JOINT ROTATION EFFECTS ON THE DYNAMICS OF MULTISTOREYED FRAMES

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SYNOPSIS

It has been the usual practice, while calculating the dynamic behaviour of multistoreyed frames, to assume that the floor systems are rigid and the columns do not undergo any rotation at their junction with the floor system. This paper examines the efficacy of such an assumption on the dynamic behaviour of multi-storeyed frames, with particular reference to forces developed due to strong ground motion.

INTRODUCTION

Notation:- The letter symbols adopted for use in this paper are defined and are listed alphabatically in the Appendix.

The physical phenomenon due to joint rotation is to introduce additional coupling between masses, thereby, increasing the flexibility of a structure. The periods get elongated and the displacements increase compared to that of a rigid structure.++

In this study, single bay frames have been considered. This makes the analysis somewhat simpler. Further, it has been shown [1] that single bay frames are more flexible than corresponding multiple bay frames. The results obtained from the dynamic analysis of single bay frames would represent the effect of joint rotation for the most severe case and therefore would be an upper bound for other cases. Also, it is possible to a arrive at single bay frames equivalent to that of multiple bay frames as far as behaviour under horizontal loads are concerned [2].

BASIC EQUATIONS OF MOTION

The building models that would be considered for investigating joint rotation is based on the following assumptions.

1. Masses concentrated at floor levels.
2. Linear spring forces (columns in multistoreyed frames act as springs. They are assumed to have linear stress strain relationships).

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++ Note: The terms 'flexible' and 'rigid' used in this paper represent relative values and not absolute values. The term 'flexible structure' is meant to indicate structures in which effect of joint rotation has been considered and the term 'rigid structure' is meant for those in which effect of joint rotation has been neglected.

3. Classical normal modes exist. It is assumed that the damping in the system is such that modal analysis is possible.

4. No base compliance.

5. There is no change in the length of columns or girders. (Axial forces are ignored).

6. There is no settlement of frame supports.

7. Joints are rigid, namely, all ends of members framing into a joint rotate an amount equal to the joint rotation.

The equation of motion for the undamped vibration of the building model, which is a n degree of freedom system, is given by

$$[K]^{-1}[M]\{\phi\} - \frac{1}{p^2}[I]\{\phi\} = \{f(t)\} \qquad (1)$$

Rubinstein and Hurty [3] have indicated a general approach to the formulation of equations of motion and a method for solution of such equations. The author [1] has worked out solutions to the problems of this study by a different method.

The solution of the free vibration problem, that is when $F(t)$ equals zero, gives the natural frequencies of vibrations (p's) and the associated coordinates of mode shapes (ϕ's).

If the structure is subjected to a strong ground motion and if the damping in the system is such that normal modes exist, then the relative displacement with respect to the base of any mass i, is given by

$$z_i = \sum_{r=1}^{n} \phi_i^{(r)} \frac{\sum_{j=1}^{n} m_j \phi_j^{(r)}}{\sum_{j=1}^{n} m_j (\phi_j^{(r)})^2} \frac{1}{p_{dr}} \int_0^t y(\tau) e^{-p_r \zeta_r (t-\tau)} \sin p_{dr}(t-\tau)\, d\tau \qquad 2$$

From equation 2, the displacement response of any mass i, in the r^{th} mode of vibration is

$$|z_i^{(r)}| = \frac{\phi_i^{(r)} \sum_{j=1}^{n} m_j \phi_j^{(r)}}{\sum_{j=1}^{n} m_j (\phi_j^{(r)})^2} \cdot \frac{1}{p_r} (S_v)_r \qquad 3$$

The inertia force acting at any floor level is $m_i \ddot{x}_i$ and from the equation of motion,

$$-m_i \ddot{x}_i = \sum_{j=1}^{n} d_{ij} z_j + \sum_{j=1}^{n} k_{ij} z_j \qquad \ldots \qquad \ldots \quad 4$$

The inertia forces are partly resisted by the damping forces and partly by the spring forces.

