EXISTENCE OF THE CRITICAL SURFACE IN EARTH DAMS DURING EARTHQUAKE

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SYNOPSIS

In the first part of the paper authors present a new method of superposition to evaluate the potentials in the earth dam during earthquake. Second part deals with the stability of an earth dam with upstream inclined filter during earthquake and rapid drawdown using the method of slices and Bishop's Method and Conclude that this type of design creates a condition whereby the critical failure surface remains the same during rapid drawdown as well as during earthquake and is independent of the soil characteristics and is the function of the upstram slope only. The design chart is presented too.

1. In recent times efforts have been made to locate a possible critical failure plane during drawdown for materials which are freely draining as well as materials which are not freely draining (highly impervious one). In such cases, the pore pressure variation should be reduced to minimum (2,3,7). This could be achieved by locating an upstream inclined filter with a slope of one vertical to two horizontal (1:2) at -0.6H from the axis of the dam (Fig. 1). The axis of the dam is referred as a vertical line passing through a point where full reservoir level (H) meets the upstream slope (3). The calculations were based on the use of slip circle method. It has been shown that in a typical earth dam, even assuming idealized elastic properties for the soil, local overstress will occur when the factor of safety by the slip circle method lies below a value of 1.8 (1). As the majority of stability problems involve factor of safety lower than this, a state of plastic equilibrium must be considered to exist throughout some of the slope. This lead to further investigation of stability using Bishop's Method (1) where the factor of safety can be given by

$$F_s = \sum\left[\left\{\frac{c'.b}{\gamma_{sat}} + (h-u/\gamma_{sat})\tan \phi'\right\} K.\sec \alpha\right] / \sum h.\sec \alpha \quad .. 1$$

which for the critical equilibrium (F_s=1) can be written as (6,7)

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$$\frac{c'}{H \cdot \gamma_{sat}} = \left[1 - \tan \phi' \frac{\sum (h - u/\gamma_{sat})\ K.\sec \alpha}{\sum h \sin \alpha}\right] / B_2 \quad .. \quad 2$$

where

h = height of the slice above the arc of the slip circle

α = the angle between the centre line of the slice and the radius of the slip circle at the arc of slice

b = width of the slice

$1/K = \tan \phi' \cdot \tan \alpha / F_s$

ϕ and c' the angle of internal friction and cohesion in terms of effective stresses

$B_2/H = \sum b \cdot K \cdot \sec \alpha / \sum h \cdot b \cdot \sec \alpha$

This analysis conclusively showed that the same zone represents the possible failure surface (3,7) (Fig.2). It is worthwhile to investigate if the same region represents the critical surface during earthquake also.

While determining the increase of water pressure on the upstream slope of the earth dam due to earthquake, it was assumed that water and soil particles are incompressible. Due to earthquake, when the dam moves downstream the water in the reservoir also flows towards the upstream slope of the dam. This is assumed to be laminar, Darcy's Law being valid; also Laplace Equation holds good. With water in contact with the upstream face, the dam moves towards its original position from its farthest downstream position. This is assumed to give maximum pressure on the upstream face. It has been shown by Zanger that the flow of water due to earthquake, considering water as incompressible satisfies the equation

$$\frac{\partial^2 p_e}{\partial x^2} + \frac{\partial^2 p_e}{\partial y^2} = 0 \quad .. \quad 3$$

which is Laplace Equation and was solved by electrical analog method for various upstream slopes and for various horizontal earthquake intensities (Fig. 3). The earthquake intensity (α) is defined as the ratio of acceleration due to earthquake to that due to gravity. It is a general practice to consider horizontal component of earthquake acceleration in case of earth dams only as the vertical component has practically no effect.

