SEISMIC EFFECTS ON EARTH FILL DAMS

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The paper deals with the effects due to dynamic seismic loads on earth fill dams. A stationary wave process and the action of a travelling wave are examined. The effects, resulting from the arising of plastic deformation zones, on the dam seismic resistance are shown. The conditions stimulating unstable vibration are defined.

1. Statement of Problem

A theory for calculation of effects resulting from horizontal seismic movements was developed in detail by the Soviet scientists Medvedev,Nazarov,Kirchinsky (1,2,3) et al. with respect to high structures of relatively small plan sizes. According to one of the authors (4,5), of a paramount importance turns to be a load arising at the moment when a surface wave passes under the toe.

An interaction of the surface wave and the structure leads to a redistribution of reactions along the toe with the resulting loads which change the safety and stability fsctors of the structure.

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2. Method of Analysis

A general method for analysis of dynamic loads acting upon the dams was developed by the author in his earlier works (6); in the present paper an application of this method for calculating various types of earth fill dams is shown.

While compiling the calculation scheme we assume the dam to be a system with several degrees of freedom, a total distributed mass of the dam is replaced by a number of concentrated masses located at the nodes of a grid plotted on the profile. The problem can be solved in terms of displacements for which purpose differential equations for motion of each concentrated mass are designed. This results in a system of differential equations of the following type:

$$y_1 = -m_1 \delta_{11} \frac{\partial^2 y_1}{\partial t^2} - m_2 \delta_{12} \frac{\partial^2 y_2}{\partial t^2} - \ldots - m_k \delta_{ik} \frac{\partial^2 y_k}{\partial t^2} + P_i(t) \Delta_{1p}$$

$$y_k = -m_1 \delta_{k1} \frac{\partial^2 y_1}{\partial t^2} - m_2 \delta_{k2} \frac{\partial^2 y_2}{\partial t^2} - \ldots - m_k \delta_{kk} \frac{\partial^2 y_k}{\partial t^2} + P_i(t) \Delta_{kp} \qquad (1)$$

Here $y_1, y_2, y_3 \ldots y_k$ are generalized displacements of masses; $\delta_{11}, \delta_{12}, \delta_{13}, \ldots \delta_{ik}$ - generalized unit displacements from unit forces applied to corresponding masses (e.g. δ_{12} means a displacement of m_1 mass due to unit force applied to m_2 mass); $P_i(t)$ is an external force applied to m_1 mass; $\Delta_{1p}, \Delta_{1p}, \ldots \Delta_{kp}$ are displacements of $m_1, m_2 \ldots m_3$ masses due to force $P_i = 1$.

Solution of the equation (1) may be written as follows:

$$y_k = \rho_{k1} A_1 \sin \omega_1 t + \rho_{k2} A_2 \sin \omega_2 t + \ldots + \rho_{kk} A_k \sin \omega_k t \qquad (2)$$

where ρ_{k1}, ρ_{k2} are the ordinates of main vibration modes; A_1, A_2, A_3 are the amplitudes of main vibration modes and ω_1, ω_2, ω_3 are natural frequencies.

To find frequencies, an equation of frequencies is made up by setting the determinant of the system of homogeneous equations, corresponding to (1), to zero.

The ordinates of the main vibration modes ρ_{ki} are the ratios of the vibrational amplitudes of separate masses; they are to be found from the homogeneous system of equations (1) for each frequency.

Thus, frequencies and ordinates of the vibration modes are determined in a routine way. For calculating the amplitudes A_k the following formula is used:

$$A_k = (A_{k1} v_1 + A_{k2} v_2 + A_{k3} v_3 + \dots) \frac{1}{\omega_1} \tag{3}$$

In this formula, A_{k1}, Ak_2 ... are the displacement amplitudes produced by unit velocities applied to separate masses. We can pass over from velocities V_1, V_2 to the impulses $S_k = V_k/m_k$ and forces, taking into account that $dS_k = P_k dt$.

Then we shall have the following formulas for the mass displacements:

$$y_1 = \left(A_{11}\frac{S_1}{m_1} + A_{12}\frac{S_2}{m_2} + \dots\right)\frac{\sin\omega_1 t}{\omega_1} + \left(A_{21}\frac{S_1}{m_1} + A_{22}\frac{S_2}{m_2} + \dots\right)\frac{\sin\omega_2 t}{\omega_1} + \dots \tag{4}$$

$$y_1 = \frac{1}{\omega_1}\left\{\int_0^t \left[\frac{A_{11}}{m_1}P_1(u) + \frac{A_{12}}{m_2}P_2(u) + \dots\right]\sin\omega_1(t-u)\,du + \int_0^t \left[\frac{A_{21}}{m_1}P_1(u) + \frac{A_{22}}{m_2}P_2(u) + \frac{A_{23}}{m_3}P_3(u) + \dots\right]\sin\omega_2(t-u)\,du + \dots\right\} \tag{5}$$

The formulas (4) and (5) allow the motion of separate masses of the dam profile to be studied and its state of deformation at various moments of time to be plotted.

