SEISMIC RESISTIVITY OF EXTENDED STRUCTURES

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When the length of a seismic wave considerably exceeds the dimensions of a structure in plan we may assume for practical purposes that the whole base of this structure will simultaneously have the same displacements. Therefore, when considering vibrations of the majority of buildings the system in the form of a flexible cantilever rod is used as a design scheme (Fig.1a). However, if the dimensions of a structure in plan may be compared with the length of a seismic wave this scheme will not be applicable since individual points of the base will not displace simultaneously. Though we may assume in this case that the motion of individual points of the base will be governed by the same law, these points will be, however, mutually displaces as to their phase in time.

For the approximate evaluation of forces induced in structures during the propagation of a seismic wave in ground there have been analysed the following two design schemes considered to be most typical. The first scheme applies to single-storey industrial buildings (single- or multi-spanned) in which the roof rests upon rather widely spaced separate columns while the second scheme applies to conventional civil buildings with continuous longitudinal walls.

In the first case the design scheme was presented in the form of a rigid solid body resting upon flexible supports (Fig. 1b) the base of which displaced with some mutual difference in time.[xx)] For simplifying the problem it was assumed that the roof was absolutely rigid, therefore the deformations produced in each post of the structure were instantaneously transmitted to the roof resulting in steel posts occurrence of corresponding elastic reactions.

In the second case it is more convenient to consider the structure as the system resting upon the single common base subjected to the action of the displaced ground the motions of which are different at various points under the base of the building. In this case various seismic effects of the ground in various parts of the base will be somehow added and averaged within the lower part of the structure. This mean effect will produce the vibrations of the structure as a whole. Hence, when the problem is formulated as

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xx) This design scheme will also be applicable when considering bridge-type structures.

above it is necessary and sufficient to evaluate the overall effect of the moving seismic wave on the base of the structure in order to take into account the length of this structure. When solving this problem it was assumed that the deformations of the base along its length might be neglected and the seismic wave be presented in the form of a certain harmonic curve moving with constant velocity. In this case the epure of the distribution of the intensity of accelerations along the length of the wave will also be harmonic, i.e. it will be similar to the epure of ground displacements.

In both cases only the motions occuring in the horizontal plane were considered.

The earthquake records show that ground motions are of a rather diversified character and they produce displacements not in one direction but in all directions irrespective of the location of the source of disturbance and, hence, the direction of the wave propagation. It frequently turns out in this case that the directions of maximum displacements and accelerations do not coincide with the direction on to the source of disturbance. Therefore, there have been considered two extreme cases assuming for the first case that the directions of displacements and accelerations coincide with the direction of the wave propagation while for the second case they are perpendicular to this propagation.

The above given assumptions used for the first design scheme (Fig. 1b) made it possible to formulate the initial equations in the following way.

Since the horizontal rod with the mass $\mathcal{M}$ is assumed as absolutely rigid, the correlation between the displacements of the bases of cantilever rods and their deflections at any moment will be:

$$y_A = y_{oa} + y_a = y_b + y_{ob} \tag{1}$$

where y_A - displacement of the mass $\mathcal{M}$;

y_{oa} and y_{ob} - displacements of the bases under the rods a and b ;

y_a and y_b - deflections of the rods a and b .

The equation of the equilibrium for this system will be as follows:

$$\mathcal{M}\frac{d^2 y_A}{dt^2} + (K_a + i\varkappa_a)y_a + (K_b + i\varkappa_b)y_b = 0 \tag{2}$$

where K_a and K_b - coefficients of rigidity of the rods a and b ;

$\varkappa_a$ and $\varkappa_b$ - coefficients characterizing damping of the rods a and b x).

Substituting by means of the equation (1) the values y_A and y_B for y_a (or y_A and y_a for y_b) and transposing all the terms containing as factors independent variables y_{oa} and y_{ob} to the right-hand side we shall obtain:

$$M\ddot{y}_a + (\sum K + i\sum \varkappa)y_a = -\left[M\ddot{y}_{oa} + (K_b + i\varkappa_b)(y_{oa} - y_{ob})\right]$$

or

$$M\ddot{y}_b + (\sum K + i\sum \varkappa)y_b = -\left[M\ddot{y}_{ob} + (K_a + i\varkappa_a)(y_{ob} - y_{oa})\right] \qquad (3)$$

where $\sum K$ and $\sum \varkappa$ - **overall rigidity and** characteristics of the resistance of the total system as a whole, i.e. in this case for both rods a and b .

