SEISMIC SHEARS AND OVERTURNING MOMENTS IN BUILDINGS

by Jorge I. Bustamante [1]

Abstract

Seismic shears and overturning moments resulting from the analysis proposed in the code of the Structural Engineers Association of California are compared with their dynamic counterparts as obtained by modal analysis of elastic shear buildings. The same response spectrum is used as the one which served to establish the criterion for overturning moment in that code. Actual and imaginary buildings are studied. The cases in which the code is either not conservative or overconservative are discussed.

Nomenclature

$B_j^{(i)}$ = displacement of the jth floor in the ith mode of vibration

C = coefficient for base shear

F_{jsta} = horizontal force on the jth floor from the static analysis of the SEAOC Code

g = acceleration of gravity

H = height of the building

h_j = height of the jth story (between floors j-1 and j)

J = factor for reducing the static overturning moment at the base according to the SEAOC Code: $J=0.5/\sqrt[3]{T_1^2}$

J_b = ratio of dynamic to static overturning moments at the base

K = coefficient to account for the type of structure. It is given in Tabla 23-C of the SEAOC Code

k_j = stiffness of the jth story

$M_j^{(i)}$ = overturning moment component in the ith mode at the jth floor

M_{jdyn} = overturning moment at the jth floor from a dynamic analysis. The zero subscript refers to the base level

M_{jSE} = overturning moment at the jth floor from the static analysis of the SEAOC Code

M_{jDF} = overturning moment at the jth floor from the static analysis of the Proposed Mexico's Federal District Code; the shear forces are computed using the SEAOC Code

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M_{jmod} = overturning moment at the jth floor resulting from applying a force of five percent of the base shear at the top of the building when the Proposed Mexico's Federal District Code is used

m_j = mass of the jth floor

n = total number of floors of the building

T_i = period of vibration of the ith mode

$V_j^{(i)}$ = shear force component corresponding to the ith mode at the jth story

V_{jdyn} = shear force at the jth story from a dynamic analysis

V_{jsta} = shear force at the jth story from the static analysis of the SEAOC Code

W = weight of building from top to base

$\ddot{X}_{ie}$ = ordinate of the acceleration spectrum corresponding to the period T_i

Z = elevation of a point of the structure measured from its base

Z_j = elevation of the jth floor measured from the base of the structure

$\bar{Z}_j$ = elevation from the jth floor to the center of mass of the portion of the building above

$(\ )^{(i)}$ = refers to the ith modal component

$(\ )'$ = is used when the dynamic response is estimated as the sum of the absolute modal responses; when this symbol is not used, the response is obtained as the square root of the sums of the squares of the modal components

Equations

The following equations are used.

$$V_j^{(i)} = \sum_{k=j}^{n} \left[\frac{\sum_{j=1}^{n} m_j B_j^{(i)}}{\sum_{j=1}^{n} m_j (B_j^{(i)})^2} m_k B_k^{(i)} \right] \ddot{X}_{ie} \qquad (j=1,2,\ldots,n)$$

$$\frac{\ddot{X}_{ie}}{C(T_1=0.25)Kg} = \left\{\begin{matrix} 0.25/T_1, \text{ if } T_1 \geq 0.25 \text{ sec} \\ 0.30+2.80T_1, \text{ if } T_1 \leq 0.25 \text{ sec} \end{matrix}\right\} ; \left(\frac{\ddot{X}_{ie}}{Kg}\right)_{maximum} = 0.079$$

$$C = 0.05T_1^{-1/3} \text{ , } T_1 \text{ in sec}$$

$$V_{jdyn} = \sqrt{\sum_{i=1}^{n} (V_j^{(i)})^2} \qquad (j=1,2,\ldots,n)$$

$$V'_{jdyn} = \sum_{i=1}^{n} \left| V_j^{(i)} \right| \qquad (j=1,2,\ldots,n)$$

$$M_{j-1}^{(i)} = \sum_{k=j}^{n} V_k^{(i)} h_k \qquad (j=1,2,\ldots,n)$$

$$M_{jdyn} = \sqrt{\sum_{i=1}^{n} (M_j^{(i)})^2} \qquad (j=0,1,\ldots,n-1)$$

$$M'_{jdyn} = \sum_{i=1}^{n} \left| M_j^{(i)} \right| \qquad (j=0,1,\ldots,n-1)$$

$$F_{jsta} = KCW \frac{m_j Z_j}{\sum_{j=1}^{n} (m_j Z_j)} \qquad (j=1,2,\ldots,n)$$

$$V_{jsta} = \sum_{k=j}^{n} F_{ksta} \qquad (j=1,2,\ldots,n)$$

$$M_{oSE} = J \sum_{j=1}^{n} V_{jsta} h_j$$

$$J = \frac{0.5}{\sqrt[3]{T_1^{\ 2}}}$$

$$M_{jSE} = \frac{H-Z_j}{H} M_{oSE} \qquad (j=1,2,\ldots,n-1)$$

$$M_{(j-1)DF} = V_{jsta} \overline{Z}_j \qquad (j=1,2,\ldots,n)$$

$$M_{(j-1)mod} = V_{jsta}\overline{Z}_j + 0.05V_{1sta}\sum_{k=j}^{n} h_k \qquad (j=1,2,\ldots,n)$$

$$\overline{Z}_j = \frac{\sum_{k=j}^{n} m_k(Z_k - Z_{j-1})}{\sum_{k=j}^{n} m_k} \qquad (j=1,2,\ldots,n)$$

$$J_b = \frac{M_{odyn}}{\sum_{j=1}^{n} V_{jsta}h_j}$$

$$J'_b = \frac{M'_{odyn}}{\sum_{j=1}^{n} V_{jsta}h_j}$$

Introduction

Buildings subjected to seismic excitations generate inertia forces that produce shears and overturning moments. Generally these are evaluated for design purposes using the requirements of a building code. The object of this paper is to gather information on the differences between the shears and overturning moments computed according to the SEAOC Code (they will be called static results in the following) and a rational dynamic analysis consistent with the hypotheses under which the overturning requirements of that code were established (1). The information obtained should lead to establish limitations of the code and to suggest improvements.