To analyse the state of stress of the dam, it is a good practice to determine the effect of individual parameters on the result, for which purpose the calculation is made for a unit effect when the force $P_k = 1$ is applied instantaneously to m_k mass at $t = 0$ and then keeps its value constant up to $t = \infty$. We can proceed to another force by means of a step approximation. From a unit effect for calculating the deflections we shall have:

$$y_{ki} = \sum_{k=1}^{k=n} \int_0^t \sum_{i=1}^{i=n} \frac{1}{\omega_k} \left\{ A_{ik}\, p_{ik}\, \frac{p_i(u)}{m_k} \sin \omega_k (t-u)\, du \right\} \tag{6}$$

From the deflections it is possible to pass over to forces using the following equations:

$$Q_{ki} = a_{i1} y_1 + a_{i2} y_2 + a_{i3} y_3 + \cdots + a_{in} y_n \tag{7}$$

Now the force Q_{ki} depends on the time. To make a practical use of the theoretical results, the formulas (6) and (7) are employed to calculate Y_{ki} and Q_{ki} for various fixed moments of time $t = \alpha T_1$ (α = 0.1; 0.2 ...), expressed as the fractions of the basic vibration period. The flexural moments of the reactions of the base and other static factors are determined according to the formulas

$$M_{ki} = \Sigma\, \eta_{ki} Q_{ki} \quad \text{and} \quad P_{ki} = \Sigma\, \xi_{ki} Q_{ki} \tag{8}$$

The factors η_{ki} and ξ_{ki} are the ordinates of the influence lines of moments or reactions.

The method developed allows us to study the state of the stress of a dam both within the elastic zone and beyound it when separate sections of the profile would be displaced and residual deformations arise.

3. Dam of Triangular Profile

The dams constructed of local materials have usually a symmetric triangular profile. Two cases should be distinguished when estimating seismic stability of such dams. With small amplitudes of the seismic waves the profile displacements will remain under the elastic limit, no residual displacements occur and, therefore, the dam profile can be treated as an elastic triangular wedge. To make a dynamic analysis of the triangular wedge, use can be made of differential equations as applied for a two-dimensional problem of the elastic theory which are solved by means of finite-differences method. Subsequently, for plotting the strained state of the profile corresponding to the given fixed moment of time, it is necessary to solve a system of finite-difference linear equations of the following form:

$$\rho \frac{(u_I - 2u_0 - u_{II})}{\Delta t^2} = (\lambda + 2\mu) \frac{(u_3 - 2u_0 - u_1)}{\Delta x^2} +$$

$$+\mu \frac{(u_2 - 2u_0 - u_4)}{\Delta y^2} + (\lambda + \mu) \frac{(v_3 - v_6) + (v_7 - v_8)}{2\Delta x \, 2\Delta y} \qquad (9)$$

For the vertical component of the seismic wave the strained state of the profile takes the form shown in Fig.1. The strained state is dependent on the wave length(4), viz.a more lengthy wave is capable of straining a rather greater section of the profile. A short wave mainly changes the state of the stress of a profile near the toe. The stress diagramm σ_k which arises in the vertical section on the symmetry axis of the profile turns to be of two-sign character which results in the fact that in the middle of the profile height tensile stresses appear which are likely to cause cracks. An analysis of the strained state of the profile shows that to increase its seismic stability, a section at the bottom of the inclined edge of the profile should be reinforced.

A strained state of a triangular profile for the horizontal component of the seismic wave has been studied in the works (7) and shown in Fig.2.

An examination of this scheme shows that maximum deformations appear at the upper part of the profile. An increase in the size of the profile slopes results in a decrease of the vibration amplitude of the apex adding, thus, to the seismic stability of the dam.

When the horizontal and the vertical components of the seismic wave act simultaneously a strained state of the profile can be obtained by a method of superposition. This is shown in Figs.3 and 4. To achieve a higher seismic stability of the earth fill dams, the toe of the slopes should be so reinforced that they be able to receive considerable vertical displacements without being destroyed. Construction of the flat slopes increases the seismic stability of the dam against the horizontal seismic load.

The rigidity of the dam apex should be also increased in horizontal sections since the combined action of the horizontal and vertical components of the seismic wave will result in a considerable rise of the shearing stresses in the horizontal sections. The diagrams of the shearing stresses are shown in Fig.5.

4. Formation of Plastic Zones

For excessive seismic efforts the analysis of the dams becomes more complicated since the links between separate parts or members of the profile are disturbed and residual deformations appear. When dealing with earth fill dams and stone fill dams of a pronounced importance is their specific feature, which we observed as early as 1942 while investigating on small models the effect of the travelling of the impact impulse through a layer of sand. The investigation revealed an interesting phenomenon: with low values of the impact loose bodies behave as elastic ones only when there is quite definite interrelation between the rigidities of the system. If the interrelation of rigidities is altered up to a certain critical value, considerable shears in the stratum of sand take place and the deformations rise markedly. The system behaves as if it were losing its stability. If we proceed to change the interrelation of rigidities in the same direction, the system will again behave as an elastic one but its rigidity factor changes. As the rigidity of the system and the value of the outer pulse change there appear three distinctive ranges where the dam profile works:

1. The range corresponding to an elastic behaviour of the system with a low outer pulse.

2. The range characterized by appearance of shears in loose medium and by appearance of large dislocations in the dam body. This can be briefly termed as a range of unstability.

3. The range corresponding to a loose medium where particles have got overcompacted which resulted in somewhat higher rigidity. Such a system can be designed by applying a method of a dynamic analysis described in Sec.2 for a system with several degrees of freedom; however, the relation between deformations and stresses will be no longer linear. This nonlinear law correlating stresses and deformations has been obtained from specially-organized experiments.