It is of interest to compare the equations (3) with the similar equation of vibrations for the system corresponding to a structure not extended in plan, i.e. to the scheme of Fig. 1a. As is known the equation of vibrations for such a system will be:

$$M\ddot{y} + (K + i\varkappa)y = -M\ddot{y}_o \qquad (4)$$

where

y_o - displacement of the base;

y - deflection of the system in Fig.1;

K - rigidity;

$\varkappa$ - characteristics of damping.

The comparison of the equations (3) and (4) shows that the equation of the system, extended in plan, as well as of the system in Fig. 1a is the equation of forced vibrations of a system with a single degree of freedom. The equations differ in that respect that in the extended system the dis-

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x) When allowing for damping forces there has been employed the hypothesis according to which the force of resistance depends not upon the velocity but only upon the value of the deformation while in time it is displaced as compared with the deformation phase by 90°.

turbing force is not only an "inertia force" $(-M\ddot{y}_{oa})$ but also an additional "elastic force"

$[(K_{\beta}+i\varkappa_{\beta})(y_{oa}-y_{o\beta})]$ characterizing the response of the post ℓ.

The analysis of the equation (3) for the case of the displacement of the base according to the law of damped sinusoid $y_{oa}=a_0 e^{-\varepsilon_0 t}\sin\omega t$ along the direction of the wave propagation showed that the forces produced thereby in the extended system were in the vast majority of cases much lower than those produced in the system, not extended in plan (Fig. 1a). Only in very rigid structures (having, as a rule, no practical significance) forces induced during the propagation of the wave can exceed forces corresponding to the case with a cantilever rod. Schematically this conclusion is shown on Fig.2.

The investigations of the work of the structure corresponding to the second design scheme were based on the following considerations.

Let us assume that the moving wave producing vibrations of the base only along the direction of its propagation (Fig. 3) confronts on its way a building the longitudinal axis of which is directed along the wave propagation. In this case at some moment of time t corresponding to the position shown in Fig. 3a the acceleration of the ground imparted to the base of the structure at any point x will be:

$$\dot{j}_x = \dot{j}_{x_0}\sin 2\pi\frac{x}{\lambda} \tag{5}$$

where

- $\dot{j}_{x_0}$ - maximum (amplitude) value of the acceleration;
- x - position of the considered point;
- ℓ - length of the building;
- λ - length of the seismic wave;
- a - value determining the location of the building with respect to the location of the wave.

The mean acceleration (   ) imparted at this moment to the building as a whole will be equal to:

$$\dot{j}_{x_{av}} = \int_a^{a+\ell}\dot{j}_{x_0}\sin 2\pi\frac{x}{\lambda}\,dx \tag{6}$$

Integrating and substituting the limits we shall obtain

$$\dot{j}_{x_{av}} = \frac{\dot{j}_{x_0}\lambda}{2\pi\ell}\left(\cos 2\pi\frac{a}{\lambda} - \cos 2\pi\frac{a+\ell}{\lambda}\right) \tag{7}$$

It is easy to see that $\ddot{j}x_{av}$ will reach maximum (and, respectively, minimum) values at the moment of time when the middle of the building coincides with $\frac{\lambda}{4}$ (or $\frac{3\lambda}{4}$) of the seismic wave.

At these moments the value of a will equal $\frac{\lambda}{4} - \frac{\ell}{2}$ and, hence,

$$\ddot{j}x_{av\,max} = \ddot{j}x_0 \frac{\sin \pi \frac{\ell}{\lambda}}{\pi \frac{\ell}{\lambda}} \qquad (8)$$

It goes without saying that the periodicity of these moments in time will be similar to that of the seismic wave.

