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ENERGY ABSORPTION BY PRESTRESSED CONCRETE BEAMS

One of the objections against the use of prestressed concrete in seismic design is that its energy absorption in a shock is smaller than for reinforced concrete. I think that this is wrong, or at least that it comes from a bad interpretation of certain tests under oscillating loads.

In fact, at rupture, the energy absorption is almost the same for the same ultimate moment and for a given cross-section of the beam (with the same resistance of the concrete) for the two materials.

Broadly speaking, the energy absorption is proportional to $M_u \times \rho_u$, M_u being the ultimate moment and ρ_u the ultimate curvature of the beam at the section of failure ($\rho = \frac{1}{r}$, r being the radius of curvature), i.e. $W = k M_u \rho_u$

Therefore the energy absorption depends upon the shape of the moment-curvature diagram (see figure 1) and at rupture is proportional to the hatched area of figure 1.

It may be shown that, theoretically, this area is the same for prestressed concrete and reinforced concrete.

Let b be the width, d the effective depth, x the depth of the compressive zone at failure, F_u the force in the steel at failure of the beam, ε_{cu} the ultimate shortening of the concrete, σ_{cu} the ultimate compressive stress in the concrete.

We have from figure 2

$$M_u = F_u \left(d - \frac{x}{2} \right) \qquad (1)$$

and the equilibrium of forces gives (both for prestressed concrete and reinforced concrete)

$$F_u = b x \sigma_{cu} \qquad (2)$$

Let us call $\bar{\omega}$ the quantity "mechanical percentage" where

$$\bar{\omega} = \frac{F_u}{b d \sigma_{cu}}$$

It is the ratio of the ultimate resistances of the steel and of the circumscribed concrete. Hence we have by equation (2)

$$\frac{x}{d} = \bar{\omega}$$

and substituting in equation (1)

$$M_u = F_u d \left(1 - \frac{\bar{\omega}}{2}\right)$$

or

$$M_u = b d^2 \sigma_{cu} \bar{\omega} \left(1 - \frac{\bar{\omega}}{2}\right) \qquad (3)$$

On the other hand, the radius of curvature at rupture, r_u, is given by (see figure 3)

$$\frac{1}{r_u} = \frac{\varepsilon_{cu}}{x}$$

Hence

$$\rho_u = \frac{\varepsilon_{cu}}{\bar{\omega} d} \qquad (4)$$

From this is deduced that, for two beams with the same cross-section (b, d) and the same concrete (σ_{cu})

(1) for the same ultimate moment $\bar{\omega}$ is the same
(2) since $\bar{\omega}$ is the same ρ_u is the same.

The first statement (same mechanical percentage for the same ultimate moment) is confirmed by rupture tests.

As for the second statement, it refers to the theoretical value of the ultimate curvature, but it is known that this identical value is reached only in the section of cracks and therefore that the <u>average</u> curvature depends upon the interval between cracks, the beam behaving as a chain of blocks, connected by small hinges of which the curvature is r_u (see figure 4).

Nevertheless, tests seem to show that, even for the average ultimate curvature, prestressed concrete is not inferior for a given percentage $\bar{\omega}$ (and therefore for a given moment M_u) to reinforced concrete, from the point of view of energy absorption (i.e. the values of $M_u \rho_u$).

I have plotted in figure 5 the results for ultimate curvature for reinforced concrete beams and prestressed concrete beams. We should have from equation (4)

$$\rho_u d = \frac{\varepsilon_{cu}}{\bar{\omega}}$$

The results of the tests are as follows.

	Beam	d metres	$\bar{\omega}$ (1)	$\frac{1}{1000} p_u$ /metre	$\frac{p_u d}{10^3}$
Reinforced Concrete	1	.245	.042	225	55
	2	.249	.17	92	23
	3	.249	.15	71	19
	5	.249	.212	(57 (86	14 21
	B5	.245	.2	55	13
	F9	.245	.054	107	26
Prestressed Concrete	AQ1	.255	.054	270	69
	AQ2	.23	.128	210	48
	Guyon (2)	.166	.318	60	10

(1) The stress in the steel at failure was taken as the elastic limit for reinforced concrete and as the ultimate tensile resistance for prestressed concrete.

(2) This beam is discussed in my book "Prestressed Concrete" Volume II.

Another beam, AQ3, has been tested and gives more favourable results, but I have for the moment discarded it since I think there has been some mistake in interpretation.

Some other beams could be reported, but it is a very long and difficult interpretation, and anyhow, I would have to check more accurately all the results (which had been obtained for a completely different purpose).

It should be noted on the other hand that

(a) due to the cracks (see figure 4) there may be a great dispersion.

