EARTHQUAKE DESIGN LOADINGS FOR THIN ARCH DAMS

by

Clair C. Crawford[1]

Abstract

A method of determining earthquake design loadings for thin arch dams is presented. The earthquake response spectrum concept is utilized and hydrodynamic effects are included. The method is applied to an actual dam design, and the significance of computed earthquake stresses is discussed.

Introduction

Earthquake loadings are dynamic loadings and must be treated as such. Determination of earthquake effects on arch dams requires an appraisal of the dynamic characteristics of the structure including hydrodynamic effects, an estimate of the character and intensity of earthquakes likely to be encountered, and a determination of the response of the structure to the earthquakes. Finally, the computed earthquake stresses must be appraised in relationship to the safety of the structure.

The purpose of this paper is to present a method of determining earthquake design loadings for thin arch dams and to comment on the interpretation of the stresses computed. The results of the method as applied to an actual structure are given.

Arch dam design is in a state of continuous refinement and updating at the Bureau of Reclamation. The method of earthquake analysis presented here represents the current (1964) approach to the problem, and it is considered that these concepts will be incorporated into Bureau design procedures.

The steps to be followed in determining earthquake stresses in arch dams are these:

1. Determine the dynamic shapes of the dam structure in the first two arch modes
2. Determine the effective hydrodynamic mass considered to move with the dam in the two modes
3. Estimate the natural periods of the dam
4. Determine the dynamic loadings as the product of the dynamic deflection at a point multiplied by the total effective mass associated with the point

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5. Adjust the value of the total load to produce a base shear equal to the desired percentage of the total effective weight of the structure and water in that mode

6. Apply these loadings separately and compute earthquake stresses for each mode

7. Combine the earthquake stresses from the two modes at any point to get the maximum effective earthquake stress

8. Algebraically combine the earthquake stresses with the static stresses

Arch Dam Dynamic Deflections

A thin arch dam is an efficient, elastic structure having very little structural damping, and under normal static loads, stresses throughout the dam will be almost entirely compressive. The longest periods of vibration of many thin arch dams are such that large excitation from strong motion earthquakes can be expected. Thus, it is important during the design of arch dams to include the effects of earthquake on the structure.

An arch dam can vibrate in many modes and because of the random nature of earthquake motion, theoretically, at least, a number of these modes should be considered. Practically, however, consideration of the two lowest frequency arch modes of the dam will produce good values for earthquake loadings. The fundamental mode of the dam will be excited primarily by the upstream-downstream components of the earthquake ground motion. The loadings for the fundamental mode are considered to provide a good envelope to embrace the effects of the first and all the higher "symmetrical" modes. The second arch mode will be excited primarily by the cross canyon components of motion and the loadings for the second mode are considered adequate to embrace the second and all the higher "unsymmetrical" modes. Extremely domed structures are not considered in this analysis and hence the effects of the vertical components of earthquake ground motion are considered negligible in relation to the effects of the horizontal motions.

To compute the shape and frequency of the vibrating dam, it is assumed that the structure is divided into a convenient number of arches formed by horizontal planes and a set of cantilevers formed by vertical radial planes. Both the arches and the cantilevers are assumed to embrace the total volume of material in the dam. The dam is assumed to deflect dynamically as a group of fixed-end arches. The deflection of the arches from the static position is assumed to be the same as that of a fixed-end primsatic beam during normal vibration. See Reference 1, page 410; Reference 2, page 452, and Figure 1.

In the vertical direction, the structure is assumed to take the shape of a vibrating prismatic cantilever. Thus, in the fundamental mode, the maximum deflections of the top arch and the crown cantilever will be identical. To find the dynamic deflection at any other location in the dam, the deflection of the crown cantilever at that elevation is computed. This becomes the maximum deflection of the arch under consideration. The deflection at any other point in the arch is computed as a function of the relative distance from the crown cantilever toward the abutment. In the second arch mode, the crown cantilever is at the arch node and so does not deflect. However, the maximum relative arch deflection is assumed to vary with elevation the same as in the fundamental mode.