Defining the sum of spring forces at any floor level as the load at the floor level, then the load 'Q' acting on any mass i is given by

$$Q_i^{(r)} = \sum_{j=1}^{n} k_{ij} z_j^{(r)} \qquad \ldots \qquad \ldots \quad 5$$

The shear force at any floor level = sum of loads above that level.

Shear V at any floor level i is

$$V_i^{(r)} = \sum_{j=1}^{i} Q_j^{(r)} \qquad \ldots \qquad \ldots \quad 6$$

(floor numbers are to be counted from top)

SPECIFICATIONS OF THE PROBLEMS

Building models of uniform mass and stiffness distributions have been considered for this study (Refer Fig 1). The ratio, S_b/S_c, of the moment of inertia per unit length of beam (floor system) to that of the column was varied over a wide range. The number of masses were also varied to consider the effect of number of storeys on the flexibility of the building. The ratio, S_b/S_c was varied from 1 to 10 and n from 1 to 20.

For each problem, frequencies (p), mode shapes (ϕ), displacement response (z) and shear response (V) have been worked out for the first four modes of vibration.

In this study, while comparing various quantities involving S_v, it has been assumed that S_v has a flat response and the damping in each mode is same.

DISCUSSION OF RESULTS

A. Natural Frequencies of Vibration

The natural frequency of vibration could be expressed as

$$p = C_1 \times \sqrt{k/m} \qquad \ldots \qquad \ldots \quad 7$$

Values of $C_1 \times n$ for the various cases have been given in Table 1.

Figs. 2 and 3 show plots of fundamental frequency versus S_b/S_c and n. It could be observed that for a ratio of S_b/S_c above five, the effect of joint rotation is negligible. Also, this effect is not much affected by increase in value of n above five. It is also observed that the effect of joint rotation is less pronounced in the case of higher modes than that of fundamental modes.

The spring constant k is a function of the length of the column and is inversely proportional to the cube of its length. It is therefore possible to choose equivalent length of columns such that the same relationship as obtained for the frequency of a rigid structure ($S_b = \infty$) could be used for the case of a flexible structure (S_b is finite).

Table 2 gives the dimensionless factor, F, by which length of the columns should be multiplied such that the fundamental frequency of a flexible structure (p_{fe}) could be obtained by using the formula for the fundamental frequency of rigid structure (p_{rd}).

This factor F, could be easily obtained to a very good degree of approximation by calculating the fundamental frequency of vibration by Rayleigh's method [4]. For the flexible structure, the influence coefficients used in Rayleigh's method could be obtained by Kloucek's procedure [2].

B. Displacement Response

The relative displacement with respect to the base of any mass i due to a ground motion could be expressed as

$$z_i = C_{2i} \cdot \sqrt{m/k} \cdot S_v \qquad 8$$

Values of C_{2T}/n, for the topmost mass, have been given in Table 3. Fig.4 shows a plot of top displacement response versus S_b/S_c and n. Fig.5 shows the displacement diagram for n=10. It could be observed that for a ratio of S_b/S_c above five, the effect of joint rotation is negligible. Also, this effect is not much affected by increase in value of n above five. It is also observed that the effect of joint rotation is less pronounced in the case of higher modes than that of fundamental mode.

In order to predict the effect of joint rotation from the values of a rigid structure, the dimensionless term z_{Tfe}/z_{Trd} is divided by $F^{3/2}$ and the results are given in Table 4. It is observed that this combination of dimensionless factors approaches unity very nearly in all the cases. This indicates that once the factor F is known, the relative displacement for a flexible structure could be evaluated from that of a rigid structure.

C. Shear Response

The shear force at any level i, due to a ground motion could be expressed as

$$V_i = C_{3i} \cdot \sqrt{k.m} \, . \, S_v \qquad \ldots \qquad \ldots \quad 9$$

Values of C_{3B}, for the base has been given in Table 5. A plot of base shear response versus S_b/S_c and n is shown in Fig.6 for the first mode of vibration and in Fig. 7 for the absolute sum of the first four modes. Fig. 8 shows the shear diagram for $n = 20$. It could be observed that for a ratio of S_b/S_c above five, the effect of joint rotation is negligible. Also, this effect is not much affected by increase in value of n above five. It is also observed that the pattern of variation of shear force along the height is more or less similar in all the cases.