This study treats actual buildings as well as hypothetical ones (imaginary buildings) whose rigidities and masses are such that the differences between the dynamic and static results are emphasized.

Since the structures studied are of the shear type, comparison with the SEAOC Code is restricted to code requirements for buildings with height to base ratio smaller than five and no reference is made to the SEAOC specification of a concentrated force to be assumed acting at the top of slenderer structures.

Dynamic Analysis

The buildings analyzed were assumed linearly elastic; damping and nonlinear effects are taken to be present in the spectrum used and in the coefficients for base shear. The structures examined were of the shear type with the masses concentrated at the floor levels. No torsional components of vibration were considered, and soil-structure interaction was disregarded. Modal analysis was used to compute shear forces and overturning moments. These were obtained as the square root of the sum of the squares of modal components, as proposed in Ref. 2. The response spectrum used for the modal analysis is drawn in Fig. 5; it is the same as the one used by Rinne(1) in the development of the overturning moment requirements of the SEAOC Code. This response spectrum is rational in that it is proportional to the envelope of normalized spectra of recorded California earthquakes.

Static Analysis

The method of analysis prescribed by the SEAOC Code is here referred to as static analysis. The fundamental period of vibration, required to compute the base shear coefficient C, is calculated here using standard dynamic theory.

Structures Studied

A total of 31 actual and imaginary structures were studied; their heights and their mass and story-stiffness distributions are given in Figs. 1 and 2. The latter shows the masses and story stiffnesses for the imaginary buildings having a fundamental period T_1=0.25sec. The masses of imaginary buildings were chosen to correspond to fundamental periods of 0.25sec (series L) and 2.0sec (series U). The masses associated to the 2.0-sec period are obtained multiplying the ones shown in Fig. 2 by 64. In the imaginary buildings the story height was assumed constant while the actual heights were used for the analysis of real buildings.

Altogether, 14 real cases were studied corresponding to 11 building (three were analyzed in two directions), 10 of them built in Mexico City and the remaining one in San Francisco. One of the buildings provided an abnormally long period (8.53sec) due to misinterpretation of the stiffness data. It is included assuming that it represents a much taller building.

The periods of imaginary structures were set at 0.25 and 2.0sec to study critical conditions of application of the Code; in the stiffer buildings, effects of the first mode are emphasized while those of the higher modes are reduced; in the longer-period structures the contributions of the higher modes are amplified. This is brought out by examining the acceleration spectrum (Fig. 5).

The distributions of mass and stiffness in the buildings are schematically represented in Figs. 3 and 4. As shown, there are imaginary buildings in which the fundamental period of the upper part was made equal to that of the lower portion; the influence of the mass was then studied by reducing the mass and the stiffness while mantaining the period. Other combinations of mass and stiffness were also studied.

Static and Dynamic Shear Forces

The ratios between static and dynamic shears are given in Fig. 6 for the real buildings and in Fig. 7 for the imaginary ones. In none of the actual buildings does the dynamic shear exceed its static counterpart. At the same time, the ratio in question increases markedly towards the top of the buildings reflecting the influence of higher modes. Accordingly, the factor of safety is much smaller at the top than at the bottom; this effect, even more pronounced, is observed in the imaginary buildings.

The type of analysis which the Code specifies gives shears at the upper stories that may considerably underestimate the dynamic shears; however, all the cases in which this happens correspond to structures with drastic reductions of mass in the upper floors. The effect is undoubtedly emphasized by the coincidence of periods of the upper and lower parts. It might seem that the setback requirements of the Code, which calls for a separate analysis of the upper portion of the building when its area is less than 75 percent of the lower part, should cover these cases; yet it does not, at least for the buildings selected. It was observed that when the analysis of the isolated part gives the same or larger values than those obtained from considering the building as a unit, the dynamic analysis no longer provides shear forces in excess of the static forces. This contention was verified in buildings 39 to 47, which have a setback and a lower part both four stories in height; it was also verified on building 35, which has an eight-story tower and only one heavy story below it.

The setback requirements of the Code, therefore, do not protect a building from large whipping of the uppermost floors under seismic excitation. Instead the results obtained indicate that a better criterion to decide if the mass of the setback is large enough to avoid the whipping effect consists in comparing the shear forces computed by assuming that the upper part stands directly on the ground, with those obtained from analyzing the building as a unit; if the latter are larger, it is unlikely that whipping action will be worth considering.

Presumably, the Code requirements for setbacks intend to cover also increased shear at the base of eccentric setbacks due to torsion. In such cases the Code may, of course, err to a greater degree on the unsafe side, since it already errs sometimes on that side without torsion.