(b) with high percentages of steel the ductility may be much reduced because the stress distribution in the compressive zone may be more like a triangle than a rectangle, giving a value of x twice and therefore a value of ρ (or $\rho\alpha$) half of what it is in the case of small percentages of steel.

Until the results of a more complete study are available I would only state that neither theoretically nor experimentally is the energy absorption at failure, for the same cross-section, the same concrete and the same ultimate moment, smaller for prestressed concrete than for reinforced concrete. And this is all which I claim for the moment.

Now in tests under oscillating loads the area of the hysterisis loop is generally greater for reinforced concrete than for prestressed concrete (see figure 6). This has to be discussed according to two assumptions.

(1) Let us suppose first that the ultimate moment is the same for the two beams which means that the same safety factor has been taken.

The two moment-curvature diagrams have the general appearance shown in figure 7. The prestressed concrete beam is not cracked up to a load which is greater than the service level, while reinforced concrete is cracked at a lower load than the service level.

(a) Therefore if the test is done at a level such as (1) of figure 7, for which the inelastic deformations are still small for prestressed concrete and much greater for reinforced concrete, the hysterisis loop will be obviously greater for reinforced concrete. But this means that the prestressed concrete has not deteriorated at all, while the reinforced concrete has deteriorated, if not detrimentally, enough to call for caution against a great number of repetitions (see the comparative tests made 20 years ago on prestressed concrete and reinforced concrete poles).

In other words, the damping due to energy absorption is a favourable factor for reinforced concrete; the same damping effect is not needed for prestressed concrete.

(b) If the level is raised, the difference between prestressed concrete and reinforced concrete will be proportionally reduced, and near the failure, the energy absorption will be of the same order for both materials.

This means that prestressed concrete will call automatically for damping when it will need it.

Thus for the same deformation (and not for the same moment), the damping will be of the same order of magnitude for both materials (see figure 8).

(2) If the safety factor is greater for prestressed concrete than for reinforced concrete (see figure 9), and I think that this is generally the case with the usual regulations, it is absolutely normal that reinforced concrete tests will show great loops while prestressed concrete will behave elastically.

Therefore when the exact characteristics of the tests (respective service and failure moments in particular) are not given, the tests are not demonstrative against prestressed concrete.

Furthermore, I would think that the aptitude to service is characterised not by damping, but by the risk of reaching the dangerous deformation.

Tests could perhaps be done according to the following principles.

Take beams of the same cross-sections in prestressed concrete and reinforced concrete.

(a) with the same ultimate moment as in figure 8.

(b) with different ultimate moments as in figure 10.

Submit them to oscillating loads with as near as possible the period of the uncracked beams. Compare the number of repetitions for which the dangerous deformation is reached, and for which the beams fail.