In order to predict the effect of joint rotation from the values of a rigid structure, the dimensionless term $\frac{V_{Bfe}}{V_{Brd}}$ is multiplied by $F^{3/2}$ and the results are given in Table 6. It is observed that this product of dimensionless factors approaches unity very nearly in all the cases. This indicates that once the factor F is known, the shear for a flexible structure could be evaluated from that of a rigid structure.

CONCLUSIONS

The effect of joint rotation is negligible and therefore may not be considered if the ratio S_b/S_c is of the order of five and above. For values of n greater than five, the effect of joint rotation remains very nearly the same as for $n = 5$. For values of n less than five, the effect is even less pronounced than that for n equal to five.

The values obtained by ignoring the effect of joint rotation, results in a conservative estimate of shear values.

If the factor F, corresponding to the equivalent length of column is known (relatively, this could be evaluated easily), then all the values for a flexible structure could be obtained from that of a rigid structure. That is, the effect of joint rotation can be taken into account in all the cases, if equivalent length of columns are properly chosen.

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APPENDIX

Notations

C_1 – frequency coefficient defined as per equation 7.

C_2 – displacement coefficient defined as per equation 8, subscript T used along with this represents values of the top mass.

C_3 – shear coefficient defined as per equation 9, subscript B used along with this represents values at base.

d_{ij} – elements of damping matrix.

$\{f(t)\}$ – column matrix whose elements are forces acting on masses.

F – factor by which length of columns are to be multiplied to get equivalent lengths.

fe – when used as subscript represents flexible structure.

h – height between floors, assumed constant.

$[I]$ – diagonal matrix whose elements are unity.

$[K]^{-1}$ – matrix whose elements are the influence coefficients of system.

k – spring constant of a column of the building model, equals to $\frac{24EI_c}{h^3}$

k_{ij} – elements of stiffness matrix.

$[M]$ - mass matrix.

m - mass concentrated at storey level.

n - number of masses.

p - undamped natural frequency of system.

p_d - damped natural frequency of system,

$= p\sqrt{1-\zeta^2} \approx p$, if $\zeta \leq 0.20$

Q - load at any floor level, defined as per equation 5.

r - index representing mode of vibration.

rd - when used as a subscript represents rigid structure.

S_b - moment of inertia per unit length of floor system.

S_c - moment of inertia per unit length of column.

S_v - response velocity spectrum,

$$= \left| \int_0^t \ddot{y}(\tau)\, e^{-p\zeta(t-\tau)} \sin p_d(t-\tau)\, d\tau \right|_{maximum}$$

t - time interval.

V - shear at any floor level, defined as per equation 6.

x - absolute displacement of mass.

$\ddot{y}(t)$ - ground acceleration.

z - displacement of mass relative to the base, defined as per equation 3.

ζ - coefficient of damping expressed as a fraction of critical damping value.

ϕ - mode shape coefficient.

TABLE 3

TOP DISPLACEMENT

$\delta_T = C_{2T} \sqrt{m/k}.\ S_v$

n	S_b/S_c	VALUE OF C_{2T}/n			
		1ST. MODE	2ND. MODE	3RD. MODE	Σ MODES +
	∞	1.0000			
	10	1.0243			
1	5	1.0472			
	2	1.1094			
	1	1.1952			
	∞	0.9472	0.0528		
	10	1.0029	0.0562		
2	5	1.0536	0.0589		
	2	1.1836	0.0646		
	1	1.3532	0.0702		
	∞	0.8795	0.0872	0.0242	0.9984
	10	0.9493	0.0948	0.0266	1.0791
5	5	1.0136	0.1016	0.0287	1.1530
	2	1.1819	0.1192	0.0336	1.3452
	1	1.4106	0.1424	0.0392	1.6040
	∞	0.8479	0.0914	0.0309	0.9855
	10	0.9216	0.0996	0.0339	1.0711
10	5	0.9893	0.1072	0.0365	1.1501
	2	1.1675	0.1269	0.0435	1.3583
	1	1.4111	0.1541	0.0529	1.6428
	∞	0.8300	0.0915	0.0324	0.9701
	10	0.9055	0.0999	0.0355	1.0584
20	5	0.9750	0.1076	0.0382	1.1400
	2	1.1577	0.1280	0.0456	1.3541
	1	1.4083	0.1559	0.0557	1.6479

+ Absolute sum of first four modes.

