac{3 \times 10^{-3}}{n} \qquad (8)$$

Substituting in 7,

$$\delta_u = 0.0002 \frac{\sigma_{cu}}{\sigma_c} \frac{l^2}{h} \qquad (9)$$

an expression giving the ultimate allowable transverse displacement of a reinforced concrete column.

6 - COMPARISON OF SEISMIC AND ALLOWABLE DISPLACEMENTS

Taking as a design condition that maximum displacements due to the earthquakes, given by expression 5, must not exceed the allowable values, given by expression 9 , the following inequality is obtained

$$0.01 \frac{\sigma_c^{1/2} \, l^{3/2}}{\sigma_{cu}^{1/4} \, h} < 0.0002 \frac{\sigma_{cu}}{\sigma_c} \frac{l^2}{h} \qquad (10)$$

Simplifying and expressing σ_c in function of σ_{cu} and l

$$\sigma_c^{3/2} < 0.02 \quad \sigma_{cu}^{5/4} \, l^{1/2} \qquad (11)$$

The maximum values of σ_c given by this inequality are indicated in fig.5.

For the usual qualities of concrete, σ_{cu} = 200 to 300 kg cm^{-2}, and lengths of columns, l = 300 to 400 cm, $\sigma_c <$ 0.19 to 0.22 σ_{cu}. This condition corresponds to setting a limit to the reduced mean compressive stress $n = \sigma_c / \sigma_{cu}$.

7 - LIMITATION OF THE DISPLACEMENTS BETWEEN FLOORS

Condition 11 ensures that the ultimate allowable strain, ε_u, will not be exceeded, but does not limit the relative displacement between floors. To avoid excessive cracking, and even rupture of the partitions such a limit has to be considered.

Expression 5 , enables to define the minimum value of h in function of the magnitudes considered

$$h = 0.01 \frac{\sigma_c^{1/2} \, l^{3/2}}{\sigma_{cu}^{1/4} \, \delta} \qquad (12)$$

For δ = 1 cm and disregarding the small influence of the concrete quality by taking $\sigma_{cu}^{1/4} \simeq 4$, expression 12 becomes

$$h = 0.0025 \, \sigma_c^{1/2} \, l^{3/2} \qquad (13)$$

which is represented in fig. 6.

For other values of δ, the allowable limits for h are obtained by dividing by δ the values given in fig.6.

Expression 13 can be easily extended to the case of columns with different cross sections. Denoting by I_k the moment of inertia of the columns and by n their number, the equivalent inertia of the columns has to be similar to the value that would correspond to h_{min} in expression 13, i.e.

$$\sum_1^n I_k = n \frac{b \, h_{min}^3}{12}$$

Assuming that the mean compressive stress does not change much from column to column

$$A_t = \sum_1^n b_k \, h_k = n \, b \, h \qquad \text{and so}$$

$$\frac{\sum_{1}^{n} I_k}{A_t} = \frac{h^2_{min}}{12} \qquad (14)$$

The above condition limits the sum of the moments of inertia of the columns in one floor in any direction.

8 - SAFETY AGAINST RUPTURE

The conditions established set limits to the reduced normal stress and to the dimensions of the columns, but do not consider the percentage of reinforcement. When the ultimate deformability of columns was studied, in 5, it was shown that ultimate value of the allowable curvature was pratically independent of the percentage of reinforcement, depending mainly on the mean compressive stress. This is only true if there exists a minimum amount of reinforcement. If no reinforcement exists the ultimate strain corresponding to steel cannot be considered and the rotation capacity would be much reduced. Then the presented expressions would be no longer valid.

The analysis of fig. 4 shows that, for the interesting volues of the reduced compressive stress, n , that range between 0 and 0.2, and for the normal percentages of reinforcement, the ratio between the ultimate displacement, δu, and the displacement that would correspond to elastic behaviour is very nearly 3.5.

As this ratio cannot exceed the indicated value, the structure must resist a horizontal force of at least 1/3.5 the force that would be induced in elastic behaviour.