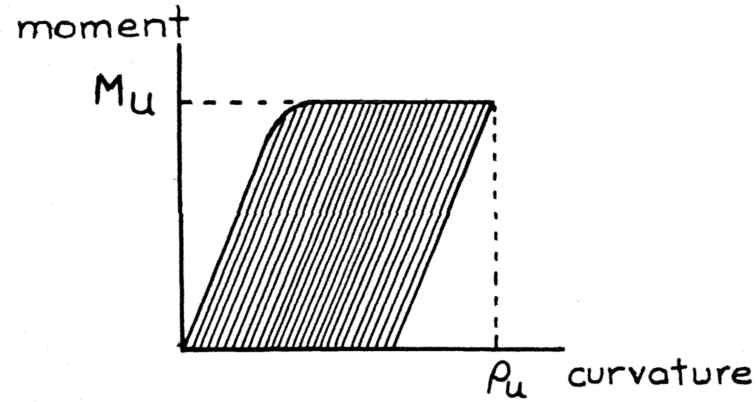


FIGURE 1

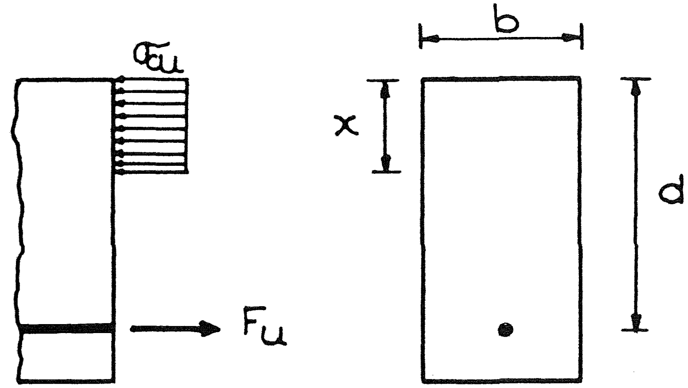


FIGURE 2

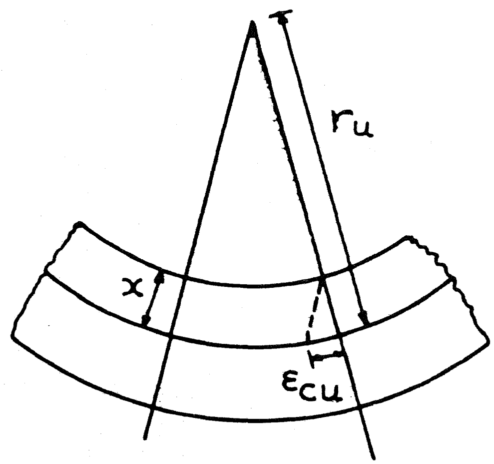


FIGURE 3

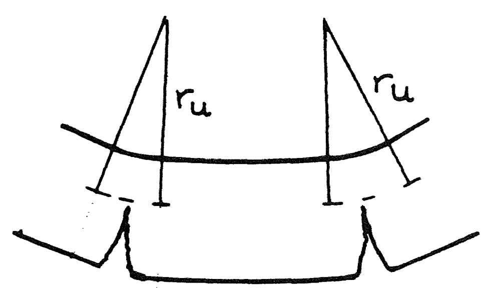
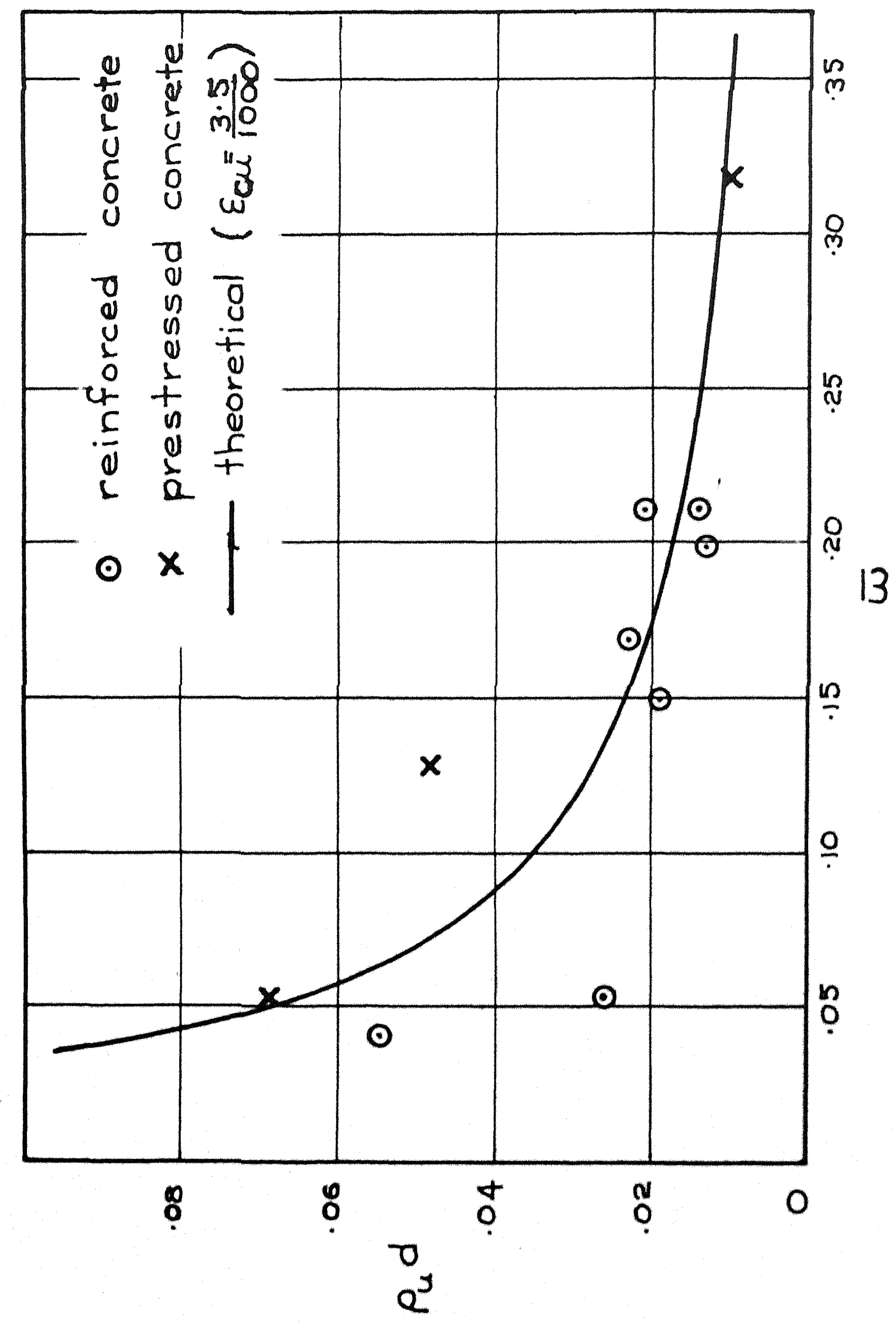