TABLE 4

($\frac{Z_{Tfe}}{Z_{Trd}} \div F^{3/2}$) versus S_b/S_c and n

n	S_b/S_c	$Z_{Tfe}/Z_{Trd} \div F^{3/2}$	
		1ST. MODE	Σ MODES +
5	∞	1.0000	1.0000
	10	1.0025	1.0039
	5	1.0044	1.0065
	2	1.0088	1.0115
	1	1.0141	1.0159
10	∞	1.0000	1.0000
	10	1.0007	1.0007
	5	1.0015	1.0016
	2	1.0029	1.0039
	1	1.0048	1.0065
20	∞	1.0000	1.0000
	10	1.0002	1.0002
	5	1.0004	1.0008
	2	1.0008	1.0015
	1	1.0014	1.0025

+ Absolute sum of first four modes

TABLE 6

($\frac{V_{Bfe}}{V_{Brd}}$ X $F^{3/2}$) versus S_b/S_o and n

n	S_b/S_c	V_{Bfe}/V_{Brd} X $F^{3/2}$		
		1ST.MODE	√MODE+	Σ MODES ++
5	∞	1.0000	1.0000	1.0000
	10	0.9905	0.9989	1.0268
	5	0.9828	0.9980	1.0505
	2	0.9664	0.9973	1.1083
	1	0.9494	0.9962	1.1807
10	∞	1.0000	1.0000	1.0000
	10	0.9936	0.9979	1.0099
	5	0.9885	0.9969	1.0175
	2	0.9778	0.9934	1.0337
	1	0.9669	0.9916	1.0531
20	∞	1.0000	1.0000	1.0000
	10	0.9962	0.9977	1.0010
	5	0.9934	0.9962	1.0018
	2	0.9872	0.9921	1.0033
	1	0.9811	0.9882	1.0055

+ Square Root of the sum of the squares of the first four modes.
++ Absolute sum of first four modes.

TABLE 5

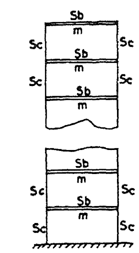
BASE SHEAR

$$V = C_{3B}\sqrt{km}.\ S_v$$

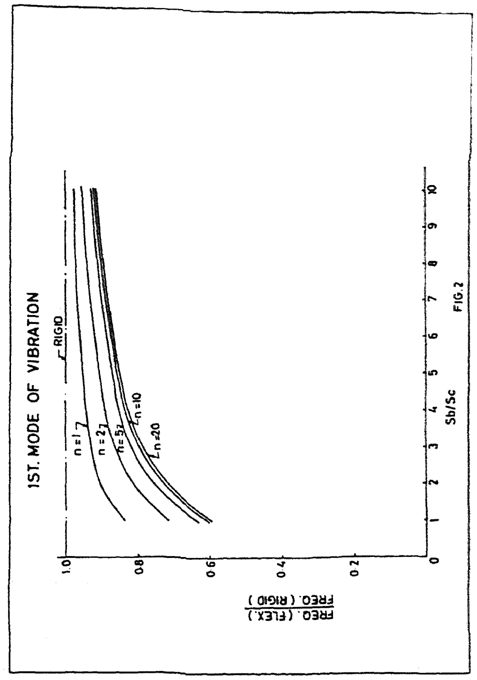
n	S_b/S_c	VALUE OF C_{3B}		
		1ST. MODE	$\sqrt{}$MODES +	ΣMODES ++
1	∞	1.0000		
	10	0.9763		
	5	0.9549		
	2	0.9014		
	1	0.8367		
2	∞	1.1708		
	10	1.1032		
	5	1.0470		
	2	0.9230		
	1	0.7955		
5	∞	1.2517	1.3140	1.8356
	10	1.1515	1.2190	1.7506
	5	1.0722	1.1430	1.6806
	2	0.9080	0.9837	1.5270
	1	0.7514	0.8278	1.3703
10	∞	1.2673	1.3570	2.0429
	10	1.1594	1.2470	1.8996
	5	1.0752	1.1610	1.7841
	2	0.9026	0.9819	1.5383
	1	0.7399	0.8125	1.2988
20	∞	1.2717	1.3720	2.1096
	10	1.1615	1.2550	1.9363
	5	1.0759	1.1640	1.8001
	2	0.9009	0.9767	1.5187
	1	0.7363	0.8002	1.2518