2. EFFECT OF INCREASED WATER PRESSURE ON PORE PRESSURE

The present practice is to transfer the increased pressure on the upstream face to the failure plane along its line of action which is at right angles to the upstream face (Line AC' of Fig. 4). Thus increase in the pressure at A will result in the same increase at the point 'C' on the failure plane. In case of highly pervious material, any change in the pressure at the upstream boundary will result in the instantaneous change in the pressure in the dam and a steady stress condition will be established instantaneously. The stress can be computed using Laplace Equation. In case of highly impervious material, sudden change in the pressure on the upstream boundary will not result in the steady condition inside the earth dam instantaneously, The increase in pore pressure, in such a case, can be coputed by the method of superposition using the following assumptions (4,5,6)

1. Instantaneous loading of saturated cohesive soil does not bring about sudden consolidation.

2. Increased water thrust due to earthquake is transmitted instantaneously throughout the body of the dam as the pressure is transmitted at the velocity of sound in water which becomes infinite when water is considered incompressible. The transmission of excess pressure would take place instantaneously, since the force transmitting it is the mass of earth dam acted upon by the earthquake acceleration. This is quite large and the celerety of the wave will be much more than the velocity of seeping water.

3. Soil being highly impervious does not permit the flow lines to change their paths instantaneously. The potentials, however, are changed due to sudden transmission of increased pressure. Thus, at any point on a flow line the new potential equals full reservoir potential plus the increase in the pressure at the starting point of this flow line.

It can be seen that the existing practice to transfer the increased pressure at right angles to the upstream face can neither be applied to pervious nor impervious soils. Instead, the method of superposition presented here should be used for highly impervious materials. The difference between these two approaches reduce by bringing the upstream inclined filter near to the earth dam axis and should be considered as the most scientific one. The equipotential lines were drawn for upstream slope of 1:2, 1:2.5 and 1:3 with the upstream inclined filter shown in Fig. 1 for full reservior condition with the help of electric analog model. Using the method of superposition, new potentials for different earthquake intensities were calculated.

3. STABILITY ANALYSIS

As already explained the earth dam with upstream inclined filter (1:2) at -0.6H from the axis of the dam represents the most scientific design for drawdown condition (Fig. 1). It was further shown that the critical slip circle passes through the upstream toe in this type of design. The same will be analysed here for the stability during earthquake to investigate if the critical plane lies in the same zone. The slip circle method being very simple and giving the lower factor of safety will be employed here. Following assumptions were made for the simplification of the calculations:

1. The reservoir water level is upto the height of the dam. In practice it will be little below the top of the dam. Thus the atual thrust due to water will be less than that considered here.

2. Material above the phreatic line (top seepage line) is considered to be saturated due to capillary rise.

3. The crest width (top width) of the dam extends horizontally till it is cut by the slip circle.

4. Small portion of the circle passing below the base of the dam has the characteristics as that in the earth dam.

The factor of safety for the cylindrical failure surface with horizontal earthquake intensity α can be written as:

$$F_s = \left[(\sigma - u - \alpha\tau)\tan \phi' + c \cdot l + P \cdot l_a / R \right] / (\tau + \alpha\sigma) \quad .. \quad 4$$

where

F_s = Factor of safety, R = Radius of the slip circle

$\sigma = F_\sigma \cdot H^2 \cdot \gamma_{sat}$ = normal stress

$\tau = F_\tau \cdot H^2 \cdot \gamma_{sat}$ = tangential stress

$u = F_u \cdot H^2 \cdot \gamma_w$ = pore pressure

$P = F_p \cdot H^2 \cdot \gamma_w$ = Resultant water thrust on the upstream face of the earth dam

$l = d \cdot H$ = length of the arc of slip circle

l_a = lever arm for the resultant water thrust from the centre of the slip circle.

$\alpha = \dfrac{\text{acceleration due to earthquake}}{\text{acceleration due to gravity}}$

γ_{sat} = saturated unit weight of the soil in the dam

γ_w = unit weight of water (taken as one)