5. Unstable Mode of Vibrations

To draw general conclusions referring to the estimation of the dam stability, we think it possible to substitute a stepped law, comprising three linear sections (Fig.6), for a nonlinear law of the change of the system rigidity as described in the paper (8).

Now the problem can be solved with the aid of the so-called "skeleton curves" which correspond to the appearance of the resonance and a considerable rise in displacements between the members of the system.

As is known, when dealing with a linear system of one degree of freedom the "skeleton curves" are in fact straight lines which are parallel to an axis of ordinates. These are shown for each frequency with dotted lines in Fig.7. On the diagrams, the ratios of frequencies are plotted as abscissas, and the amplitudes as ordinates. When in the process of movement, the rigidity of the system drops the "skeleton curve" bends first to the left and then to the right following a rise in the rigidity of the system. This is accompanied with an appearance of a range of unstable movement and shears.

The estimation of the general stability of the dam can be made, as a first approximation, determining the relationship which characterizes physical properties of the dam and constructing the stresses - a strain diagram of the system considered. This diagram is then to be approximated with the three linear sections as shown in Fig.6. This simplified diagram is used for determination of three values of frequency corresponding to three different sections of the diagram. Thus, a band within which the "skeleton curve" of the whole system can be plotted and the range of unstable movement corresponding to this range may be obtained. This stage of investigation refers to studying the properties of the dam profile and allows us to answer the question whether the dam is stable or not under the given seismic action. A study should be made of separate harmonics which make up a spectrum of a seismic wave and refer to the vertical component of the surface wave. Some of the harmonics, whose frequencies are within the limits of the unstable band, shown in Fig.7, may cause shears in the dam profile if an additional condition is fulfilled, namely , if the amplitudes of these harmonics are above the horizontal straight line which corresponds to the ratio

$$\frac{A}{A_o} = A^x = 1.$$

Thus, if the spectrum of the seismic load is of the amplitudes which are outside the unstable range plotted for the given profile of the dam this profile is considered to be reliable.

For the practical purposes, the curvilinear areas of unstability can be replaced by the stepped ones. The diagram shown in Fig.7 is constructed within the dimensionless coordinates; on the x-axis are plotted the reduced amplitudes which are equal to the ratio of the amplitude of the given term of expansion of spectrum to the amplitude obtained from this load in the elastic region. On the y-axis are plotted the reduced frequency which is equal to the ratio of the frequency of the given term of the spectrum of action of the seismic wave to the profile frequency calculated for the elastic zone.

When performing practical calculations one has to confine himself to a few overtones corresponding to the highest frequencies of forced vibrations; therefore we shall have several points which should be plotted on the drawing and see whether all of them are outside the unstable range.

We described the calculation method as applied to the system with one degree of freedom which is the most simple one, and, as experience has shown, is accurate enough for practical estimation of the dynamic properties of the dam.

A higher accuracy can be obtained by using a system with several degrees of freedom for which purpose the profile of the dam should be divided into the sections which will be interconnected through nonlinear links corresponding to physical properties of materials used for constructing the sections concerned and the forces (friction and cohesion) which arise at the boundaries between these sections. An analysis of such a system will be much more complicated but no essential difficulties will be met with if electronic computers are used for the purpose. An increase in the number of degrees of freedom will mean a corresponding increase of the number of "skeleton curve" on the diagrams of unstability and intersection of these "skeleton curves" is possible in such cases.

The points of intersections are of peculiar properties, i.e., the effect of instability is governed not only by the parameters of the exterior dynamic effect but also by the characteristics of mutual associations which are to be found between some of the members of the system. This conclusion is of a paramount practical value since it shows that the stability of the dam with respect to seismic loads will drop sharply if separate portions of the dam profile are conjugated in a poor way. Thus, the proposed method enables us to estimate a total stability of a triangular-shaped earth-fill dam.

6. A. Comparison to Results Observed

A detailed study of vibrations in the slopes of earth fill dams has been made by S.V.Medvedev (1). Vibrograms obtained for horizontal displacements of points located at various levels from the slope toe have forms that characterize a system with nonlinear characteristics. Fig.9 shows vibrograms for horizontal vibration of the ridge and the toe of a 20 m high earth slope with an inclination of about 45°. The vibrations were excited by exploding small charges placed far enough from the slope. Fig.9 also shows the results obtained in the study (8) for a "three-linear" system with one degree of freedom for two cases: (a) when the phases of the sub-harmonics of the third order and of the basic harmonics are the same, and (b) when these phases are opposite to each other. A qualitative comparison of the above diagrams reveals their close similarity. This can serve as a preliminary confirmation that an earth fill dam, under certain conditions, becomes a nonlinear system and some pulsations can arise in it which are characteristic for nonlinear systems.

7. Stresses in Dam Base

A designer of earth fill dams must pay great attention not only to the amount of inertia forces that is expressed in terms of seismic factors. It is also of significance to know numerical values of strains and stresses and vibration velocities to be found at the dam bases during heavy earthquakes. Maximum values of ground vibration velocities, strain and stresses are given in (9) for earthquakes of various intensity.

The values of vibration velocities have been obtained by means of integrating the accelerations or differentiating the shifts. Fig.9 shows a diagram of the vibration velocities

V, in cm per sec. The velocities are shown for periods from 0.1 to 2.0 sec. The numbers on the diagrams 9, 8, 7 and 6 denote the intensity of vibrations according to a seismic scale. The vibration velocity for each value of intensity keeps within the limits confined by lines shown in Fig.9. With the periods from 0.1 to 0.5 sec. the vibration velocity rises with an increase in period. With the periods from 0.5 sec to 2.0 sec. the velocity value keeps constant.