Table 1 gives the relative deflections of the arches and the crown cantilever. The arches are divided by the location numbers into 10 equal segments from abutment to abutment. The crown cantilever is divided by the location numbers into 10 equal segments starting at the top of the dam. Thus, a point located 0.3 of the vertical distance from the top of the dam and 0.2 of the arc length from the abutment would have a relative deflection of (0.5909)(0.3901) = 0.231 in the fundamental mode and (0.5909)(0.8027) = 0.474 in the second arch mode.

Effective Hydrodynamic Mass

When a submerged body is accelerated, the force necessary to produce the acceleration is larger than that necessary to accelerate the mass of the body not submerged. This is equivalent to the submerged body causing a certain volume of the enveloping fluid to be accelerated with it. See Reference 3. The effective volume of fluid accelerated with the submerged body is determined only by the size of the body and its configuration relative to the direction of acceleration. Neither the magnitude of the acceleration nor the viscosity of the fluid is pertinent.

In the case of a reservoir restrained by a dam the problem is considerably different because the dam has water only against one face, there is a free surface involved, and the walls and floor of the canyon introduce additional boundary conditions. However, the concept of effective volume of water being accelerated with the dam is valid. Westergaard (4) has solved the two-dimensional case of a very wide, straight gravity dam with vertical upstream face subjected to upstream acceleration. By electric analog methods, Zangar (5) extended the work to include the effects of various slopes on the upstream face of the dam. More recently, Zienkiewicz and Nath (6), using somewhat different electric analog methods, have further extended the work to include arch dams, canyon walls, and cross canyon motions.

From consideration of all the above, the horizontal dimension "b" of the volume of water considered to be accelerated with the dam in the fundamental mode is given by:

$$b = \frac{7}{8}\sqrt{hy} \qquad (1)$$

where h is the maximum water depth and y is distance measured vertically downward from the water surface. See Figure 2.

In the second arch mode of vibration, excited primarily by cross canyon motions, the dimension "b" of the volume of water considered to be accelerated with the dam is given by:

$$b = \frac{1}{3}\sqrt{hy} \cdot \frac{2x}{L} \qquad (2)$$

where L is the length of the intersection of the water surface with the face of the dam and x is distance measured from the vertical centerline of the dam. Equation (1) and particularly Equation (2) are admittedly imperfect, but both are considered adequate to predict the effective volume of water being accelerated by the dam.

Shape of Dynamic Loadings

The dynamic load at any point in the dam is directly proportional to the product of the dynamic deflection of and the mass associated with that point. The associated mass, of course, includes the mass of the concrete of the structure plus the mass of the effective volume of water accelerated with the structure. In the first arch mode, the dynamic loadings are alternately directed radially upstream then downstream at the natural frequency for that mode. In the second arch mode, the loadings on opposite sides of the vertical centerline of the dam are in opposite direction and these loadings alternate direction during every cycle at the natural frequency for this mode. The shape of the dynamic load is computed as the product of the relative dynamic deflection at a point multiplied by the total effective weight associated with that point. This loading is integrated over the whole structure to get a total radial load. This load is then adjusted to give the base shear determined to be proper for design. In the second arch mode of vibration, the loading is integrated from the centerline node to one abutment and the loads are adjusted to produce the desired base shear on half the structure. See Figures 3 and 4.

Response to Earthquake

Use of the earthquake response spectra concept is considered proper to determine earthquake loadings for thin arch dams. See References 7, 8, and 9. This concept demonstrates that a firmly founded, elastic structure, having a single degree of freedom when subjected to the ground motion of a certain earthquake, will respond in a predictable manner. At some time during the earthquake excitation, the structure will develop maximum values of relative displacement, relative velocity, and acceleration. These maximum values are determined by three variables: the earthquake motion, the period of the structure, and the damping present in the structure. The spectra are the plots of these maximum responses as ordinates against the period of the structure as abscissas. See Reference 10.

The earthquake response spectra are computed for structures having a single degree of freedom. The maximum base shear developed by this single-degree-of-freedom structure is given by the product of the total mass of the structure and the value of the spectral acceleration. For structures having many degrees of freedom, the maximum base shear in any mode can be obtained by applying an "effectiveness factor" multiplier to the base shear computed for a single-degree-of-freedom structure having the same total mass and the same natural frequency. See Reference 7.