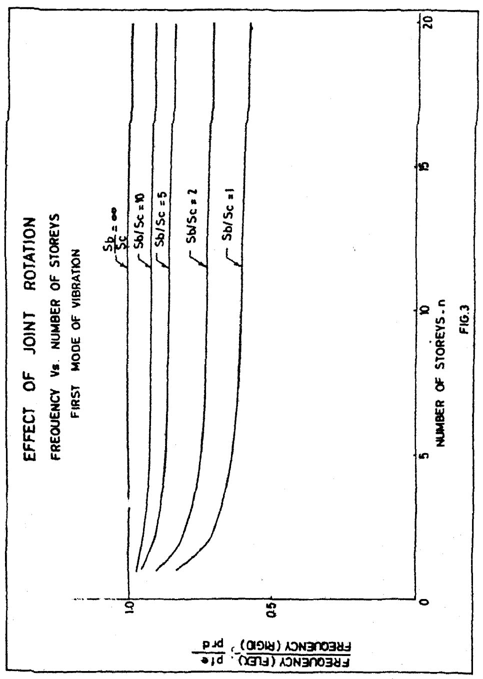
\+ R.M.S. value of first four modes

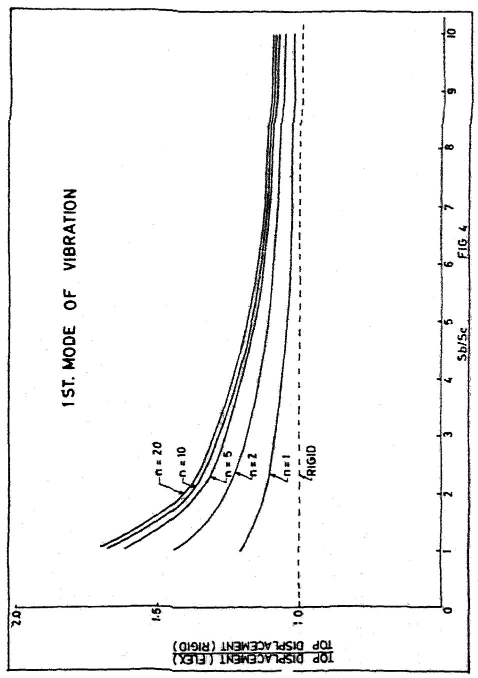
++ Absolute sum of first four modes



BUILDING MODEL FOR FINDING EFFECT OF JOINT ROTATION