With an increase of intensity by a unity the velocity value rises by factor 2. Thus, with the intensity of 8, the ground vibration velocity is characterized by the values from 8 to 16 cm per sec.

To determine the stresses in grounds, use was made of data on the wave propagation velocities for basic kinds of grounds. In addition, the data on the density of these grounds were taken into consideration. Fig9, in its left-hand side, also shows the values of maximum stresses appearing at the base of the dam during earthquakes. The stresses are shown for the earthquakes of various intensities (6,7,8 and 9). The stresses differ from type to type of ground. Fig.9 illustrates six types of ground.

As seen from the diagrams, for the same intensity of an earthquake, the stresses in solid rocks are 25 times as high as in loose grounds. The stress value during the earthquakes of intensity 9, for example, may vary from 5 to 10 kg per sq.cm.

8. Deformations of Dam Base

The amount of maximum relative deformations appearing in grounds of various types at the bases of earth fill dams are illustrated in Fig.10. On the figure are presented the values for the above six different types of ground. To determine deformations, use is made of the values of the wave propagation velocities and those of the ground vibration velocities. As seen from Fig.10, rock bases exhibit dynamic deformations whose values are 15 times as low as those of loose grounds.

Both the top and the bottom slopes of the earth fill dams in seismic districts are made flat. This accounts for tremendous inertia forces which develop in earth fill dams owing to much construction material involved. However, the length of a seismic wave is of the same order of magnitude as is the size of an earth fill dam; that is why at the same moment in different points of a dam the accelerations differ not only with respect to the amount but also with respect to the sign. A seismic factor in the calculations normally increases only with height from a base to a ridge though it might be expected that this factor must change along two horizontal axes of a dam as well.

Conclusion

Approximate calculations made herein belong to the study of an important and new problem which is of a great practical significance. The calculations include an attempt at throwing light on the problem of nonlinear vibrations in earth fill dams by using simplified engineering methods. Further investigation in this direction will contribute to finding the best profile to be chosen when constructing an earth fill dam.

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Captions to Figures

Fig.1. Strained state of a dam profile due to a vertical component of seismic load when t = 0.25 To(1) and t = 75 To (2).

Fig.2. Strained state of a dam profile due to a horizontal component of seismic load (Ishizaki (7)).

Fig.3. Strained state of a dam profile due to a combined action produced by vertical and horizontal components of the seismic load when t = 0.25 To.

Fig.4. Same, when t = 0.50 To.

Fig.5. The diagrams of shearing stresses at the dam slope along horizontal platforms

1 - due to horizontal load (Ishizaki and (7)).

2 - due to vertical load

3 - due to combined action of horizontal and vertical loads

Fig.6. Stress-strain diagram

Fig.7. Unstability ranges

Fig.8. Vibrograms of motion obtained by experiments

Fig.9. Vibration velocity, V, in cm per sec. and stress, σ, in kg per sq cm. during earthquakes of intensity 6,7,8 and 9. Velocities are dependent on period T. Stresses differ for six types of ground: 1 - granite, 2 - limestone, 3 - marl, 4 - gravel, 5 - sand and clay, 6 - loose ground.

Fig.10. Relative strains, ε, during earthquakes of intensity 6,7,8 and 9 in grounds of six types: 1 - granite, 2 - limestone, 3 - marl, 4 - gravel, 5 - sand and clay, 6 - loose ground.

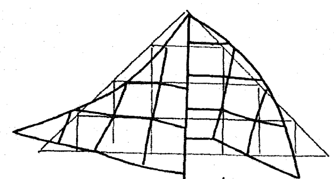

Fig.1

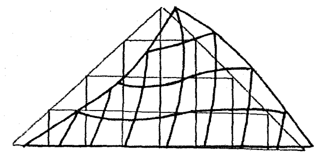

Fig.2

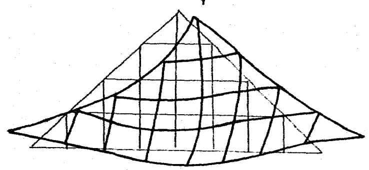

Fig.3.

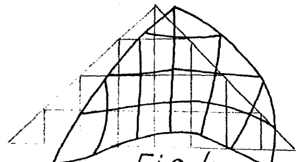

Fig.4.

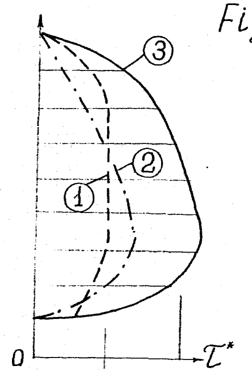

Fig.5.

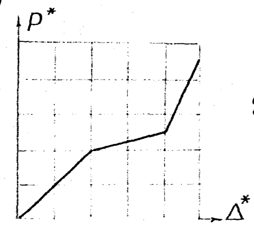

Fig.6.

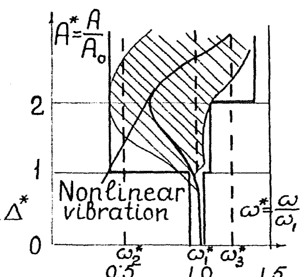

Fig.7.