Spectral plots for the recorded strong-motion earthquakes, for small values of damping, show marked similarities. These similarities promote the concept of "standard" earthquake response spectra to predict the effects on structures of future earthquakes to which the structures might be subjected.

Measurements made by Okamoto and Takahashi (11) show the equivalent viscous damping in two arch dams to be about 4 percent to 5 percent of critical with the reservoir empty and only slightly higher with the reservoir full. It is considered that an equivalent viscous damping of 5 percent of critical is a good value to use for a thin arch dam.

The frequencies of vibration of a thin arch dam can be estimated by energy methods. The square of the circular frequency is obtained by equating the maximum potential energy of the structure during vibration to the maximum kinetic energy. The following formula for natural frequency includes the potential energy of bending and rib shortening of the arches, and bending of the cantilevers. The effective hydrodynamic mass accelerated with the vibrating structure is also included.

$$\frac{\omega^2}{E} = \frac{C_1 \sum_{i=1}^{n} \frac{I_i K_i^2}{L_i^3} + C_2 \sum_{i=1}^{n} \frac{A_i L_i K_i^2}{R_i^2} + C_3 \sum_{j=1}^{m} \frac{b_j K_j^2}{H_j^3} \left[a_j^3 + 4(a_j c_j)^{3/2} + c_j^3\right]}{C_4 \sum_{i=1}^{n} \mu_i L_i K_i^2 + C_5 \sum_{i=1}^{n} \frac{B_i W d_i L_i K_i^2}{g}}$$

where

ω = circular frequency of dam, radians/sec

i = number of the arch, 1st, 2nd, 3rd, etc.

j = number of the cantilever, 1st, 2nd, 3rd, etc.

n = total number of arches

m = total number of cantilevers

E = modulus of elasticity, lb/ft^2

I_i = moment of inertia of arch cross-section about vertical axis, ft^4

A_i = area of arch cross-section, ft^2

K_i = maximum relative deflection of arch i, ft

K_j = maximum relative deflection of cantilever j , ft
L_i = length of arch i , ft
H_j = length of cantilever j , ft
R_i = radius of curvature of centerline of arch i , ft
a_j = thickness at base of cantilever j , ft
b_j = width of cantilever j , ft
c_j = thickness of top of cantilever j , ft
μ_i = mass per unit length of concrete in arch i , $\text{lb-sec}^2/\text{ft}^2$
B_i = horizontal dimension of effective water volume at abutment of arch i , ft
d_i = vertical depth of arch i , ft
W = unit weight of water, lb/ft^3
g = acceleration of gravity, ft/sec^2
C_1, C_2, C_3, C_4, C_5 are constants resulting from integration of the energy expressions. See Table 2

If the cantilever is not trapezoidal, the term $4(a_j c_j)^{3/2}$ may be replaced by $4\ s_j^3$ where s_j is the thickness of the cantilever j at midheight.

Computed frequencies of thin arch dams using this formula compare well with measured values.

A thin arch dam located in an area with no known earthquake history or with a history of only minor earthquake activity should be designed to resist earthquake loadings of moderate intensity. For this purpose, moderate intensity loadings will be considered caused by a moderately strong earthquake located relatively close to the structure. Quantitatively, this moderately strong earthquake will produce a maximum ground acceleration of 0.1g in sound rock foundation at the damsite. A study of available strong-motion earthquake response spectra gives an average maximum acceleration magnification of about 3-1/2 for 5 percent critical damping. The maximum spectral acceleration response occurs in the range of periods from about 0.15 seconds to 0.6 seconds. The longest periods of thin arch dams will usually lie in this range. The base shear effectiveness factor of a thin arch dam is approximately 0.6 in both of the first two modes. Hence, the maximum base shear produced by dynamic loadings will be that caused by an acceleration of

$$(0.1g)(3.5)(0.6) \approx 0.2g$$

A dam subjected to a high intensity earthquake might have to sustain loadings three or four times as great as this value, provided the structure remained elastic during the motion.

If the configuration of an arch dam is such that the two longest vibration periods fall outside the range 0.15 to 0.6 seconds, the problem of earthquake loadings may require a somewhat different approach than that presented in this paper.