FIGURE 4



reinforced concrete
prestressed concrete
theoretical ($\varepsilon_{cu} = \frac{3.5}{1000}$)
$\rho_u d$
.08
.06
.04
.02
0
.05
.10
.15
.20
.25
.30
.35
$\overline{\omega}$

FIGURE 5

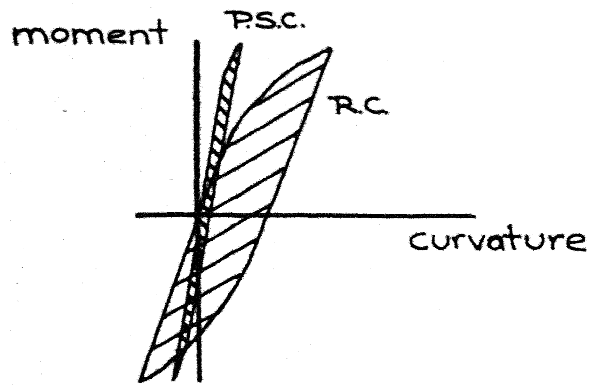


FIGURE 6

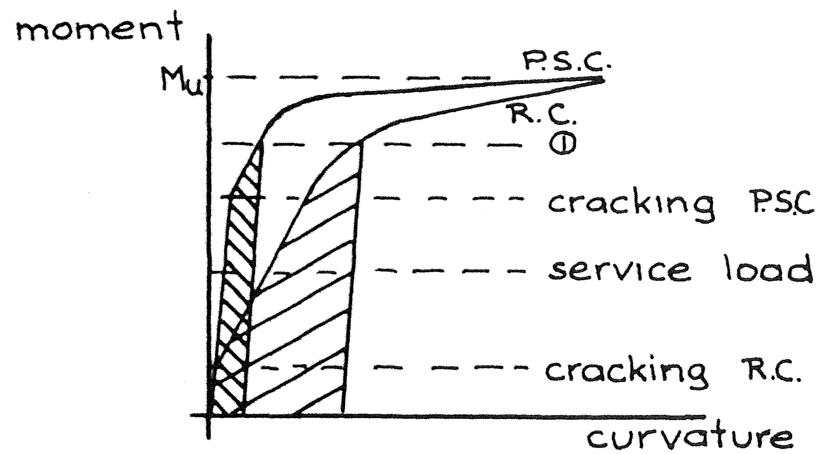


FIGURE 7

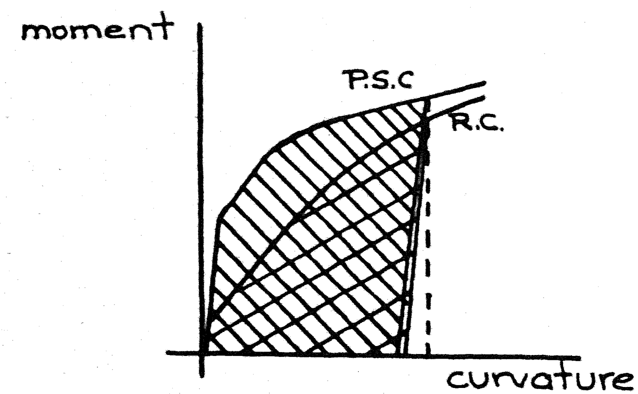


FIGURE 8

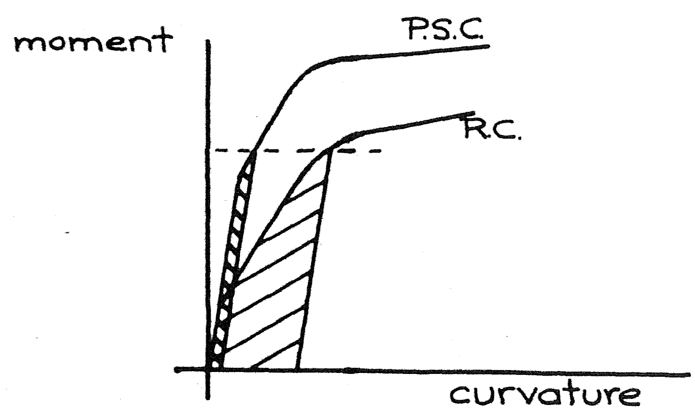


FIGURE 9