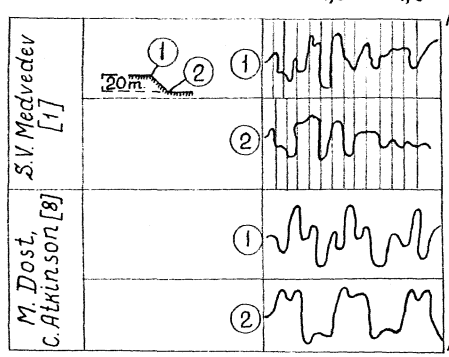

Fig.8

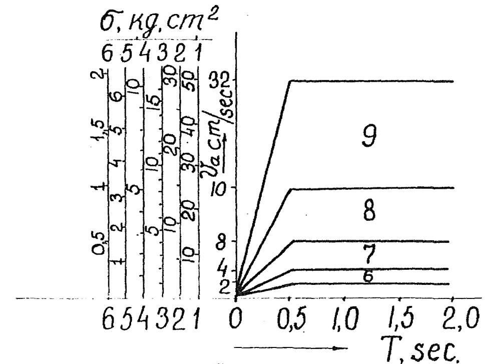

Fig.9.

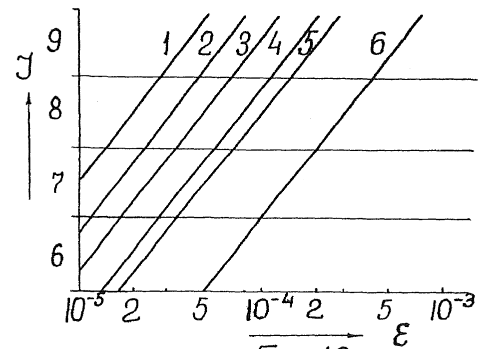

Fig.10.

A REVIEW OF THE SEISMIC STABILITY METHODS OF EARTH DAMS

DISCUSSION

BY N.N. AMBRASEYS*

In principle, any method of stability analysis of earth or rock-fill dams involving earthquake forces is merely an extension of the stability analysis methods under static condition; and unless the latter method is rigorously understood and investigated, the seismic stability analysis method cannot be applied with confidence.

Let us discuss first the basic principles of the static stability analysis of earth dams. Broadly speaking, this involves the principle of limiting equilibrium, with which the stability of an earth dam is investigated by comparing:

(D) The stresses induced within the structure and in its foundation by body and external forces, with

(R) The strength that can be mobilised in the structure under these conditions,

where (D) and (R) are interrelated.

If such a comparison is carried out, and the average stress along a potential failure surface set up by gravity and other external forces does not exceed the mean strength mobilised by these stresses, the factor of safety of the structure against failure along this potential failure surface is greater than one. The factor of safety thus defined expresses the ratio of the actual mean strength of the structure and foundation along a failure slip surface to that required to maintain limiting equilibrium.

For a factor of safety (F) greater than one, the mean strength exceeds that required to maintain equilibrium and the structure, though it may be locally overstressed and irreversibly deformed, a kinematic slip mechanism will not arise.

When the factor of safety is less than one, the mean strength is less than that required to maintain limiting equilibrium and part of the dam will slide on a failure surface. The sliding mass will move out, and will come to rest at a position where the new mean stresses induced by gravity, kinetic and external forces will not exceed the average shear strength of the material on the failure surface. The amount of relative movement may amount to a few inches, a few feet or many yards.

In practice, the comparison of the stresses induced in a dam by various forces, with the mean strength that can be mobilised under these stresses, can be carried out in a number of different ways. All these

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different ways involve basically (D = driving forces) and (R = resisting forces) and differ only in the assumption for the particular way (D) and (R) should be interrelated, the exact interrelation being missing in all stability analysis methods (see equations 4). Different methods, therefore, employ different assumptions concerning the way (D) should be related to (R), some of which, from the engineering point of view appear to be very satisfactory (1) (2).

All methods of stability analysis in common use have one important principle in common; that is, the principle of "limiting equilibrium". If this principle is violated, a portion of the structure will slide along a slip surface, and no matter how small the movement, on the basis of this principle a failure has occurred. In all these methods of stability analysis this is observed and the design is always controlled by the fact that the factor of safety under the most critical conditions should never be less than one.

Now let us turn to the stability analysis under seismic conditions, maintaining the same principle, that is of "limiting equilibrium". In this method if we include seismic body forces, conditions (D) and (R), though quantitatively now modified, will in principle still hold together with the assumed interrelation. The analysis of a typical dam section that involves seismic forces is shown in Figure 1. Formally, the equilibrium of the element (abcd) requires satisfying the following system of equations:

Horizontal forces : $$\frac{dE}{dx} + k\gamma\left[a(x) - b(x)\right] + \sigma_n'\frac{db(x)}{dx} - \tau = 0$$

Vertical forces : $$\frac{dX}{dx} - \gamma\left[a(x) - b(x)\right] + \sigma_n' + \tau\frac{db(x)}{dx} = 0 \qquad \ldots.(4)$$

Moments about M : $$X + \frac{d}{dx}\left[Ez(x)\right] - \frac{dE}{dx}b(x) + \tfrac{1}{2}k\gamma\left[a(x) - b(x)\right]^2 = 0$$

$$\tau = c' + \sigma_n'\tan(\phi')$$

The five unknown functions in (4) are the position of the line of thrust $z(x)$, the integral of stresses along (ac) or (bd) $X(x)$ and $E(x)$, and the stresses at M, τ and σ_n'

It is obvious that system (4) is indeterminate. If the actual stress distribution within the body of the dam or the position of the line of thrust were known, a solution of (4) would be possible. Moreover, we have to estimate the distribution of the seismic coefficient $k(x)$ inside the dam, as well as the possible effects of the shock on the properties of the material (c') and(ϕ'). This we will discuss later.