Computation of Stresses

The first and second mode loadings should be applied separately to the dam and the stresses computed for each loading. Then, to obtain the effective earthquake stress at any point in the dam, the first and second mode stresses are combined using the equation

$$\sigma_e = \pm \sqrt{\sigma_1^2 + \sigma_2^2} \tag{3}$$

where σ_e is the effective earthquake stress and σ_1 and σ_2 are the stresses computed for Modes 1 and 2. See Reference 12. Finally, the effective earthquake stress is combined algebraically with the static stress to get the total effective stress at any point in the structure.

In order to test the effects of these earthquake loadings, the method was applied to Morrow Point Dam, under construction by the Bureau of Reclamation. Morrow Point Dam is on the Gunnison River in Colorado and is an important feature of the Bureau's Colorado River Storage Project. The dam will be a 465-foot high, thin, double-curvature arch dam, and will have a volume of approximately 360,000 cubic yards. The top of the dam is 12 feet thick; crest length is 710 feet. The radius of the axis is 375 feet, and the base of the crown section has a thickness of 52 feet. See Figure 5. Stress computations were made by the trial load method.

Morrow Point Dam will be located in an area of low to moderate seismic activity. The most intense earthquake on record in the vicinity occurred in October 1960, and was estimated to be of maximum intensity VI on the Modified Mercalli Scale. See Reference 13. The computed natural frequencies of the first two modes are 2.7 and 3.0 cycles per second with reservoir full. The dynamic earthquake loading applied to Morrow Point Dam was such as to produce a maximum base shear of 0.2g in each of the first two modes.

For static loading including temperature effects, the maximum compressive stress was found to be 900 psi at the crown extrados at elevation 6940, and the maximum tensile stress 90 psi at the abutment extrados at elevation 6865. Including earthquake effects, the maximum compressive stress was computed to be 1,700 psi at the crown extrados at elevation 7015, and the maximum tensile stress was 900 psi at the crown extrados, elevation 7165 (top of the dam).

In the top portion of the dam, where the large tensile stresses are computed, if cracking occurs it will undoubtedly manifest itself as slight, temporary openings in the vertical contraction joints. These will close as the structure goes into the opposite phase of the oscillation and will be permanently closed when vibration ceases.

The maximum total compressive stress, which is the important indicator of the ultimate strength of the dam, is less than 40 percent of the 4,500-psi utlimate compressive design strength of the concrete. For this type of dynamic earthquake loading, it is considered proper design to accept a maximum total compressive stress of one-half the ultimate compressive strength of the concrete.

Conclusions

Thin arch dams are elastic structures that can be excited to resonance by earthquakes, thus developing dynamic loadings in the dam.

A method of quantitatively determining these dynamic loadings and of combining the stresses developed therefrom has been presented.

Using the methods presented, the maximum compressive stress in the dam, including earthquake, should be limited to one-half the compressive strength of the concrete.

There is need for additional technical data regarding the natural frequencies and damping present in arch dams, spectral response curves for strong-motion earthquakes with damping appropriate for dams, effective hydrodynamic mass of water accelerated with arch dams, and the tensile strength of concrete when subjected to rapid loading.

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** The substance of this paper was presented in the unpublished paper "Proposed Earthquake Loadings for the Design of Thin Arch Dams" by Clair C. Crawford and Merlin D. Copen, presented at the Annual Meeting and Structural Engineering Conference of the American Society of Civil Engineers, held at San Francisco, California, October 14-18, 1963"

Table 1

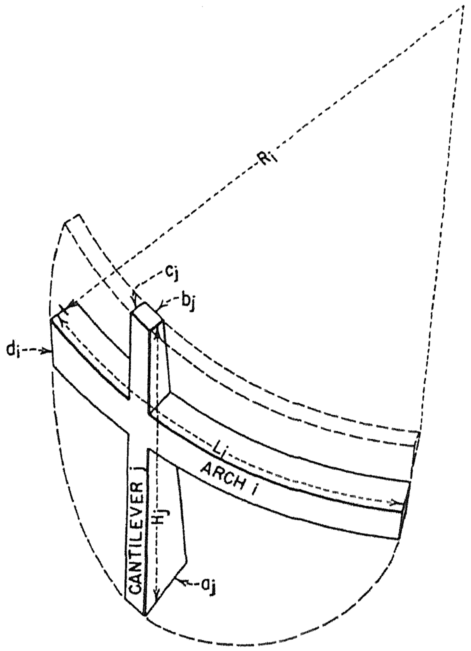
Relative Arch and Cantilever Deflections