The expression (8) show that for the ratio $\frac{\ell}{\lambda} = 0$ it reaches its maximum since at

$$\lim \ddot{j}x_0 \frac{\sin \pi \frac{\ell}{\lambda}}{\pi \frac{\ell}{\lambda}} \underset{for\ \frac{\ell}{\lambda} \to 0}{=} \ddot{j}x_0 \cos \pi 0 = \ddot{j}x_0$$

and as $\frac{\ell}{\lambda}$ increases the value of $\ddot{j}x_{0av}$ decreases and at $\frac{\ell}{\lambda} = 1$ becomes zero. Hence, when the direction of accelerations coincides with the direction of the wave propagation the effectiveness of the action of the moving wave on the extended structure will be lower than that on the cantilever rod.

Now let us see what will occur if the directions of displacements and accelerations are normal to the direction of the wave propagation. In this case as it can be clearly seen from Fig. 3b the building will be effected by accelerations tending not only to displace it progressively but also turn the building in plan. In fact, for the moment of time corresponding to the position shown in Fig. 3b when the torsional moment caused by inertia forces produced by the ground reaches its maximum, we shall have:

$$M_{t\,max} = \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \ddot{j}y_0\, m \sin 2\pi \frac{x}{\lambda}\, x\, dx = \frac{\ddot{j}y_0\, m}{2} \left(\frac{\lambda}{\pi}\right)^2 \left[\sin \frac{\pi \ell}{\lambda} - \pi \frac{\ell}{\lambda} \cos \frac{\pi \ell}{\lambda}\right] \qquad (9)$$

where

m - linear mass (along the length of the building).

If this moment is divided by moment of inertia of the building with respect to the vertical plane passing through the middle of its length $\left(\frac{m\ell^3}{12}\right)$, we shall obtain the value of the mean angular acceleration for the base of the structure as a whole, i.e.

$$\ddot{J}_{\varphi_{av}\,max} = \frac{6\ddot{J}_{y_0}}{\ell}\left[\frac{\sin\pi\frac{\ell}{\lambda}}{\left(\pi\frac{\ell}{\lambda}\right)^2} - \frac{\cos\pi\frac{\ell}{\lambda}}{\pi\frac{\ell}{\lambda}}\right] \tag{10}$$

This expression, unlike the expression (8), becomes zero at $\frac{\ell}{\lambda} \to 0$ and reaches a maximum at $\frac{\ell}{\lambda} = 0{,}5$ (Fig. 3c) equalling:

$$\frac{24\ddot{J}_{y_0}}{\pi^2 \ell} \approx 2{,}4\,\frac{\ddot{J}_{y_0}}{\ell}$$

Simultaneously with the angular acceleration $\ddot{J}_{\varphi}$ the building will be effected by the progressive acceleration $\ddot{J}_y$ reaching its maximum value, similarly to the above given case, in the position shown in Fig. 3d:

$$\ddot{J}_{y_{av}\,max} = \ddot{J}_{y_0}\,\frac{\sin\pi\frac{\ell}{\lambda}}{\pi\frac{\ell}{\lambda}} \tag{11}$$

Though the periodicity of both accelerations, just as before, is the same as that of the moving wave they will be mutually displaced in phase by the angle $\frac{\pi}{2}$

Hence, for the given case when ground accelerations are normal to the direction of the wave propagation the extended structure will resist two loadings and, therefore, additional torsional vibrations will occur in the building though progressive accelerations $\ddot{J}_{y_{av}}$ will be lower than those for a cantilever rod. Therefore, unlike the first case, we cannot be sure here of the favourable effect due to the length of the structure.

The availability of tosional vibrations in structures with no eccentricities between the centres of masses and rigidities was observed by V.S. Pavlyk /7/ and V.S. Preobrazhensky (8) when they experimentally investigated vibrations of actual buildings produced by microseisms. Fig. 4 illustrates, as the example, the forms of vibrations of a symmetrical brick building recorded by V.S. Preobrazhensky in the nature by measuring the vibrations produced by microseisms.

If we apply the same considerations for the case when the longitudinal axis of the building is normal to the direction of the propagation of the seismic wave (Fig. 3e) we may easily prove that the effectiveness of its action is somewhat similar to that for a cantilever rod. In fact, the component $\ddot{J}_x$ will not produce a different effect since the transverse dimensions of the building are not large as compared with the wave length. At all the points along the length of the structure the component $\ddot{J}_y$ at the same moment will have one and the same value since all the points of the base, being located along the wave front, must be in the same phase of motion.