For elastic behaviour the seismic coefficients $c_e = \frac{F}{N}$ can very easily be obtained in function of the natural frequency of the structure. In fact, the elastic stiffness is $F/\delta = 12\ EI/l^3$ and from expression 1 and 2

$$c_e = 0.245\ f \qquad (15)$$

Considering that, due to non-linear behaviour, the ultimate moment needs only to be 3.5 smaller than the elastic one,

$$c = 0.07\ f \qquad (16)$$

The ultimate reduced moment must then be

$$m = \frac{c\ N\ l}{2\ b\ h^2\ \sigma_{cu}} = 0.035\ \frac{\sigma_c\ l}{h\ \sigma_{cu}}\ f \qquad (17)$$

and considering expression 4

$$m = \frac{21\ \sigma_c^{1/2}}{\sigma_{cu}^{3/4}\ l^{1/2}} \qquad (18)$$

which shows that the ultimate reduced moment is a function of the mean compressive stress in the concrete, the length of the column and the quality of the concrete alone. As the percentage of reinforcement is also a function of the reduced moment and normal force alone, this percentage does not depend on the geometry of the columns but only on the parameters indicated.

Taking $n = \sigma_c / \sigma_{cu}$ and $\sigma_{cu}^{1/4} \simeq 4$ as before

$$m = 5\ \frac{n^{1/2}}{l^{1/2}} \qquad (19)$$

that is represented in fig. 7.

Usual tables giving the percentage of reinforcement in function of n and m can be used to compute the percentages of reinforcement required.

For values of n between 0.19 and 0.22 and the usual lengths of columns ($300 < l < 400$ cm), m ranges between 0.11 and 0.14. So, for instance, if steel of the considered quality 40 is used, the total percentage of reinforcement for symmetrically reinforced columns must be

$$\varpi_t \simeq 2 \times 10^{-5}\ \sigma_{cu} \qquad (20)$$

For σ_{cu} equal to 200 and 300 kg cm^{-2}, expression 20 gives percentages of reinforcement of 0.4 and 0.6% respectively.

9 - CONCLUSIONS

The design criteria presented allows to define a seismic coefficient that can be compared with those of building regulations.

The coefficient c (expression 16) is a function of the natural frequency alone, fig. 8, and is 3,5 times smaller than the one corresponding to elastic behaviour.

Fig.8 indicates that for structures of very low frequency very small seismic coefficients could be adopted, but it must be considered that the limitation of relative displacements sets a limit to the minimum natural frequency. For $l = 300$ cm and $\delta < 3$ cm, expression 1 gives $f > 2$ Hz and from expression 16 a value $c > 0.14$ results.

If the deformability condition was disregarded, a new condition, represented by curve 1 in fig. 8, would be required to avoid that the moments due to the relative displacements of the structure would exceed those that can be resisted by the columns.

This shows that, in practice, if the intensity of El Centro, 1940, earthquake is taken for reference, the seismic coefficients to be considered must vary, according to the natural frequency of the structure, between about 0.15 and 0.35.

The conditions represented by expressions 11, 12 and 19 enable a direct determination of the maximum allowable reduced compressive stress in the concrete, the minimum height of the columns and the minimum percentage of reinforcement respectively and can be directly used in the design of reinforced concrete buildings of the considered type.

REFERENCES

1. Ravara, A. – Spectral Analysis of Seismic Actions , Paper to be presented at the III World Conference on Earthquake Engineering, New Zealand, 1965.

2. Pereira, J.Jervis – Behaviour of an Elasto-Plastic Oscillator Acted by Random Vibration, Paper to be presented at the III World Conference on Earthquake Engineering, New Zealand, 1965.

3. Borges, J. Ferry — Statistical Estimate of Seismic Loadings, Preliminary Publication, V Congress of the International Association for Bridge and Structural Engineering, Lisbon, 1956.

4. Borges, J.Ferry — Structural Behaviour and Safety Criteria, Preliminary Publication, VII Congress of the International Association for Bridge and Structural Engineering, Rio de Janeiro, 1964.

5. Veletsos, A.S. and Newmark, N.M. – Effect of Inelastic Behaviour on the Response of Simple Systems to Earthquake Motions, II W.C.E.E., Tokyo, 1960.

