SPECTRAL THEORY OF SEISMIC FORCES

BY A. ARIAS* AND L. PETIT-LAURENT*

ABSTRACT

PART 1. - SPECTRAL CHARACTERISTICS OF STRONG-MOTION ACCELEROGRAMS

INTRODUCTION

The following characteristics of the accelerograms of strong-motion earthquakes appear as obvious even on superficial examination:

a) Earthquakes are transient and hence non-stationary phenomena of finite duration;

b) Acceleration as a function of time exhibits a high irregularity;

c) Different parts of the same record show strong variations in their properties, specially in the extreme values of acceleration.

These characteristics might lead to the conjecture that the only common property of different accelerograms is precisely their randomness.

In 1947, Housner proposed a mathematical model of strong-motion accelerograms in which the earthquake is considered as a random sequence of pulses. Following the same line of thought several investigators have represented the accelerograms as white noise. The adequateness of these models of seismic motion has been tested by comparing the maximum response spectra of simple linear oscillators subjected to these pseudo-earthquakes with the maximum response spectra of the same oscillators for real earthquake records.

Some shortcomings of this procedure may be pointed out:

a) The maximum response spectra of simple linear oscillators do not represent the properties of ground motion directly, because the

---

* Universidad de Chile, IDIEM.

frequency response of the oscillator is involved in the computation of this kind of spectra; thus, the differences between real earthquakes and pseudo-earthquakes obtained as explained above, are not revealed to their full extent by this method of comparison;

b) Maximum response spectra of simple linear oscillators are obtained by operations on the accelerogram that are not linear. The computations include the calculation of the extreme values of Duhamel's integral. Thus, it is impossible to trace back the steps from the final result to obtain the properties of ground motion itself. In particular, the power density spectra of ground motion cannot be obtained by tracing back the operations through which the maximum response spectra are reached. This implies, also, that it is not possible to evaluate exactly the influence of the frequency response characteristics of the recording instruments on the ordinates of the maximum response spectra of linear oscillators.

The maximum response spectra technique is very adequate for the earthquake design of one-degree-of-freedom linear structures by modal analysis, but it fails altogether when we come to non-linear structures. The difficulty of earthquake design problem of non-linear structures arises from the fact that the principle of superposition of effects does not hold, so the maximum response spectrum technique, based on this principle, cannot be applied, and we have to go back to the accelerogram itself considered as a forcing function, to study the response of non-linear systems.

Unfortunately, with very few exceptions, the number of earthquake records available at a given place is at present very scarce, and altogether insufficient for statictical studies of the response of non-linear structures. The designer cannot wait until the necessary experimental information is accumulated. It is then desirable to have a mathematical model to represent the accelerograms of the earthquakes to be expected at a given locality. This model should be compatible with the principles of dynamics and with the experimental information on real strong-motion earthquakes available at the present time.

This paper deals with the analysis of real strong-motion earthquake records, The purpose of this analysis is to obtain an experimental foundation for a mathematical model of the accelerogram to be proposed in a companion paper (Part 2). The second paper contains a mathematical analysis of ground motion, following an idea of Kanai's, that leads succesively to a mechanical model of ground motion, a mathematical model of the accelerogram, and a formula for maximum response spectra - this last one being applicable, of course, only for the design of linear structures - that agrees very satisfactorily with the standard spectra proposed by Housner.

CONCLUSIONS

a) An examination of 38 power spectra obtained from acceleration records of strong-motion earthquakes shows that real earthquakes are far from being representable as white noise.

b) The correlograms of the acceleration records obtained in the U.S.A. and in Santiago de Chile are strongly "damped"; this means that for these earthquakes the frequency band width is rather large. The same conclusion can be obtained by an examination of the corresponding power spectra.

c) The correlograms of the records obtained in Mexico City show weak "damping"; this means that a substantial part of the energy is delivered in a relatively narrow frequency band. The same conclusion may be reached on examining the power spectra.

d) Mexico City records seem to contain important deterministic components.

e) The frequencies of the higher peaks that appear in the spectra of Mexico City are low if compared with the rest of the spectra. This may be - and very probably is - related with quality of soil in Mexico City. A somewhat similar remark is valid for the records obtained in Seattle in 1949.

f) Some of the spectra (v.gr. El Centro) contain important parts that resemble a rectangle. This implies that the corresponding accelerograms may be analyzed as a band limited white noise plus a residual.

g) Several spectra show peaks at frequencies that are multiples of some frequency ω_0 , or very nearly so. Several explanations may be offered:

First: the peaks may arise from the abrupt interruption of the auto-correlation function at a point where $\hat{R}(\tau)$ is not zero, or from the fact that the auto-correlation function has values of some importance near the end of the interval for which it was computed.