Arch deflections			Crown cantilever deflections
Location	Mode 1	Mode 2	
0	0	0	1.0000
0.1	0.1191	0.3033	0.8624
0.2	0.3901	0.8027	0.7255
0.3	0.6902	1.0000	0.5909
0.4	0.9165	0.6837	0.4611
0.5	1.0000	0	0.3395
0.6	0.9165	-0.6837	0.2299
0.7	0.6902	-1.0000	0.1365
0.8	0.3901	-0.8027	0.0639
0.9	0.1191	-0.3033	0.0168
1.0	0	0	0

Table 2

Values of Constants for Use in Frequency Equation

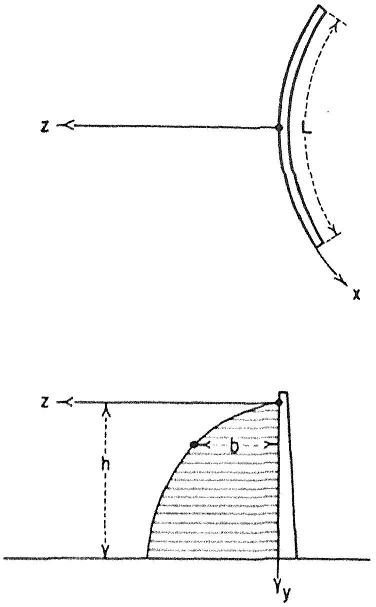
Constant	Mode 1	Mode 2
C_1	99.1	875
C_2	0.137	0
C_3	0.0278	0.0278
C_4	0.198	0.230
C_5	0.198	0.095



DEFINITION SKETCH

ARCH AND CANTILEVER
FOR FREQUENCY COMPUTATION

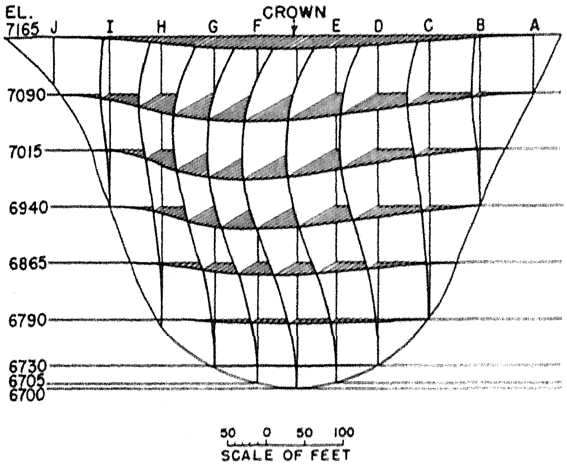
FIGURE 1



DEFINITION SKETCH

BODY OF WATER CONSIDERED
TO MOVE WITH DAM

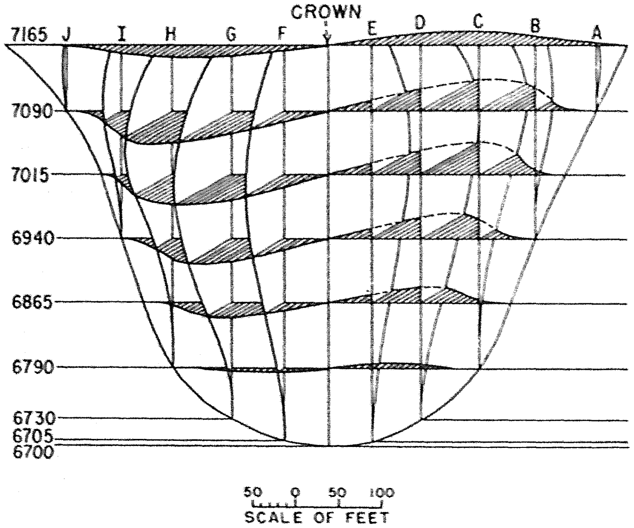
FIGURE 2



EARTHQUAKE INTENSITY LOADS
FIRST MODE OF VIBRATION
PROFILE ON AXIS
LOOKING DOWNSTREAM (DEVELOPED)