6. Berg, G.V. and Thomaides, S.S. – Energy Absortion by Structures in Earthquakee, II W.C.E.E., Tokyo, 1960.

7. Blume, J.A., Newmark, N.M and Corning, L.H. – Design of Multistory Reinforced Concrete Buildings for Earthquake Motions, Portland Cement Association, Chicago, 1961.

8. SERAC – Non-linear Response Analysis of Tall Buildings to Strong Earthquake and its Application to Dynamic Design. Report n°.4, Tokyo, 1964.

9. Borges, J.Ferry – Non-linear Analysis of Reinforced Concrete Structures, Publications, International Association for Bridge and Structural Engineering, Volume 23, Zürich, 1963

10. Borges, J.Ferry, Pereira, J.Jervis, Ravara, A. – Comportamento do betão armado em regimen dinâmico. Relatório interno, Laboratório Nacional de Engenharia Civil, Lisboa, 1963.

11. Borges, J.Ferry, Arga e Lima, J., Coelho, J.T. and Monteiro, V. – – Studies on the Non-linear Behaviour of Reinforced Concrete Structures, Report, Laboratório Nacional de Engenharia Civil, June, 1964.

NOMENCLATURE

b - width of a rectangular section

c - seismic coefficient

f - natural frequency of an oscillator

g - acceleration of gravity

h - depth of a reinforced concrete rectangular section (distance between the center of gravity of the tensile reinforcement and the opposite face)

h_t - total depth of a rectangular section (distance between opposite faces)

k - correction factor

l - length of a column

$m = \dfrac{M}{b\ h^2\ \sigma_{cu}}$ - reduced bending moment

$n = \dfrac{N}{b\ h\ \sigma_{cu}}$ - reduced compressive force

q_y - yielding factor

A - reference cross-sectional area of concrete

A_a - total cross-sectional area of steel

E - modulus of elasticity of concrete

F - horizontal force

I - moment of inertia of the cross section of a column

M - bending moment

N - normal force

S - power spectral density of acceleration

δ - relative displacement

δ_s - relative displacement due to earthquake action

δ_u - ultimate relative displacement

δ^* - elastic relative displacement

$\varepsilon = \varepsilon_1 - \varepsilon_2$ difference between strains at extreme fibers

ε_c - strain in concrete

ε_s - strain in steel

ε_u - ultimate strain difference

$\sigma_c = \frac{N}{b\ h}$ - mean compressive stress

σ_{cu} - ultimate compressive stress determined in cylinder tests

$\varpi_t = \frac{A_a}{b\ h}$ - total percentage of reinforcement

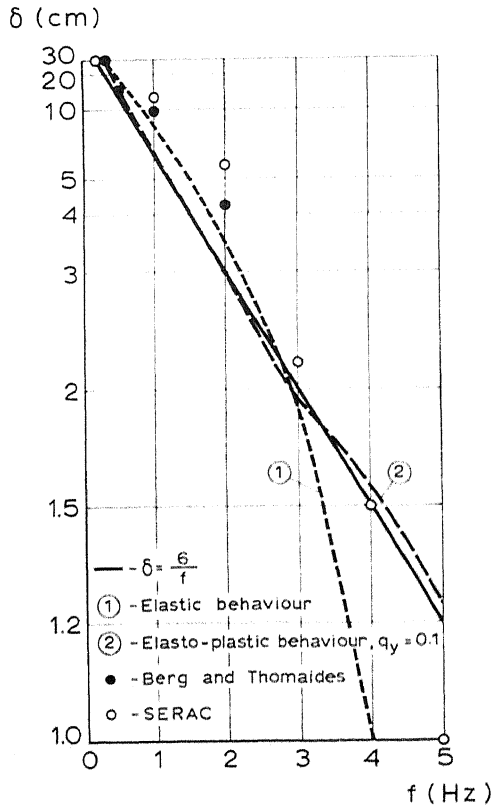
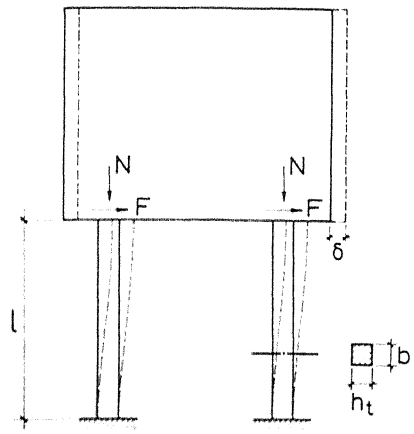