The critical cases presented here require such a large reduction of mass from one part of the building to the other that they cannot represent usual structures. One can expect mass reductions of this magnitude only in television towers and other light structures on top of buildings.

It was noticed that the deflections of the critical structures under the static loads show an abrupt change in slope between the two bodies of the buildings(3). Such a pronounced change was not apparent in the other real or imaginary buildings.

Despite their varity, the extreme cases studied do not constitute limiting conditions. If masses in the upper part are further reduced, the ratio of dynamic to static shears will increase accordingly. Yet these large ratios need not be a source of danger or of costly structural solutions in practice, if adequate provisions are adopted, as the shears in question are actually quite small. This suggests the convenience of adding a small force at the top floor to obtain static shears larger than the dynamic ones, even for the critical cases. A static force of five percent of the base shear provides forces larger than the dynamic ones in all the cases studied. After a preliminary design, the foregoing deflection criterion is useful to discern the need for assuming this force. In any event the increment of a small force will not appreciably increase the cost of a structure designed for large shears, and for cases with abnormally small shears the addition of such a force is indeed advisable. This criterion seems adequate for shear forces alone. It will be shown that it is also valid for overturning moments.

Shear Force at the Base

In all the cases studied, real and imaginary, the static base shear was always larger than or equal to the dynamic shear. This is strictly the case for buildings with fundamental period equal to or less than 0.25sec, series L(4). For tall buildings with long fundamental periods a gross computation is feasible; if it is assumed that the only modes which contribute significantly to the base shear are those having periods equal to or greater than $T_1/5$, then one can assure that, for T_1 not smaller than 2.8sec, the static analysis gives an upper bound to the base shear. For most buildings, moreover, only the first two modes contribute measurably to the base shear. In this common case, if the second period is not less than $T_1/3$, the dynamic shear will be less than the static one provided $T_1 = 1.3$sec. These conclusions are applicable to the spectrum used and to the base shear coefficient of the Code. They are based on the fact that, if the base shear coefficient is not smaller than the acceleration spectrum ordinates divided by g, then the static shear is an upper bound to the dynamic one(4).

The exclusive use of the first mode as design criterion is inadequate, even at the base, since it may seriously underestimate dynamic shears. For the real buildings the ratio $V_{1dyn}/V_1^{(1)}$ was 1.1 to 1.3 and exceptionally 1.5; for the imaginary ones, 1.3 was a common ratio but it reached the surprising values of 9.5 and 41.1 for buildings 29 and 37, respectively, both of series U. This matter will be discussed later.

The first mode may give estimates of the dynamic shears that are considerably in error on the unsafe side. The errors may be much larger at the upper floors. There are cases when the static analysis yields more accurate results than taking only the first mode of vibration.

The ratios 9.5 and 41.1 mentioned above can be explained by compar ing buildings 29 and 37 with building 25. The eight floors of the latter are virtually the same as the corresponding portions of buildings 29 and 37 (Fig. 2). The ninth mass on top of these two buildings causes their fundamental modes practically to have no displacements throughout most of the structure, while the second modes resemble the fundamental in building 25. In both exceptional cases it is precisely the second mode which produces the exceedingly large contribution to shear. In contrast, the fundamental mode for building 27 is, in its first eight stories, almost identical with that of building 25, despite the apparent similarity between buildings 27,29 and 37.

Overturning Moment at the Base

Overturning moments obtained from static shear (moments which are called static in this paper) normally constitute gross overestimates, as their computation ignores the fact that maximum shears do not act simultaneously, nor even with the same sign, at all stories. To account for this situation the SEAOC Code specifies the reduction factor $J=0.5T_1^{-2/3}$ but not less thar 0.33 nor more than 1.00, where T_1 = fundamental period of the building, in seconds.

The ratios of the dynamic to the static overturning moments appear in Fig. 8 for all buildings studied, real and imaginary. The latter can be recognized by their fundamental periods of 0.25 and 2.00sec. The two most widely accepted forms of combining modal responses are included; the resulting ratios are called J_b for the square root criterion and J_b' for the sum of absolute values. The SEAOC J factor is drawn on the same figure.

It is concluded that there may be large differences between the computed ratios and the reduction factor of the SEAOC Code. The errors involved may be as large as 100 percent, even for moderate periods, of less than 35sec; however, all are on the safe side. These considerations are formulated assuming that there are no errors in the estimation of the fundamental periods of the buildings.

In Fig. 8 it can also be noticed that J_b and J_b' are relatively close to each other.

Overturning Moment Distribution

The ratio of dynamic to SEAOC overturning moments for all stories of the buildings are plotted in Figs. 9 (real buildings) and 10 (imaginary structures, series L and U). Fig. 9 also shows the ratios obtained using the static analysis of the proposed Mexico's Federal District Code(5), which will be called the Mexican Code. From these figures one can conclude that the SEAOC moments are always larger than the dynamic moments, even for the buildings where dynamic shears were considerably greater than SEAOC shears. The ratio of static and dynamic moments increases pronouncedly towards the top. This tendency is opposite the one concerning shears, for which the larger factors of safety were at the bottom.