It will be appropriate here to ask what practical conclusions may be drawn on the basis of the above given analysis.

As it is known, seismic motions of the ground are a complicated vibration process containing numerous components with various amplitudes and frequencies and, hence, containing a number of waves with different lengths. Naturally, it is extremely difficult in practice to take into account the effect of all these components. Therefore, it is necessary, first of all, to decide what components produce the strongest effect on the building. It seems that it is most correct (as it was assumed by the authors before) (4) to assume that the effectiveness is the strongest by the component whose frequency is most near to the frequency of natural vibrations of the structure. Since at earthquakes a very wide range of frequencies is usually observed (at least within 0.1 - 2-3 sec) we may assume for practical purposes that in the vibration motion of the base there can always be found the component whose period equals the period of natural vibrations of the building. Then, the velocity of the propagation of seismic waves in the ground (v) being known, we can find that for the structure with the period of natural vibrations (T) the length of the wave which ought to be taken into account should be equal to

$$\lambda = v T \qquad (12)$$

It follows from this assumption that the effectiveness of the length of the structure must depend not only upon its length proper but also upon the period of its natural vibrations. In fact, if we take the ratio of the mean acceleration for the whole building ($\ddot{x}_{av}$) to the value of the acceleration of the ground ($\ddot{x}$) we shall obtain the value characterizing the effectiveness of the reduction of the action of an earthquake on the extended structure (with due regard to its longitudinal component), i.e.

$$\gamma_x = \frac{\ddot{x}_{av}\,max}{\ddot{x}_0} = \frac{\sin \pi \frac{l}{\lambda}}{\pi \frac{l}{\lambda}} \qquad (13)$$

or

$$\gamma_x = \frac{vT}{\pi l} \sin \frac{\pi l}{vT}\,, \qquad (14)$$

It follows from the last expression that in systems with short periods of natural vibrations we may expect a greater reduction of the seismic effect (connected with accelerations directed along the wave propagation). The qualitative confirmation of the above said may be found in the article by Karapetян B.K. /3/ containing the values of reduced accelerations[x] measured during the explosion on the building and the ground by means of pendulums with various periods of natural vibrations. The data taken from this article are presented in Table 1.

The Table shows that the reduction of accelerations due to the length of the building was equal to $\frac{0.021}{0.039} \approx 0.54$ in the system with the period of natural vibrations 0.2 sec and $\frac{0.462}{1.463} \approx 0.32$ in the system with the period 0.05 sec.

Allow us to point out in this connection that G.Housner in his work (2) also notes that the effect of the length of the building should be stronger as regards the action of high-frequency components of ground motions, substantiating his view by the considerations similar to the above given.

This problem was further investigated by V.S.Preobrazhensky and A.B. Grossman in their works. The former studied the problem of torsinal - to and fro vibration of buildings, characterized by the above given schemes (Fig. 3), but when allowing the presence of eccentricity between the centres of mass and rigidity. As the result of the carried investigation it were received the formulae for determining the frequency and form of free vibration of buildings and formulae of design of seismic load, that is determined not only by the horizontal force S, but by the torsinal moment M.

Table 2 gives the comparison of circular frequencies of free vibrations of buildings, designed and measured in nature at vibrations produced by microseisms.

Formulae, received by V.S. Preobrazhensky for determining the value of seismic forces and torsional moments, are of the same character, that the formulae existed according to Norms in the USSR (1), i.e.

$$S_K = K_c \beta \eta_K \bar{Q}_K$$

$$M_K = K_c \beta \eta_K \frac{\bar{\theta}_K}{R_K}$$

where

K_c - coefficient of seismisity;

β - dynamic coefficient;

η_K - coefficient of forms at level K;

$\bar{Q}_K$ - reduced weight at level K.

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x) Under the notation "reduced acceleration" the author of the article /3/ implies the value of acceleration occurring in the mass of the pendulum of the measuring device.

$\bar{\theta}_K$ - reduced moment of inertia systems at level K ;

R_K - rotating radius of system at the level K .

Unlike the formulae, that takes into consideration only progressive vibrations, values l_K, Q_K and θ_K have some other meanings.