Fig. 1



$$\delta = \frac{F\,l^3}{12\,E\,I}$$

Fig. 2

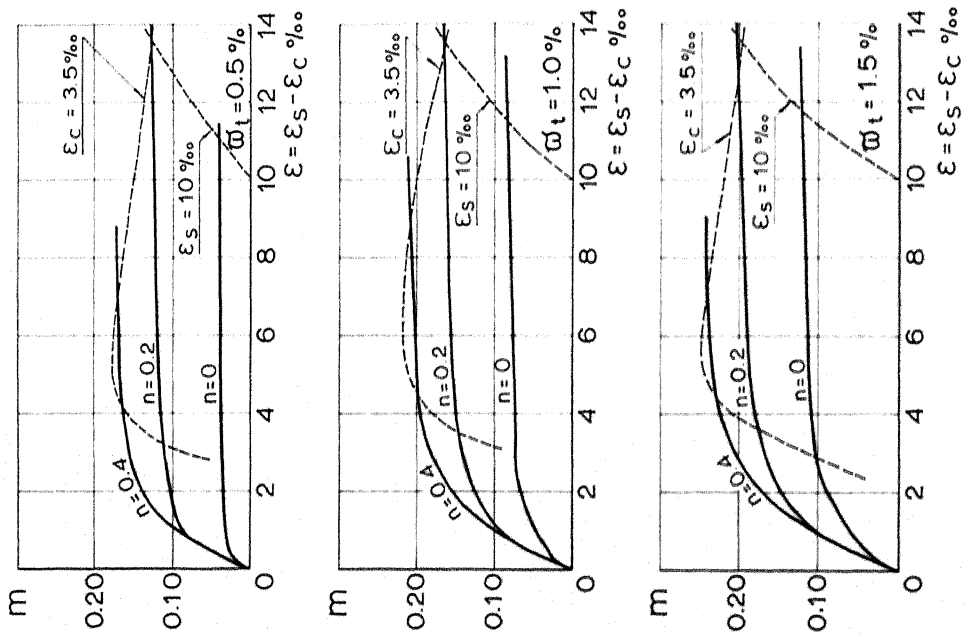


Fig. 3

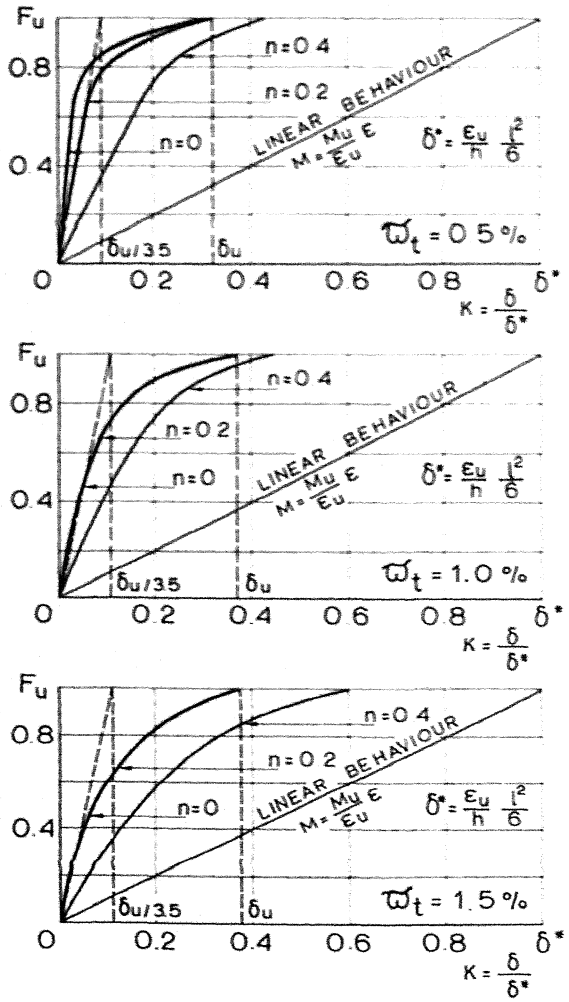

Fu
0.8
0.4
0
0.2
0.4
0.6
0.8
δ*
K = δ/δ*
n=0.4
n=0.2
n=0
LINEAR BEHAVIOUR
M = Mu/εu ε
δ* = εu/h l²/6
δu/3.5
δu
ϖt = 0.5 %
ϖt = 1.0 %
ϖt = 1.5 %

Fig. 4

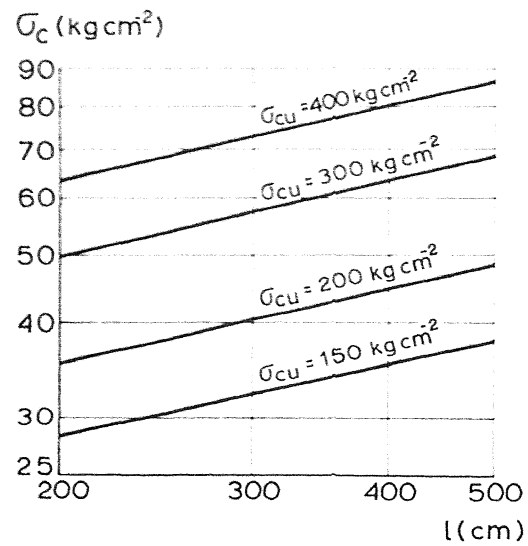

σc (kg cm⁻²)
90
80
70
60
50
40
30
25
200
300
400
500
l (cm)
σcu = 400 kg cm⁻²
σcu = 300 kg cm⁻²
σcu = 200 kg cm⁻²
σcu = 150 kg cm⁻²