As seen in Fig. 9 the Mexican Code requirements ask for larger factors of safety at the base than at the upper levels at least for buildings whose important modes have periods longer than 0.25sec for the SEAOC spectrum. The moments are obtained by multiplying the story shear by the distance to the center of mass above the elevation considered. For each building, these moment ratios match closely the corresponding shear ratios, giving therefore about the same factor of safety for shear and overturning at the base. A proposal similar to the Mexican Code may, therefore, seriously underestimates the overturning moments in the critical buildings discussed in relation to shear forces. The criterion proposed in connection therewith, which consists in applying at the top a force of five percent of the base shear, will cover the shear as well as the overturning moment requirements and will also provide much more uniform factors of safety throughout the building than the ones given by the SEAOC Code.

To substantiate this discussion, Fig. 12 shows the ratios of dynamic to static overturning moments, computed according to the foregoing paragraph, for the imaginary buildings, series L.

It is a moot question whether the factor of safety against overturning should preferably increase or decrease with height in a given building. Against shear it whould no doubt decrease, as only structural failure is involved, but overturning moment may involve partial lifting of the foundation, a phenomenon which may or may not actually constitute a type of failure, and the prevention of which often involves expenditures that are out of proportion compared with those required to insure a reasonable degree of safety against failure in upper stories. The question is complicated by the fact that, if the structure is securely anchored at the foundation elevation, the factor of safety against overturning should preferably be a decreasing function of height, but if base rotation can take place (and the foundation rigidity is sufficient to prevent secondary types of damage), overturning moments in the lower stories will be bounded and usual criteria will lead to overdesign in the lower portion of the building. The large factors of safety against overturning which the SEAOC provides in the upper stories may not be objectionable, except when overturning moment governs the design. Further study of the matter is clearly warranted.

Conclusions

The study presented has been confined to some real buildings of different heights and some imaginary buildings of 8 or 9 stories, all of the shear type, analyzed elastically. It was assumed that the estimation of the fundamental period was accurate. Despite these restrictions the study gives some insight into the problem. The following conclusions are drawn therefrom.

1. The requirements of the SEAOC Code for buildings with height to base ratios smaller than five provide shear forces that are in general larger than the dynamic seismic forces. This was true

for all the real and most of the imaginary buildings examined.

2. The only cases when the foregoing conclusion does not hold, are those of structures with setbacks having a prominent reduction in mass. The problem is especially acute when these portions, if considered isolated, have natural periods of vibration equal or close to the ones of the rest of the building. However, the mass reduction must be such that it is not to be expected in a usual building, save for television or radio towers, elevated tanks, and similar structures standing on top of the building. The setback requirement of the SEAOC Code will not cover this contingency.

3. If a force of five percent of the base shear is assumed at the top of the building, in addition to the forces required by the SEAOC Code for nonslender buildings, the shears obtained in all the structures studied are larger than the dynamic forces. This will usually not increase the cost of the structure appreciably.

4. For most practical buildings the SEAOC Code will provide a base shear force larger than the dynamic one.

5. Overturning moments obtained according to the SEAOC Code are always larger than those computed dynamically. This is so for all the real and imaginary buildings. The factors of safety are quite large near the top and much smaller near the base.

6. Factors of safety provided by the Code for base shears are much larger than for overturning moments at the base, save for buildings having a fundamental period smaller than 0.25sec.

7. An overturning moment computation as prescribed by the Mexican Code, plus the moments due to a force of five percent of the base shear, applied at the top of the building, provides static moments larger than the dynamic ones. This artifice leads to a more uniform factor of safety than the SEAOC provisions throughout the height of the building. The factor of safety is then approximately equal to that for story shears.

8. The dynamic analysis of a structure permits in general, the use of shear forces and moments smaller than those required by Code; it also leads, usually, to a better distribution of stiffnesses throughout the height and, consecuently, a better seismic behavior of the structure. It is pertinent to mention the objectionable effects of partial overdesign under nonlinear behavior.

9. A dynamic analysis based on only the first mode of vibration may give shears and moments with larger errors than a static analysis and on the unsafe side throughout the entire height of a building.

Acknowledgement

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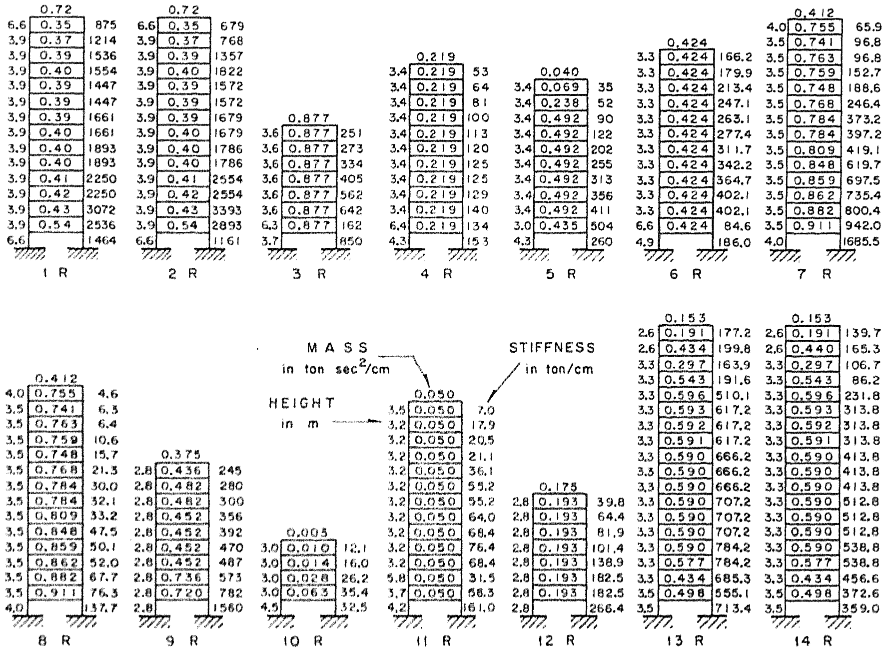