Now, if the instability is brought about by the action of seismic body forces (F less than 1), this instability will be of short duration since seismic forces act for a comparatively short periods of time. During that period of instability, the principle of limiting equilibrium is obviously violated and part or parts of the structure will slide on failure surfaces. But since, as we have just said, the earthquake forces will act for a very short period of time, it is conceivable that sliding will stop after a certain amount of relative displacement has occurred (assuming, of course, that no liquefaction or extreme reduction of the shear strength of the material on the failure surfaces will occur).

If we consider that the violation of the principle of limiting equilibrium under seismic conditions is not permissible, then we have to design the dam in such a way so that its factor of safety under all conditions will be greater than one.

In computing the maximum earthquake forces acting on the dam to be used in the limiting equilibrium stability analysis method, we consider the dam to possess linear and fully reversible stress-strain characteristics. For small earthquake movements earth and rock-fill dams respond almost as elastic oscillators; in general, however, the response of such structures to strong ground motions would be non-elastic. The device of using elastic behaviour is to some extent justified by the fact that, irrespectively of whether dams behave as elastic or elastoplastic bodies, their initial response is elastic. Also, the use of highly damped spectra for the natural periods involved justify to some extent the use of linear theory.

Concerning the last of equations (4) what is usually not appreciated is that when the fill of a dam is shaken by an earthquake, the available shear strength to resist the dynamically imposed shear stresses is not the full strength of the material, since for the static stability, before the earthquake, a certain amount of its strength had been mobilised by static forces. The strength available to resist the seismic stresses will not, in general, be the remaining strength on a static basis. The reason for this is that the application of the dynamic forces will lead to a change in the pore water pressures. The time of loading by the seismic forces, is so short that no drainage or dissipation of excess pore water pressure can occur, and the failure conditions in these circumstances will involve a pore pressure change different from that implicit in the assumptions made when computing the static stability. Hence, the strength available to resist the ground motions will be that which the fill will have if, after being stressed to the point which represents the stress conditions for the static equilibrium, it is then sheared under undrained conditions at a high rate of strain.

In these circumstances, a saturated fill material will appear to behave as though the angle of shearing resistance were zero, and the available strength will be independent of the magnitude of the stress changes induced by the earthquake.

To illustrate this point, consider a point P in the dam (Figure 2) before the earthquake. The element P is consolidated under an effective pressure p' and a lateral pressure Kp'. The magnitude of the major principal effective stress p' can be taken, as a first approximation, as being equal to the effective vertical head of soil above point P, i.e.

$$p' = z\left[\gamma_d - (1 - n)\gamma_w\right] \qquad \ldots\ldots\ldots\ldots\ldots$$

Thus, element P in the fill before shaking is subject to a deviator stress $p'(1-K)$. If the pore pressure coefficient A for the fill is known* it can be shown that the available undrained strength at point P will be given by:

$$c_u = \frac{p'\sin(\phi')\left[K + (1-K)A\right]}{1 + (2A-1)\sin(\phi')} \quad \text{.................(1)}$$

where in equation (1), which is valid for $c'=0$, ** and for materials close to full saturation,

$$K = \frac{1 - \sin(\theta)}{1 + \sin(\theta)} \quad \text{with} \quad \tan(\theta) = \tan(\phi')/F$$

where F is the factor of safety at P before the earthquake. K is obviously the ratio of minor to principal effective stresses at the point whose factor of safety is greater than one. This value of K lies between the coefficient of earth pressure at rest (K_o) and the ratio σ_3'/σ_1' in a condition of limiting equilibrium corresponding to failure.

Equation (1) can be used in a stability analysis in terms of total stresses. In terms of effective stresses, the stability analysis can be carried out by computing the pore pressure change due to the seismic forces. Assuming that the dam will behave elastically, the earthquake may be considered to produce a horizontal shear stress $\pm\Delta\tau$. Its magnitude can be computed from

$$\Delta\tau = \left\{\sum_n \left(\frac{1}{\lambda_y^2} P_n S_{an}\right)^2\right\}^{1/2} \gamma h \quad \text{.................(2)}$$

in which λ_y denotes the zeros of the Bessel functions, S_{an} is the acceleration spectrum in the n-th mode, h is the height of the dam, y refers to the distance below the crest level; γ is the bulk density of the material, and P_n is a transcendental function of y, λ_y, h, and n, that has been tabulated in reference (3), Figure (3).

For the condition of plane strain, in a saturated material, the pore water pressure change due to $\Delta\tau$ is:

$$\Delta u = \sqrt{3}\,(A - 1/3)\,\Delta\tau \quad \text{.................(3)}$$

where A is the pore pressure coefficient for plane strain.

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* If the factor of safety at P is comparatively small, it is permissible to assume $A = A_f$ (at failure).

** If $c' \neq 0$, the term $c'\cos(\phi') / \left[1 + (2A-1)\sin(\phi')\right]$ should be added to (1).

For partly saturated materials equations similar to (1) and (3) can be derived.