H = Height of the earth dam

Dividing eqn. 4 by $F_\tau \cdot H^2 \cdot \gamma_{sat}$ and putting factor of safety equal to unity for the critical equilibrium and after rearranging, it can be written as:

$$\frac{c' \cdot B_3}{H \cdot \gamma_{sat}(1+\alpha \cdot B_1)} = 1 - \tan \phi' \frac{B_1 - B_2/\gamma_{sat}}{1 + \alpha \cdot B_1} + \frac{B_4/\gamma_{sat}}{1 + \alpha \cdot B_1}$$

or

$$\frac{c'}{H \cdot \gamma_{sat}} = \frac{1 + \alpha \cdot B_1}{B_3} \left[1 - \tan \phi' \frac{B_1 - B_2/\gamma_{sat}}{1 + \alpha \cdot B_1} + \frac{B_4/\gamma_{sat}}{1 + \alpha \cdot B_1} \right] \quad .. \quad 5$$

where

$$B_1 = F_\sigma/F_\tau \quad , \quad B_2 = F_u/F_\tau \quad , \quad B_3 = l_a \cdot H/F_\tau \cdot H^2$$

$$B_4 = (F_p/F_\tau) \cdot (l_a/R)$$

The left hand side of the equation 5 is a non-diamentional number and is very similar to Taylor's Stability Number for dry slopes (3) with the difference that the right hand side in the present case has a term γ_{sat} which varies with the soil. The stability number for various slip circles using the average saturated unit weight of the soil of 2 tons/cu.m. were calculated for upstream slopes of 1:2, 1:2.5 and 1:3 taking number of slip circles and contour lines for the same were drawn. Fig. 2 shows the contour lines of the stability number for various slip circles for the upstream slope of 1:2.5 for tan ϕ'= 0.2 for (a) rapid drawdown using slip circle method, (b) rapid drawdown using Bishop's Method (1), (c) earthquake with α=0.05 and tan ϕ'=0.1 and (d) earthquake for α=0.15 and tan ϕ'=0.2. It can be seen that in all cases the nature of the contour lines of the stability numbers are same and the critical circle lies very near to the one during rapid drawdown case. The postion of these are shown in Fig. 1.Looking to the eqn. 5 it can be seen that the stability number will depend upon the value of saturated unit weight in the right hand terms of the equation. The saturated unit weight of the soil will vary between 1.8 to 2.2 tons/cu.m. For all practical purpose, an average value of 2.0 tons/cu.m. can be taken for calculations. Effect of the change in the unit weight of saturated soil on the location of the critical slip circle is investigated in details for the rapid drawdown case in freely and non-freely draining materials and is conclusively established that the change in the unit weight has no effect on the location of the critical slip circle (3). The stability number given in Fig. 5 can be used for selecting preliminary section of the earth dams. This can be better understood by taking one example.

EXAMPLE

Calculate the critical height of an earth dam with upstream slope of 1:2.5 and earthquake intensity of $\alpha = 0.05$, 0.1 and 0.15 for the following soil characteristics:

$$\tan \phi' = 0.2,\ c = 7.5 \text{ tons/sq.m.},\ \gamma_{sat} = 2.0 \text{ tons/cu.m.}$$

Using Fig. 2 and 5 it can be tabulated as follows:

	Rapid Drawdown		Against earthquake		
	slip circle Method	Bishop's Method	α=0.05	α=0.1	α=0.15
$c'/H\gamma_{sat}$	0.08	0.066	0.057	0.082	0.106
critical height in meters	46.9	56.8	66.00	46.00	35.50

4. CONCLUSIONS

Following conclusions can be drawn

1. The centres and the arcs of the critical slip circle during earthquake lie very near to those of rapid drawdown case leading to the conclusion that the same zone represents the critical failure surface during rapid drawdown as well as during earthquake.

2. Looking to Fig. 5, it can be seen that the difference in stability number for earthquake with low intensity and that for rapid drawdown case is not appreciably great especially for soils having low angle of internal friction.

3. The variation in the stability against the angle of internal friction is approximately constant in case of earthquake.

Effort is made to present the critical height of saturated earth slopes during earthquake in a full reservoir case rather than to present the factor of safety as the factor of safety depends largely on the factor of ignorance and the courage to design and on the construction authority. This type of design with upstream inclined filter or thin clay cores sloping upstream and placed near the upstream slope needs more investigation and encouragement. A slope protection of about 3 meters thickness consisting of pervious material should be provided on the upstream side of the dam to avoid the formation of cracks due to wetting and drying cycles in the clayey soils. Necessary thickness of stone pitching on the upstream face should be provided to stand against the wave action.