FIG. 1 HEIGHT, MASS, AND STIFFNESS DISTRIBUTIONS OF THE REAL BUILDINGS

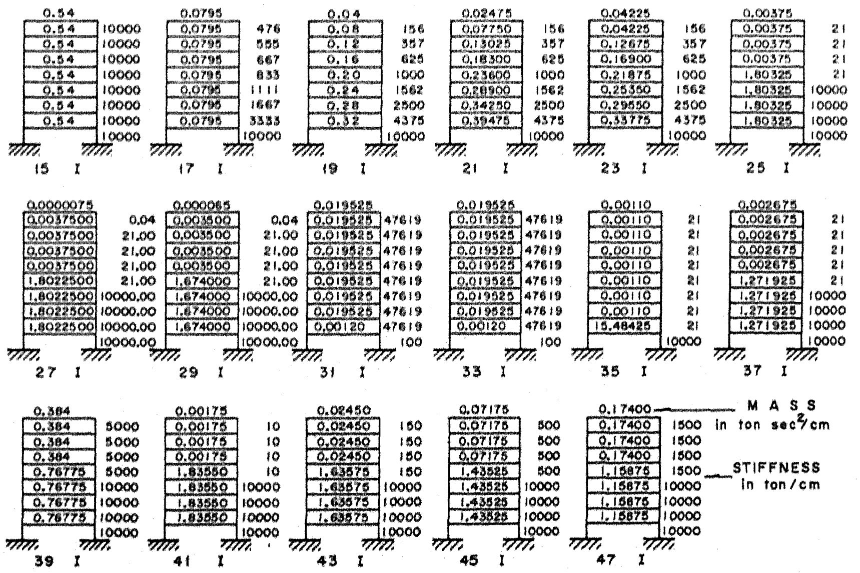

FIG. 2 HEIGHT, MASS, AND STIFFNESS DISTRIBUTIONS OF THE IMAGINARY BUILDINGS

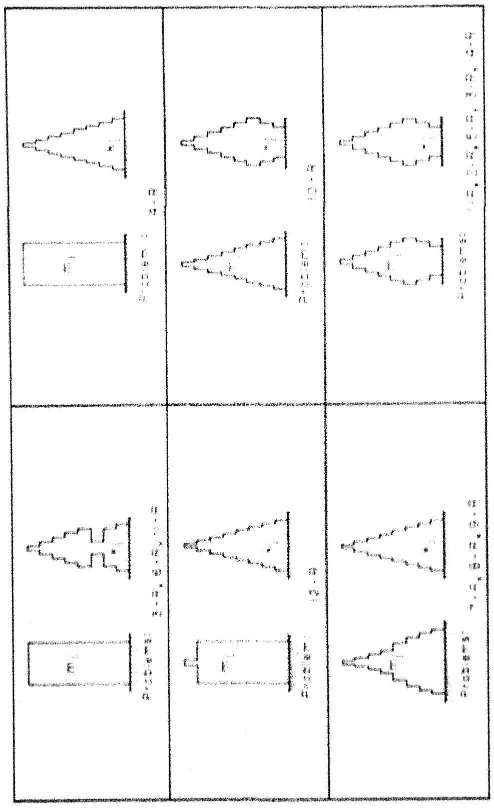

FIG.3 SCHEMATIC REPRESENTATION OF MASS AND STIFFNESS VARIATIONS IN THE REAL BUILDINGS

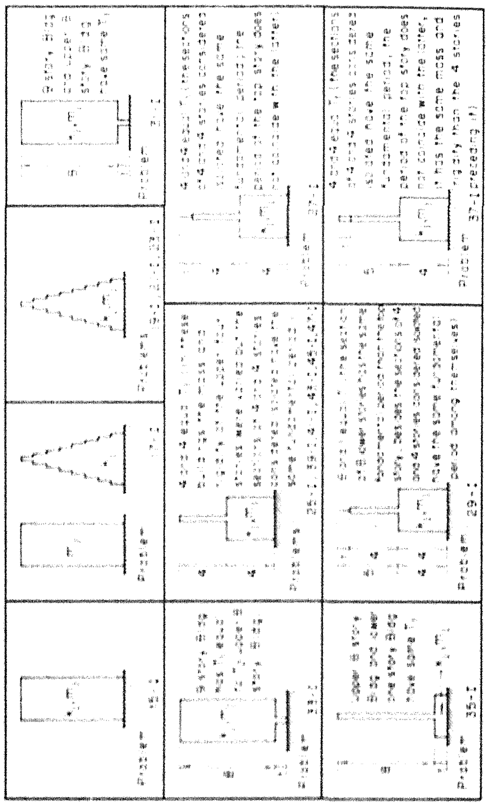