In the limiting equilibrium method the maximum seismic forces on the dam can be evaluated, using one of the methods available (3,4,5) depending on the complexity of the problem. These forces are then applied in a stability analysis, assuming that they will act permanently. The result of the analysis should show a factor of safety greater than or equal to one.

If we consider that the instability of an earth dam that will be brought about by the action of a strong seismic force will be of short duration, and probably of non-destructive consequences, we may, for the brief duration of the application of the seismic forces abandon the principle of "limiting equilibrium" and attempt to compute the magnitude of the displacements and the deformations that the structure will undergo. For the sake of convenience let us name this method of instability analysis the "displacement" method.

The displacement method, therefore, is concerned with determining the magnitude of this finite deformation of the dam. It assumes that, for a particular structure, (D) and (R), as well as their appropriate interrelation under seismic conditions, are known. The deformation of the dam will be made up of the individual movements associated with earthquake shocks that will create instability (F less than one). These movements, will occur along the same or different failure surfaces, as shown in Figure 4 (6 p.38). Recent studies (7,8) show that it is not difficult to compute the order of magnitude of the movement of sliding that is likely to occur during a particular earthquake.

Recapitulating, for the earthquake resistant design of earth and rockfill dams, in the <u>limiting equilibrium</u> method, after taking into consideration the appropriate soil properties of the materials involved, together with the static and the dynamic forces imposed on the structure, we design the dam in such a way that its factor of safety against complete failure will always be greater than one.

In the <u>displacement</u> method, we allow the factor of safety (as defined in the limiting equilibrium method) to drop below one for the short duration of the earthquake pulses involved in an actual earthquake which can induce failure. We then compute the cumulative displacements produced by sliding (Figure 4). If, for the strongest probable earthquake, the total displacements do not exceed a certain predetermined value, the design is considered adequate.

However, it must be borne in mind that the use of the latter method requires the establishment of criteria of allowable displacements. These criteria will depend upon the soil properties, importance of structure, available free-board, internal drainage facilities, erodability during overtopping, facility of repair, aftershock activity, and insurance rating. These factors need detailed study. It should be noticed that little progress has been made in formulating design procedures based upon displacement criteria under static conditions, let alone under the complex dynamic conditions of seismic loading.

Both methods have their merits, and much can be said about the validity of the principles involved in the displacement method. It must be borne in mind however, that the failure of a big dam with the reservoir full has appalling consequences, and that with the present state of knowledge of the behaviour of the fill and foundation materials and of the failure mechanism of such structures one must be conservative. Either method of approach to the seismic stability of an earth dam involves assumptions which may or may not be reasonable, but the fact remains that the limiting equilibrium method (with $F > 1$) is more conservative method of design.

I believe that under seismic conditions, a dam with a full reservoir should be designed on the basis of the principle of limiting equilibrium with $F > 1$ and that it should not be allowed to deform. Immediately after construction, with the reservoir empty, or at rapid draw-down with the reservoir level very low, the displacement method may be used.

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Note: Figures 2 and 3 as well as the values of (A) refer to the models used by:

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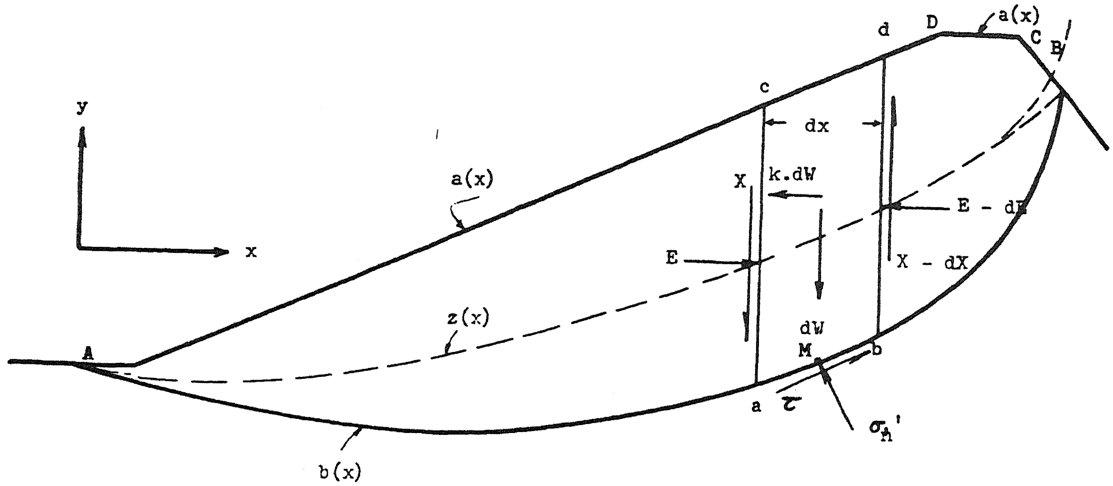
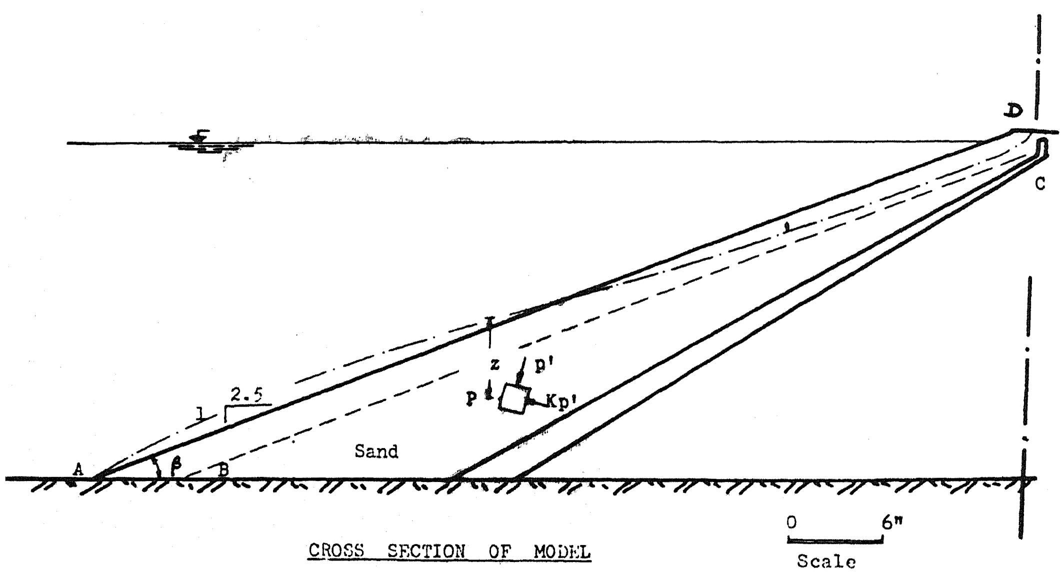