5. REFERENCES

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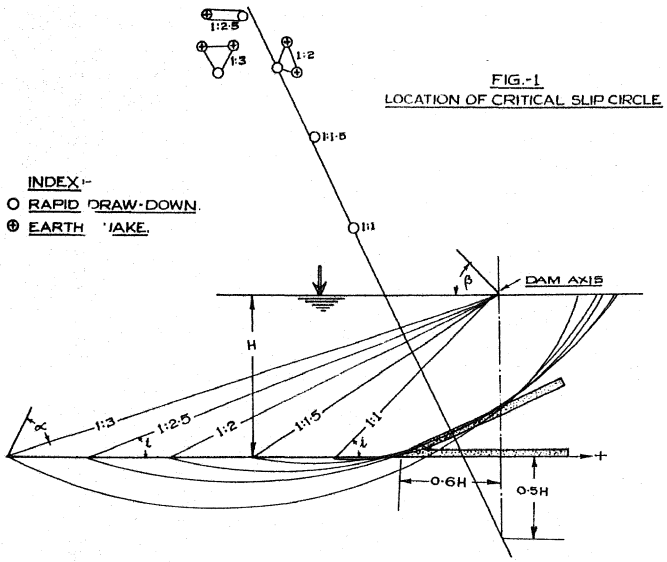


FIG.-1
LOCATION OF CRITICAL SLIP CIRCLE

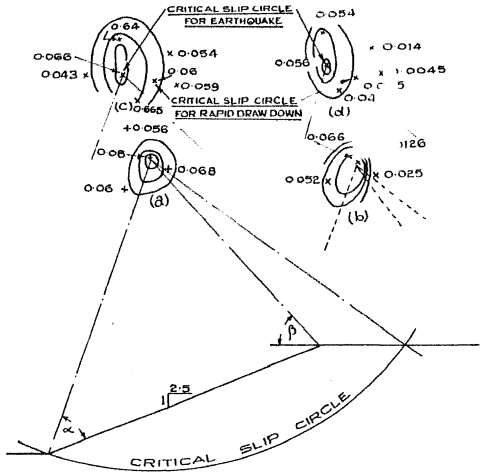


FIG.2.
a). RAPID DRAW-DOWN SLIP CIRCLE FOR $\tan\phi = 0{\cdot}2$
b). RAPID DRAW-DOWN, BISHOP'S METHOD $\tan\phi = 0{\cdot}2$
c). EARTH QUAKE, $\tan\phi = 0{\cdot}1$, $\alpha = 0{\cdot}05$.
d). EARTH QUAKE, $\tan\phi = 0{\cdot}2$, $\alpha = 0{\cdot}05$.

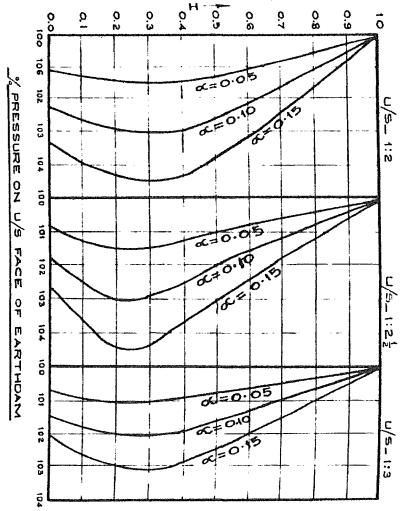


FIG. 3.

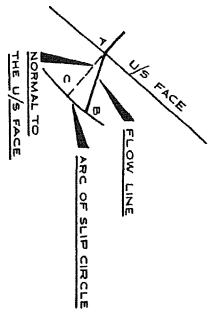
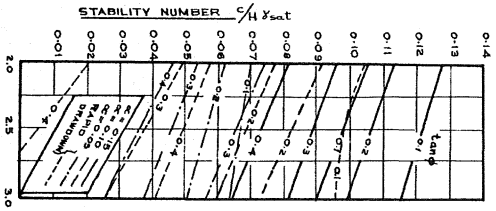


FIG. 4.



STABILITY NUMBER FOR DIFFERENT UPSTREAM SLOPE FOR DIFFERENT VALUE OF tan φ

FIG. 5.