Fig. 5

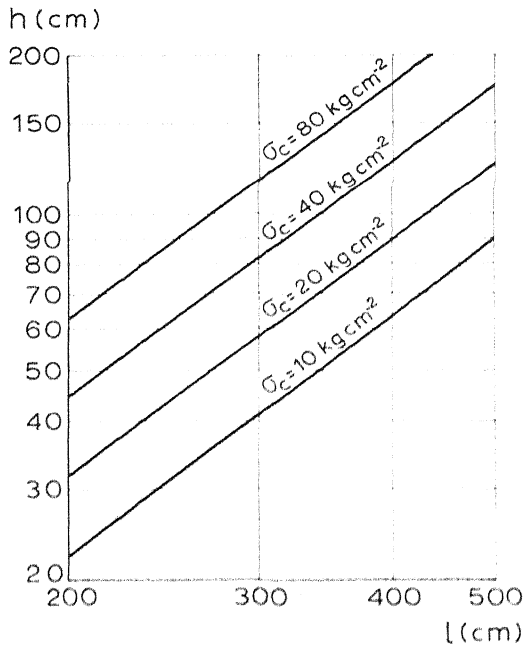


Fig. 6

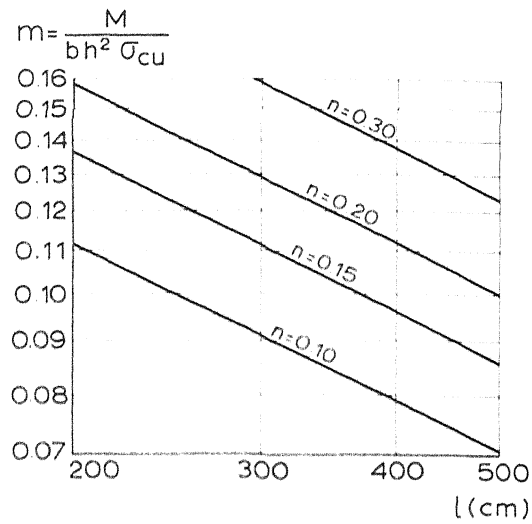


Fig. 7

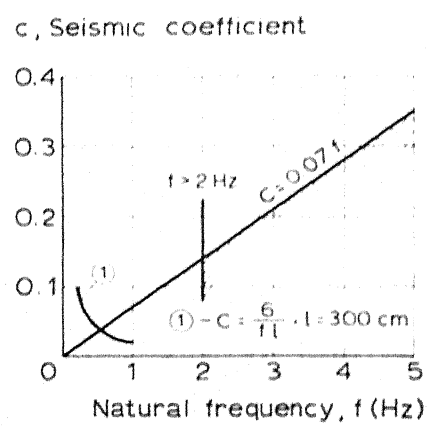


Fig. 8

SEISMIC DESIGN CRITERIA FOR REINFORCED CONCRETE BUILDINGS

BY J.F. BORGES

QUESTION BY: E. Del VALLE - MEXICO

I think that the assumption of fixed ends for the columns is not a very real one, as the trends in architecture are to reduce the depth of the beams rather than to increase it, wherefrom the stiffness at the ends of the columns is largely reduced from the fixed-end assumption. Have you extended your studies to include such a condition?

AUTHOR'S REPLY: The hypothesis of perfect built-in columns was considered as an extreme case. I agree that it is of much interest to study other conditions, and particularly to consider the deformability of the beams.

QUESTION BY: O.A. GLOGAU - NEW ZEALAND

In addition to the effect of beams in relieving the columns mentioned in the previous question would the author say that in buildings that do not have such a sudden change in stiffness more joints can take part in the energy absorption and we get a different case from the one presented?

AUTHOR'S REPLY: The behaviour of real buildings is in general more involved than the one assumed for design purposes. If there are supplementary connections their effects are on the safe side.

QUESTION BY: A. TORK - NEW ZEALAND

1. What are the allowable strains?
2. How is the minimum ductility factor of 3.5 guaranteed? What is the type of reinforcing for this? Are experimental data available to demonstrate that columns can take many reversals of loading into a ductility range of 3.5?

AUTHOR'S REPLY:

1. It was assumed that ultimate values of the strains in concrete and steel must not be surpassed and from this the limit values of the difference of the strains, at extreme fibers (that correspond to ultimate values of curvature) were obtained. These values were used to determine the allowable displacements.