FIGURE 1



CROSS SECTION OF MODEL

FIGURE 2

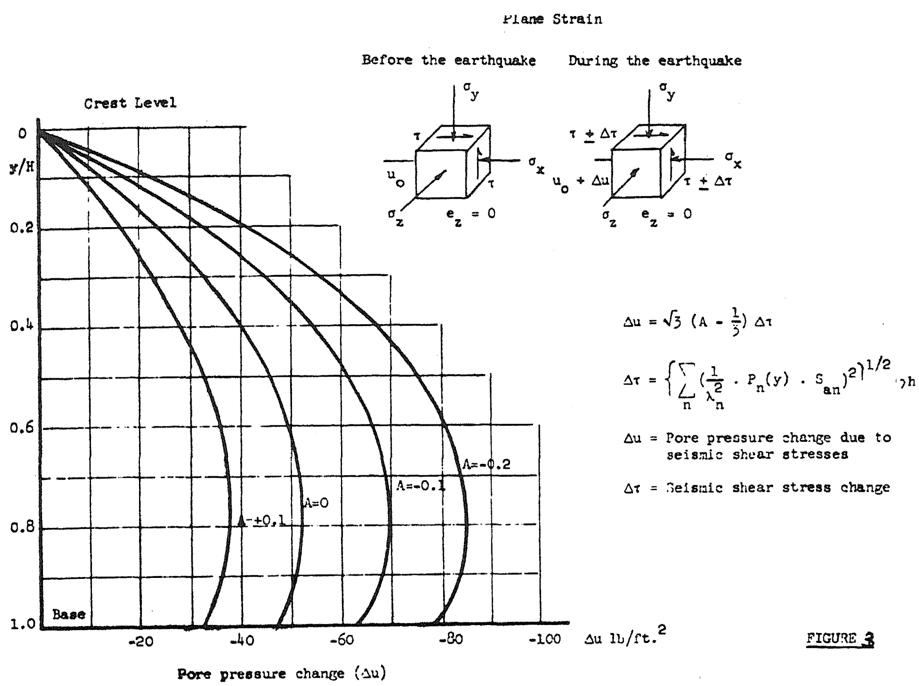


FIGURE 3

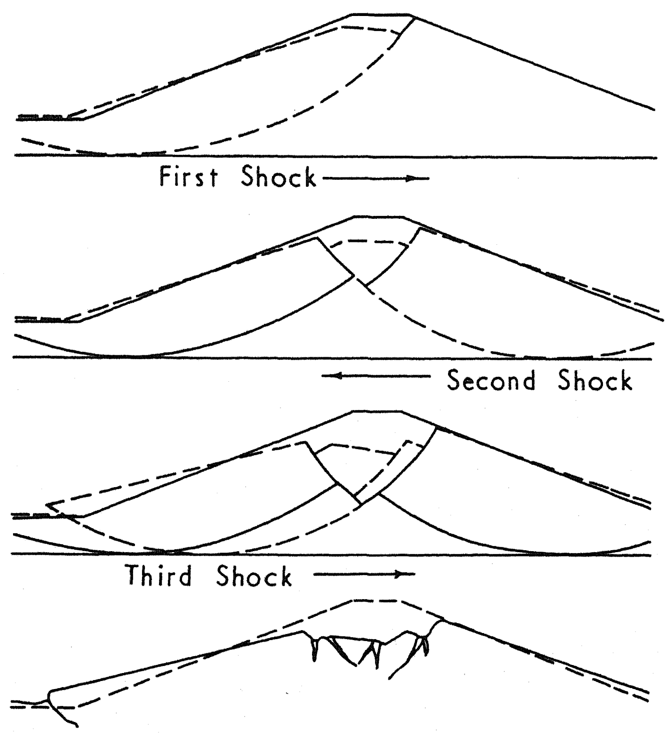


FIGURE 4