FIG. 1







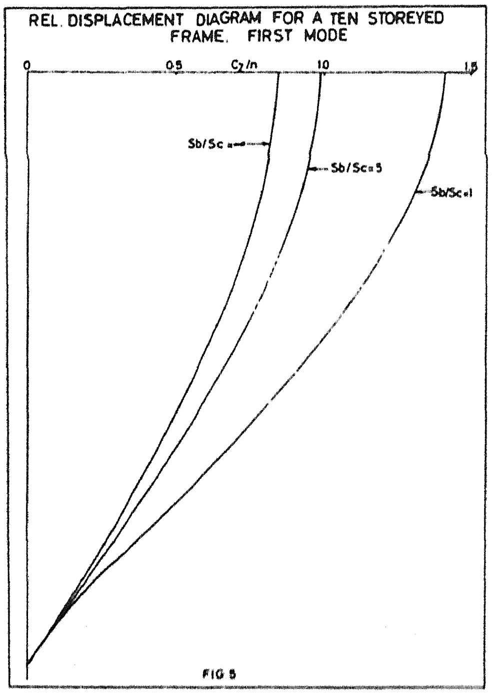


FIG 5

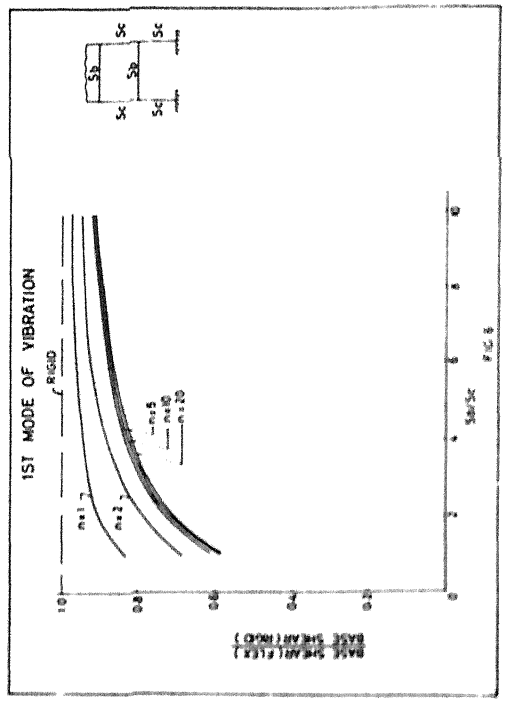


FIG 6

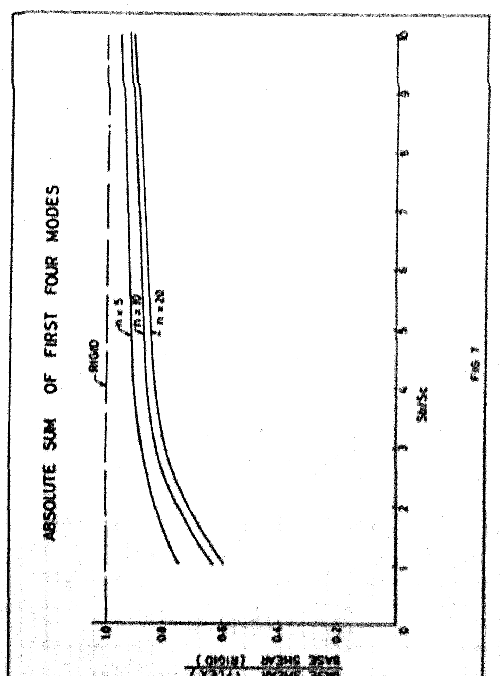


FIG 7

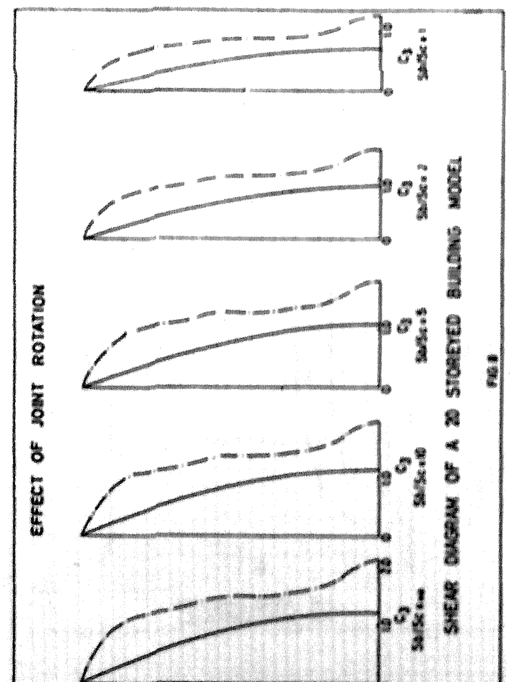


FIG 8