2. For built-in columns the ratio between the ultimate displacements and that which for the same load, would correspond to elastic behaviour is not much influenced by the qualities of concrete and steel. The adopted value of 3.5 seems to be suitable for practical applications. Concerning the number of reversals of loading in the ductility range, it must be considered that during an earthquake this number is always small. Some model tests are being performed for studying the influence of these reversals.

QUESTION BY: E. ROSENBLUETH - MEXICO

The following additional matters seem worthy of consideration in buildings of the type studied in this paper.

1. Shearing stresses will also limit the column dimensions under some conditions.
2. Even in nominally symmetric buildings the possibility of accidental torsion is ever present, chiefly because of unavoidable asymmetry of stiffness and strength. The physical model tests offer an excellent opportunity for a study of this question.
3. Additional bending moments in the columns are caused by vertical loads acting with the eccentricity of lateral displacements. Simultaneously, the axial forces on the columns oscillate because of the overturning moment and in tall buildings these forces may temporarily become tensions.

It is hoped that the author will include these matters in his method of design. Perhaps he has already given them thought.

AUTHOR'S REPLY:

1. Concerning the shearing strength of columns, I perfectly agree that it must be considered and this can be done by conventional methods for the defined actions.
2. Torsion effects may in many cases be, in fact, of paramount importance. The general principles presented would also apply to these effects.
3. The additional bending of columns due to the lateral displacements was considered and it was seen that it is only important for structures of very low natural frequency, say smaller than 0.5 c.p.s.

In the presented paper several simplifying hypotheses had to be considered to arrive at results of practical

interest, but it is hoped that in the near future the same lines of thought may be followed for more general cases.

COMMENT BY: J.A. SBAROUNIS - U.S.A.

Mr. Sbarounis pointed out that Dr. Borges' paper referred to "unconfined" concrete rather than that normally considered "confined" by stirrups or column ties.