Second: the accelerogram might contain a non sinusoidal periodic component, of frequency ω_0, that, because of the computational procedure, is analyzed into sinusoidal components of frequencies ω_0, $2\omega_0$, $3\omega_0$, ...

Third : damped sinusoidal components of frequencies ω_0, $2\omega_0$, $3\omega_0$, ... may be present in the accelerogram; this possibility is related with the transmission of the earthquake waves through a medium having these natural frequencies.

For reason already given, the first explanation must be discarded in those cases where the spectra show several important peaks of similar heights.

h) The peaks that appear in the power spectra occur at the same frequencies at which peaks appear in the corresponding velocity response spectra of linear oscillators computed by Alford, Housner and Martel, Jennings, Lepe and Torres. However, at high frequencies, peaks that do appear in the velocity response spectra, do not appear, or are less marked, in the power spectra. This difference in behaviour of the two kinds of spectra can be explained because the gain of the one-degree-of-freedom linearly damped oscillator is not the same as the gain of Fourier's filter. Besides by definition, the response spectra represent the maximum response, therefore peaks are enhanced.

i) Power spectra are obtained from accelerograms by means of linear operators. This does not apply to maximum response spectra, that are obtained by computing the extreme values of Duhamel's integral. Therefore it is not possible to trace back the steps that lead to response spectra to obtain some of the statistical properties of ground motion itself. On the other hand, power spectra show some of the statistical properties of the accelerogram directly, and are independent of the properties of linear oscillators.

j) Once the power spectrum for ground acceleration is known, it is possible to obtain the power spectra for ground velocity and ground displacement, dividing the ordinates of the first by w^2 and w^4, respectively.

k) Power spectra for ground acceleration must be equal to zero for $w = 0$; furthermore, power spectra for ground acceleration must be of order w^2 near the origin. If the first condition is not fulfilled, the mean value of the acceleration would be different from zero and the ground velocity would increase infinitely with the time. If the second condition is not fulfilled, the mean value of ground velocity would not be zero, and ground displacement would increase infinitely with the time.

l) It is not necessary that the power spectrum for ground acceleration be of order w^4, or of higher order, near the origin. If it were of order w^4, the final displacement would tend to zero as the time increases, and this does not happen necessarily in real earthquakes.

m) Acceleration power spectra tend to zero when w increases infinitely. As a consequence, velocity and displacement power spectra tend to zero when $w \rightarrow \infty$. This condition is necessary, because if it is not satisfied, the total power would be infinite. In the range covered by the computations given in this paper, this condition appears to be satisfied by real earthquakes.

n) Kanai has proposed the following power spectrum for velocities:

$$S_{\dot{x}}(\omega) = G^2(\omega) = \frac{1 + 4h_g^2 \frac{\omega^2}{V_g^2}}{\left(1 - \frac{\omega^2}{V_g^2}\right)^2 + 4h_g^2 \frac{\omega^2}{V_g^2}} B \qquad (37)$$

where

h_g = damping ratio of ground
V_g = natural frequency of ground
B = constant $>$ o

This ground velocity power spectrum corresponds to a ground acceleration power spectrum given by:

$$S_{\ddot{x}}(\omega) = \omega^2 S_{\dot{x}}(\omega) \qquad (38)$$

Then

$$\lim_{\omega \to \infty} S_{\ddot{x}}(\omega) = 4h_g^2 V_g^2 B \neq 0 \qquad (39)$$

Then condition stated in m) is not fulfilled. Total power turns out to be infinite.

o) An alternative that seems to meet the objection put up in the preceding paragraph is to put:

$$S_{\ddot{x}}(\omega) = \frac{4h_g^2 \frac{\omega^2}{V_g^2}}{4h_g^2\left(1 - \frac{\omega^2}{V_g^2}\right)^2 + \frac{\omega^2}{V_g^2}} B \qquad (40)$$

This is obtained by computing the output acceleration power spectrum of a simple linear oscillator in which spring and dashpot are in series, subjected to a forcing function (input) whose velocity power spectrum is a constant.

The spectrum proposed by Kanai is obtained under the same assumption for the forcing function, but considering that the spring and the dashpot of the simple linear oscillator are in parallel.

The spectrum proposed (eq. 40) is of order w^2 at the origin and tends to zero when $w \to \infty$. Total power is finite now.

The idea contained in the last paragraph is further explored and justified in a companion paper.

x x x x

PART 2. - A THEORETICAL MODEL FOR STRONG-MOTION EARTHQUAKES

This companion paper has not been abstracted.