MORROW POINT DAM

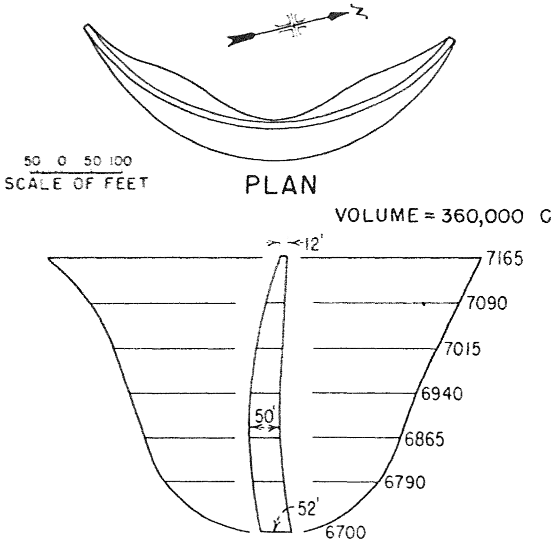
FIGURE 3



EARTHQUAKE INTENSITY LOADS
SECOND MODE OF VIBRATION
PROFILE ON AXIS
LOOKING DOWNSTREAM (DEVELOPED)

MORROW POINT DAM

FIGURE 4



PROFILE LOOKING DOWNSTREAM
AND CROWN CANTILEVER SECTION

MORROW POINT DAM

FIGURE 5

EARTHQUAKE DESIGN LOADINGS FOR THIN ARCH DAMS

BY C.C. CRAWFORD

QUESTION BY: **N.N. AMBRASEYS - UNITED KINGDOM.**

Were there any assumptions concerning the fixing of your dam with abutments. You assume fixing of some degree and under an earthquake one would expect some relaxation. What would be the effect of relaxation in which the greatest effect was water thrust.

AUTHOR'S REPLY: For the purpose of developing earthquake design loadings, it is assumed that the dam is fixed at the abutments. Of course, deformation of the abutments is included in the analysis of the dam after the design loadings have been determined.

QUESTION BY: **J.F. BORGES - PORTUGAL**

I think that the interest of model tests for dams design must also be emphasized. These tests may be particularly useful if they are performed using random noise vibrations according to a technique already described. Borges, Pereira, Ravara and Pedro - "Seismic studies on concrete dam models", Symposium on Concrete Dam Models, Laboratorio Nacional de Engenharia Civil, Lisbon, October 1963. The comparison of analytical and experimental results is in general very important for improving design.

AUTHOR'S REPLY: The author agrees that model tests can be an important aspect of dam design. He agrees also that comparison of analytical and experimental results is important for improving design. Discussion of these aspects of dam design, however, is outside the scope of the subject paper, which presents an analytical method for determining earthquake design loadings.

QUESTION BY: **T. HATANO - JAPAN**

Eq. (1) is derived from rigid vibration and two dimensional analysis. Thin arch dams also vibrate very elastically and the shape of the vibration is not as you state.

AUTHOR'S REPLY: In developing the earthquake loadings for the dam, the normal modes of vibration are studied separately. In the first arch mode, wherein all parts of the structure move upstream in phase, it is immaterial that the structure is very flexible rather than rigid. The effective hydrodynamic mass of water that can be assumed to move with the dam is determined by the fact that the structure is undergoing accelerated motion. The effective mass is not dependent upon the magnitude of the acceleration. In the higher modes, of course, less effective water mass is accelerated with the dam.

QUESTION BY: F.F. MAUTZ - U.S.A.

1. To what extent is the possible effect of seismic waves (or fresh water tsunamis) considered in the seismic stress analysis of an arch dam?
2. What is the effect of neglecting vertical cantilever action (modes) in the analysis where arch action modes alone are considered?

AUTHOR'S REPLY:

1. Waves generated by motion of the dam are not considered in developing earthquake loadings on the structure. The effect of the waves is considered negligible compared to the other phenomena involved.
2. Vertical cantilever action is included in the analysis, however, only the first cantilever mode is considered. The higher cantilever modes would relatively small loadings on the structure. In the interest of simplicity, only the modes developing the largest loadings are used. The effects on the structure of the smaller loadings are considered to be adequately embraced within the envelope provided by the large loadings.