FIG.4 SCHEMATIC REPRESENTATION OF MASS AND STIFFNESS VARIATIONS IN THE IMAGINARY BUILDINGS

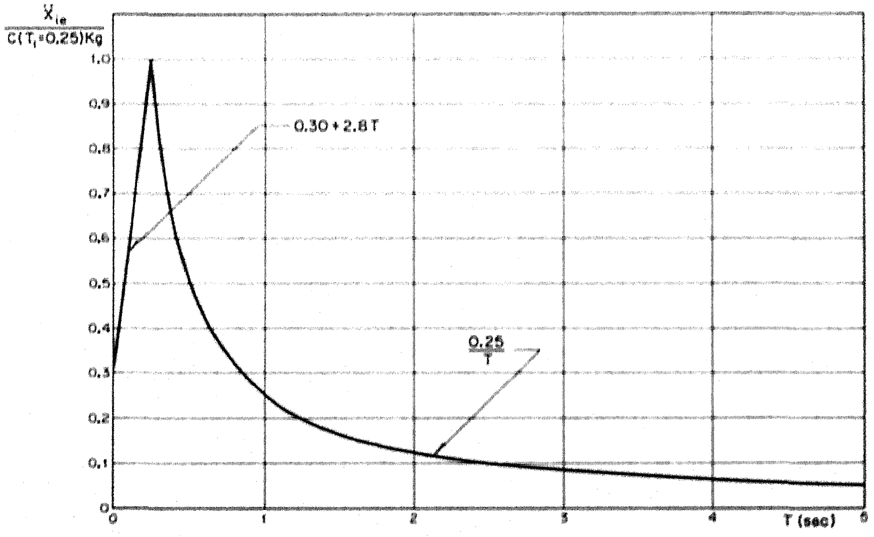

FIG. 5 ACCELERATION SPECTRUM

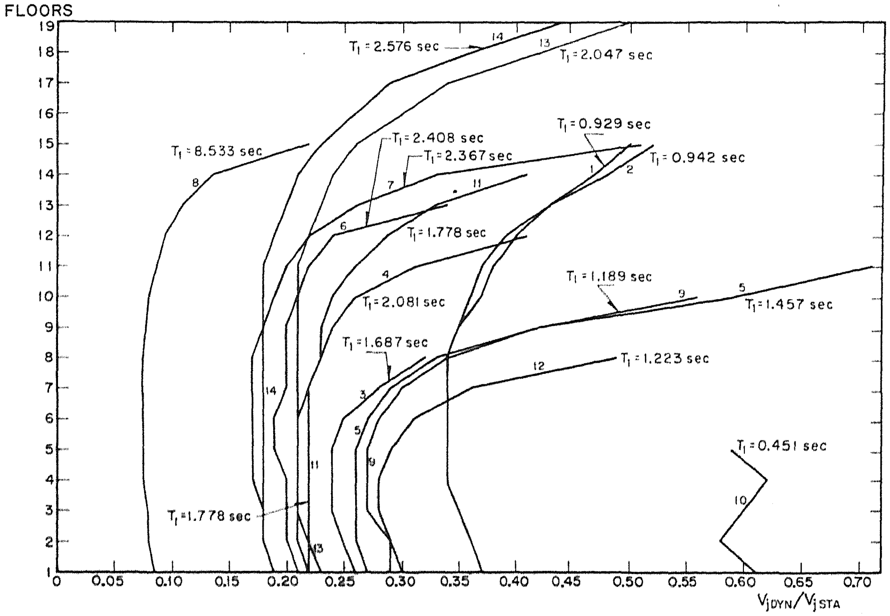

FIG. 6 RATIO OF DYNAMIC TO STATIC STORY SHEAR. REAL BUILDINGS

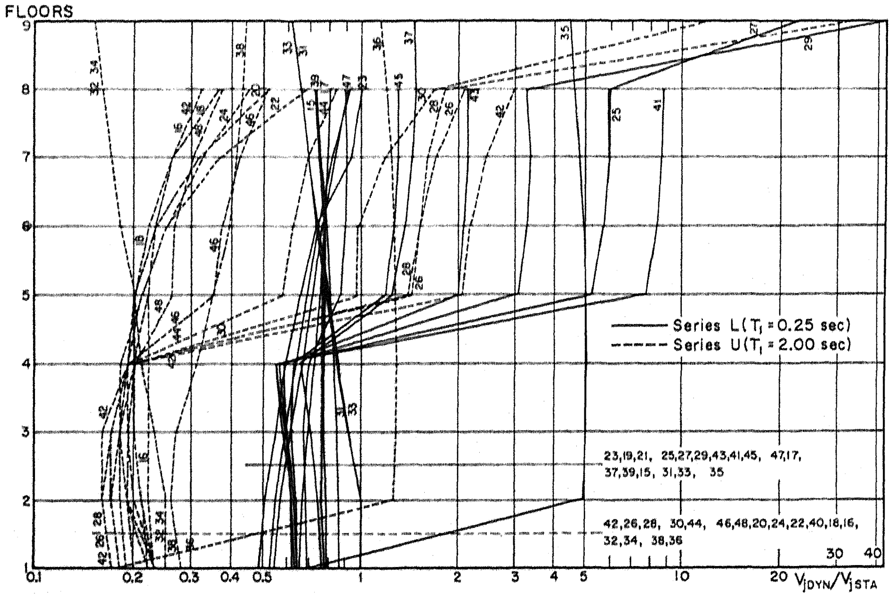

FIG. 7 RATIO OF DYNAMIC TO STATIC STORY SHEAR. IMAGINARY BUILDINGS, SERIES L(T_1=0.25 sec) AND SERIES U(T_1=2.00 sec)