The investigations, carried out by A.B. Grosman considered vibrations of multispanned expended structures corresponding to the scheme of Fig.1b. Here it was considered disturbance produced by different laws of base motion (stationary harmonic, impulse and damped sinusoid) and as the result of confrontation of behavior of expended (Fig. 1b) and cantilever (Fig. 1a) systems it was found the effect of reduction of vibrations of expended constructions in comparison with the cantilever system.

For the purpose of cheking the results of theoretical investigations one of the authors of this paper (I.L. Korchinsky) gave an idea of invention of setting for testing models of expended structures. The principle of this setting was following:

Instead of ordinary vibroplatform, that was like one table, supported on flexible pendulum bearings (Fig. 5a) it were made row of mutually isolated tables, each supported on its bearing (Fig. 5b). Then each table could receive its own law of motion that in particular may be singl-type but mutually advanced by phase on any angle. In this case it is convinient to give motion to these tables by means of one common roller, that has contacts with different tables at diffirent points along its circle. These contacts may be as of continious action, when the stationary regime of motion is given, and as well of short action, when it is necessary to give damped free vibrations to the base (tables).

Taking into consideration this principle the group of specialists, i.e. P.A. Burdelov, A.B. Grosman, P.A. Makarov, V.S. Menkevich and the authors of this paper worked out a project of testing setting that now have already been prepared and ready to operation. Fig.6 illustrates the general view of this testing setting, on which the model of structure in the form of flexible posts, supporting the mass of roof, is situated. Fig. 7 illustrates the vibrogramms of vibrations of tables of test setting (modeling the motion of base structure) and masses of model, that is the roof of extended structure. The consideration of vibrogramms shows, that this setting allows quite accurate to give displacement in phases of motion of its separate tables.

Table 1.

Places of device mounting	Values of reduced accelerations in fractions of g — Period of natural vibrations of device pendulum (sec)	
	0.05	0.2
On ground	1.463	0.039
In building	0.462	0.021

Table 2.

Object of investigation	Experimentally received frequencies		Design values of frequencies		Coordination of frequencies			
	radian in sec.				$\frac{P_p}{P_p}$ -design -experimental	$\frac{P_t}{P_t}$ -design -experimental	experimental $\frac{P_t}{P_p}$	design $\frac{P_t}{P_p}$
	progressive P_p	torsional P_t	progressive P_p	torsional P_t				
1	2	3	4	5	6	7	8	9
1. Five-storey brick building	20,3	25,4	22,0	24,1	1,07	0,95	1,25	1,11
2. Five-storey brick building	19,4	23,4	21,7	23,7	1,12	1,09	1,20	1,09
3. Five-storey large-panelled building x)	22,7	24,8	–	–	–	–	–	–

x) Because of the lack of data of this object, the determination of frequence according to design formulae was not done.

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FIGURE CAPTIONS

Fig.1 Design scheme of structures.

a - limited extension.
b - great extension.

Fig.2 Graph of dynamic effect.

- for the system of limited extension
- for the system of great extension.

Fig.3 Scheme of orientation of buildings according to motion direction of seismic wave. (The direction of displacement and acceleration are coincided with motion propagation direction of wave for Fig. a; Fig. b, c, d, e - the direction of displacement and acceleration are perpendicular to propagation of wave).

a,b - arbitrary position of wave.
c - wave position, at which the zero point is placed at the centre of the span.
d - wave position, at which the maximum amplitude is placed at the centre of the span.
e - wave position at which the maximum amplitude is out of the structure.

Fig.4 Forms of vibration of symmetrical extended brick building.

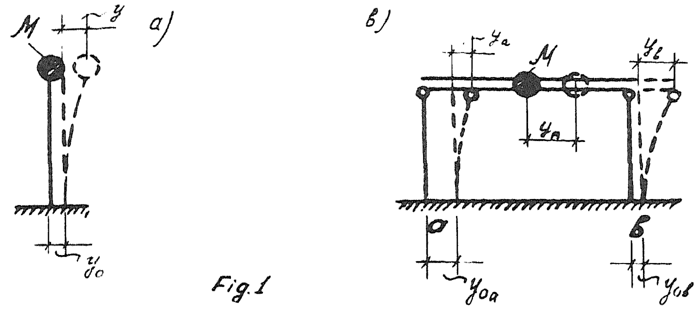
Fig.5 Scheme of setting for test models of extended structures.

a - tables, supported on flexible bearings.
b - kinematic scheme of separated vibrotables.