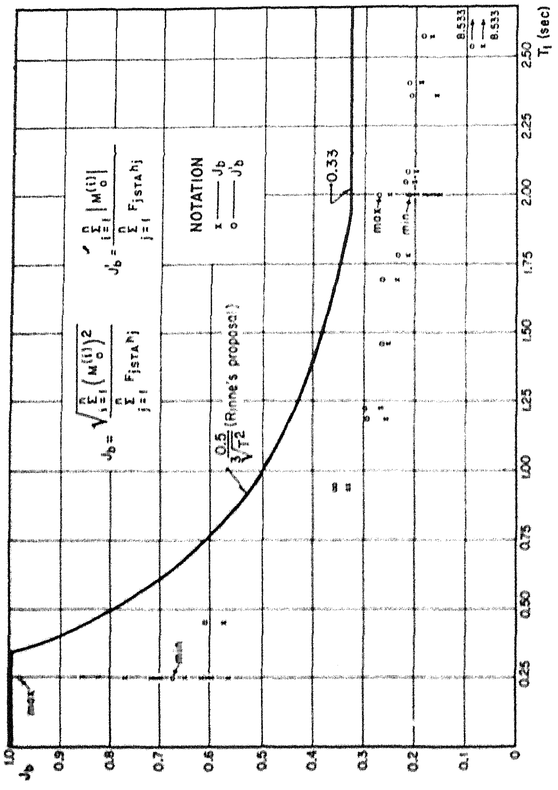

FIG. 8 REDUCTION FACTOR FOR ALL BUILDINGS

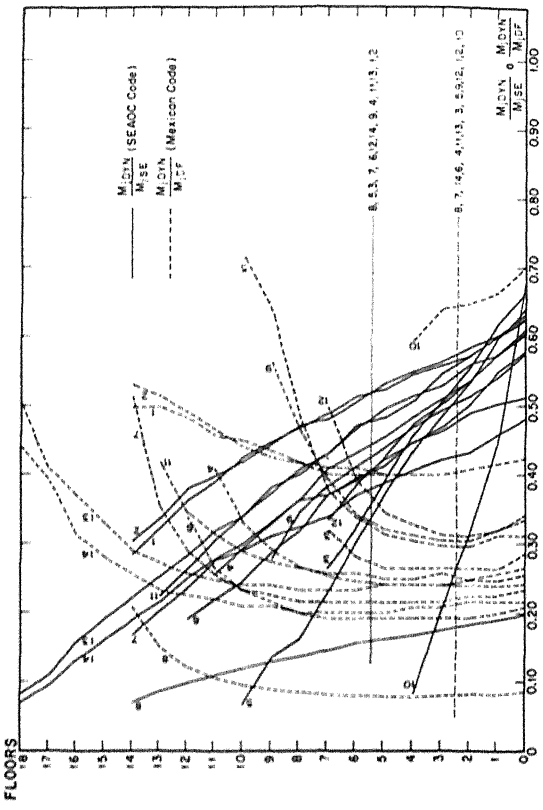

FIG. 9 RATIO OF DYNAMIC TO STATIC OVERTURNING MOMENTS AS OBTAINED USING THE SEAOC AND MEXICO'S PROPOSED FEDERAL DISTRICT CODE. REAL BUILDINGS

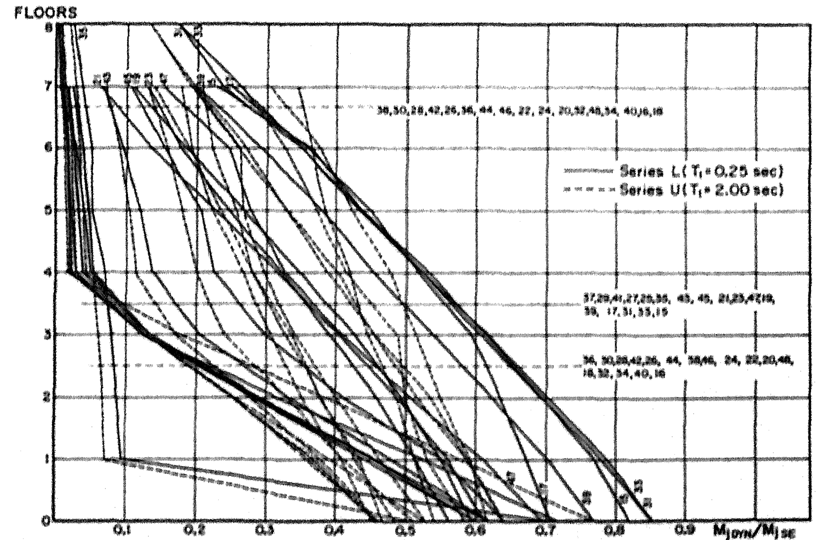

FIG. 10 RATIO OF DYNAMIC TO STATIC OVERTURNING MOMENTS AS OBTAINED USING THE SEAOC CODE. IMAGINARY BUILDINGS, SERIES L(T_1=0.25 sec) AND SERIES U(T_1=2.00 sec)

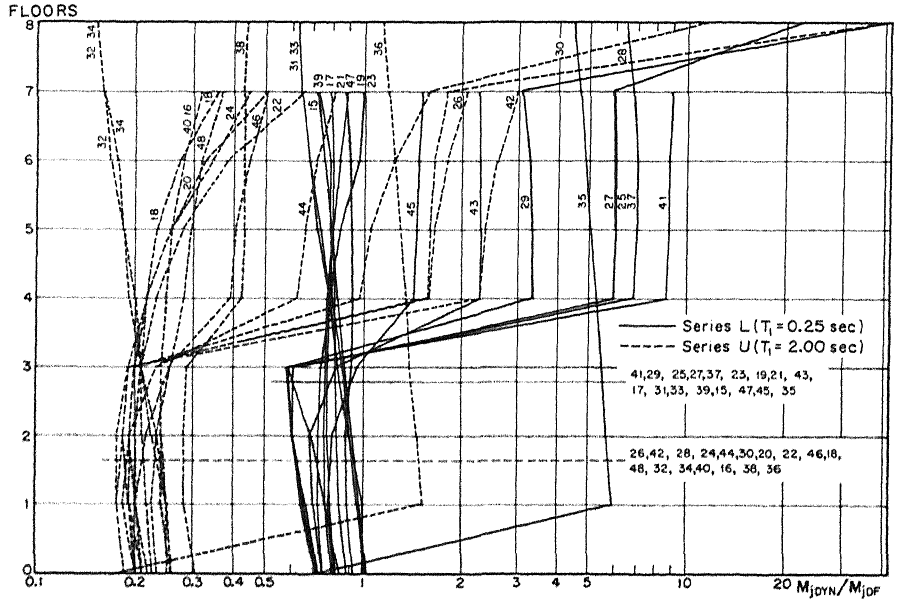

FIG. 11 RATIO OF DYNAMIC TO STATIC OVERTURNING MOMENTS AS OBTAINED USING MEXICO'S PROPOSED FED. DIST. CODE. IMAGINARY BUILDINGS, SERIES L(T_1=0.25sec) AND SERIES U(T_1=2.00sec)

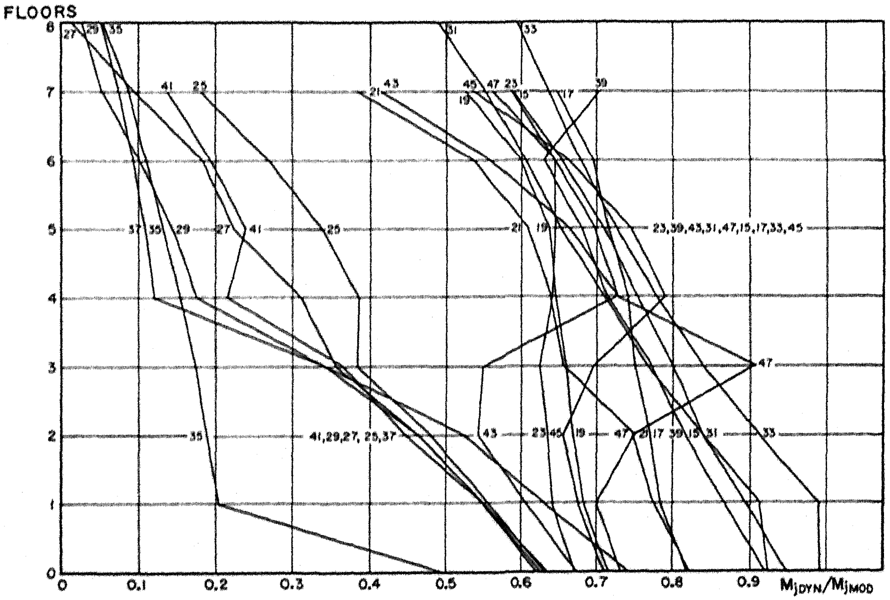

FIG. 12 RATIO OF DYNAMIC TO STATIC OVERTURNING MOMENTS, THE LATTER WERE OBTAINED USING THE CRITERION MEXICO'S PROPOSED FEDERAL DISTRICT CODE AND A FORCE OF 5% OF THE BASE SHEAR APPLIED AT THE TOP OF THE BUILDINGS. IMAGINARY BUILDINGS SERIES L(T_1=0.25 sec)

SEISMIC SHEARS AND OVERTURNING MOMENTS IN BUILDINGS

BY J. I. BUSTAMANTE

QUESTION BY: R.W. BINDER - U.S.A.

The work of SEAOC may be worth noting in connection with the use in analysis of a concentrated force at the top of the structure, making the ratio of this force to the base shear of a continuous function of the height-to-base ratio in bending type structures. As you may know the basic code makes provision for a minimum concentrated force of 10% at the top of all types of structures when the height-to-base ratio is 5 or more.

AUTHOR'S REPLY: The SEAOC requirement for the case of height-to-base ratio of 5 or more is proved in this paper to be somewhat conservative for shear-type buildings. For bending type structures, specially chimneys I will hesitate to comment about specific numbers without further research. The study presented here is not applicable for those structures.

COMMENT BY: E. ROSENBLUETH - MEXICO

Could you correct all your studies for bending type buildings such as Binder refers to i.e. those which deflect in flexure. The errors pointed out in the comparison between the reduction factor proposed, on the basis of those proposed by J. Rinne lead us to the conclusion that the latter are on the safe side.

AUTHOR'S REPLY: Apparently it was not properly emphasized during the oral presentation the J. Rinne's proposal gives reduction factor that are on the safe side, assuming the ordinate spectrum proposed by the SEAOC code is adequate.