Fig.6 Appearance of test setting.

Fig.7 Oscillogram of vibrations of four vibrotable setting and models.

1,2,3,4 - vibrations of 1-4 vibrotables.
5 - vibration of mass model representing the roof of extended building.



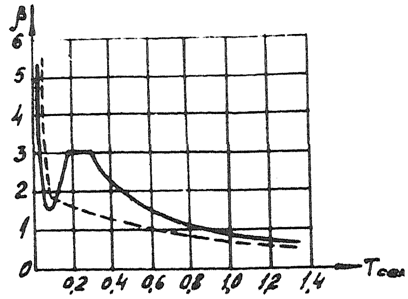


Fig. 2

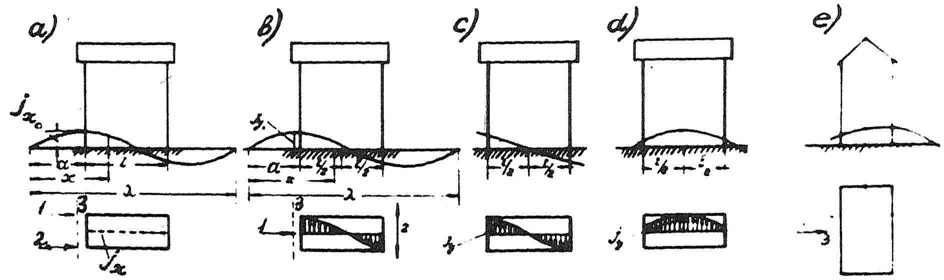


Fig. 3

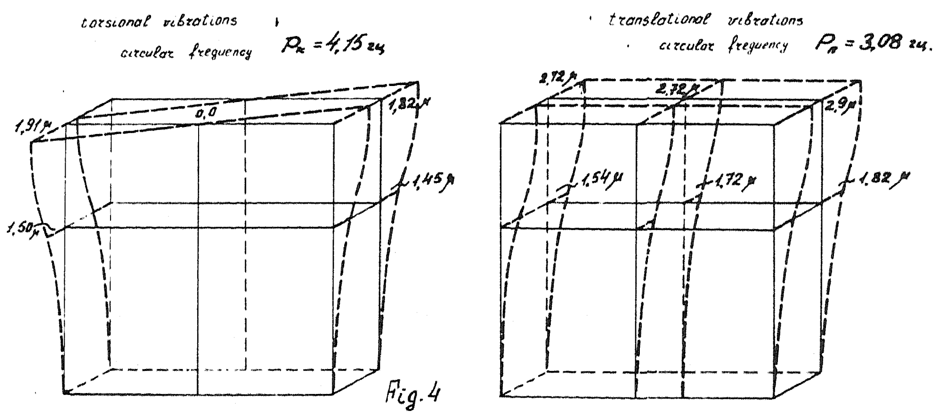


Fig. 4

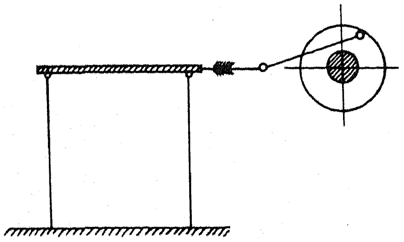


Fig 5a

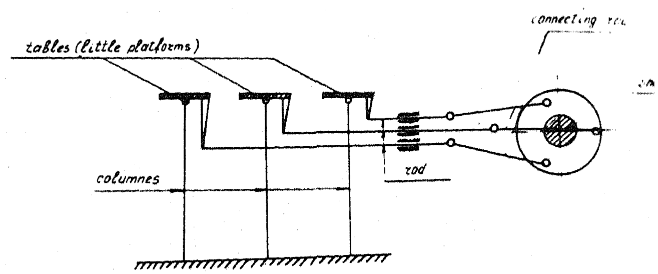


Fig 5b

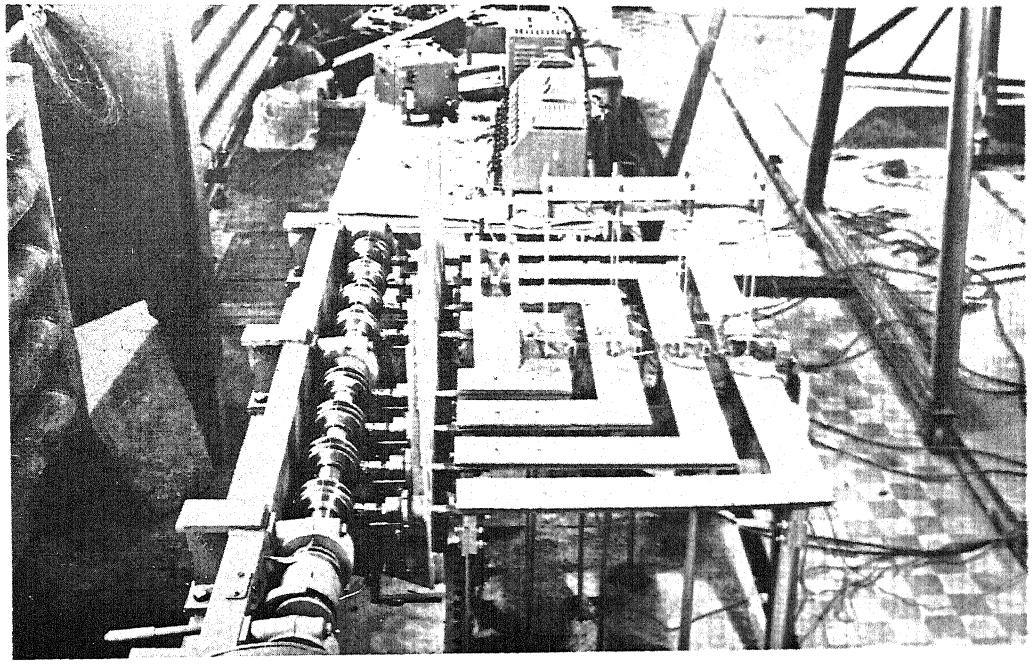


Fig.6

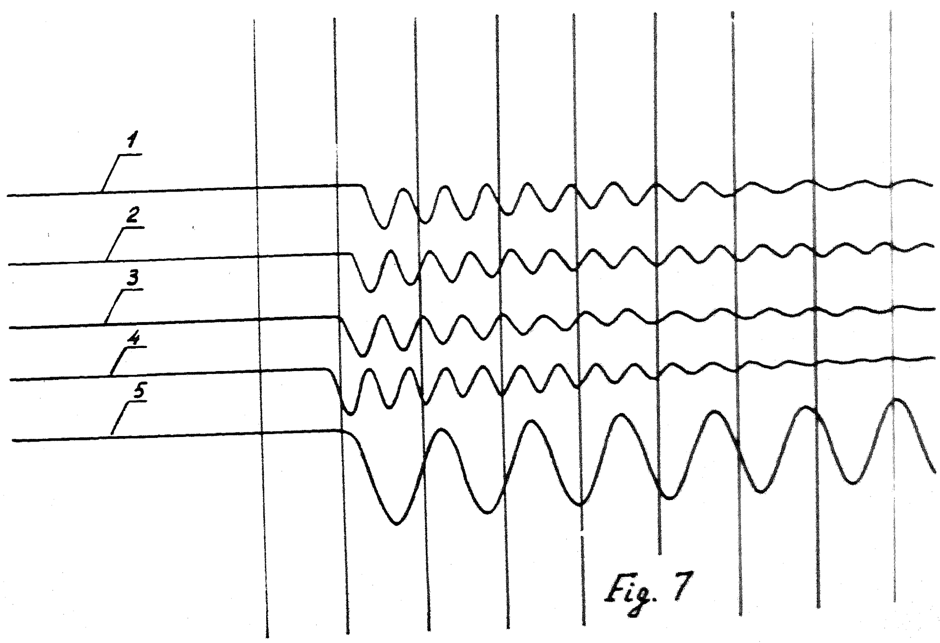


Fig. 7