INELASTIC EARTHQUAKE RESPONSE OF TALL BUILDINGS

by

Ray W. Clough*, K.L. Benuska** and E.L. Wilson***

ABSTRACT

A digital computer procedure for evaluating the inelastic forces and deformations developed in each column and girder of any arbitrary building frame subjected to earthquake motions is described. A special bi-linear moment-rotation property may be prescribed independently for each member. The distribution of maximum deformations and forces produced in two different 20 story building frames by the El Centro 1940 earthquake, computed by this program, are discussed and compared with results obtained in a purely elastic analysis. Three different earthquake intensities, approximately 2/3, 3/3 and 4/3 of El Centro, are considered.

INTRODUCTION

Great advances have been made during recent years toward a more complete understanding of the behaviour of structures subjected to earthquake excitation. The introduction two decades ago of the elastic response spectrum concept(1), which provides a convenient means for representing the elastic behaviour of simple structures, was followed by recognition of the fact that the forces predicted by such spectra far exceed normal design requirements(2). Because structures having much less strength than is prescribed by the spectral values were observed to have performed satisfactorily in rather severe earthquakes, it became apparent that the elastic response spectrum is not a direct measure of the significant earthquake behaviour of many structures. Even moderate earthquakes may be expected to produce inelastic deformations in typical buildings, and it is now understood that the plastic energy absorbed by the structure has a controlling influence on the deformation amplitudes which it may develop.

Recognition of the important role played by ductility in the earthquake performance of structures led to initiation of research programs directed toward the quantitative study of simple elasto-plastic systems subjected to earthquake motions(3,4,5). These investigations demonstrated that the maximum structural displacement amplitudes produced by a given earthquake tend to be reasonably independent of the yield strength of the structures(3). In other words, the maximum displacement in a simple structure was found to be about the same whether it remained elastic or yielded.

---

* Visiting Professor, University of Cambridge, on leave from Department of Civil Engineering of California, Berkeley.
** Associate, T.Y. Lin and Associates, Van Nuys, California.
*** Senior Research Engineer, Aerojet General Corporation, Sacramento, California.

On the basis of this observation, the ductility factor concept was introduced[6], this factor being defined as the ratio of total deformation (elastic plus plastic) to the elastic limit deformation. For structures in which the deformation amplitude is independent of the yield strength, the yield strength required to provide any given ductility factor may be found by dividing the elastic spectral response force by that ductility factor. Thus the ductility factor concept extends the applicability of elastic response spectra to include any elasto-plastic structure which responds as a true one degree of freedom system,i.e. to any simple system for which the plastic mode of deformation is similar in shape to the elastic.

Unfortunately, this extension still leaves very severe restrictions on the usefulness of response spectra in practice. It is seldom true, even in very simple structures, that plastic deformations are distributed similarly to the elastic deformations. For more complex structures, in which several modes of vibration may be excited significantly by an earthquake, even purely elastic behaviour cannot be predicted precisely by response spectrum procedures[7]. Computer studies have demonstrated that various superposition techniques, notably the root-mean-square method, will give reasonable estimates of the maximum response to be expected in regular elastic structures[8,9]. However, it is quite unlikely that similar procedures, even when combined with the ductility factor concept, can lead to useful information about the inelastic response of arbitrary tall buildings. In practice, yielding may range from a general to an extremely localized phenomenon; it may be expected to destroy completely the elastic vibration mode characteristics which form the basis of mode superposition techniques, and the relationship between total inelastic energy absorption and the maximum local yield amplitudes must be exceedingly complex.

In the past, this problem has not been of critical importance. Undoubtedly, significant quantities of energy were absorbed in inelastic deformations by most structures subjected to severe earthquakes. However, traditional buildings have great capacity for inelastic energy absorption. A major part of the stiffness and strength of such structures is provided by non-structural partitions and exterior walls,and vast quantities of energy will be absorbed by these elements before the basic structure is stressed even to normal design levels[10]. In such cases, the empirical reduction of seismic coefficients from elastic response spectrum indications to values commensurate with experience (as has been done in the code recommended in 1958 by the Structural Engineers Association of California[11]) is fully justified.

On the other hand, the tendency in the design of modern, high-rise buildings is towards the use of minimal quantities of non-structural materials. Interior partitions often are light-weight elements, completely detached from the structural system, and the exterior curtain walls may be entirely of glass. Thus the entire strength of the building must be provided by the basic structural system, which also must provide the full inelastic energy absorption capacity. Clearly a more complete understanding of the inelastic earthquake behaviour of such structures is necessary if they are to be designed economically and with adequate factors of safety in regions of intense seismic activity. It was the purpose of the investigation described herein to shed some light on this problem.

METHOD OF ANALYSIS

Outline of Procedure

The IBM 7090 computer program employed in this investigation was developed originally by the junior author while employed by T.Y. Lin and Associates under contract with the U.S. Office of Civil Defense(12). It is designed to evaluate numerically the non-linear response of multi-story buildings to arbitrary time-varying lateral forces. The dynamic analysis is carried out by a step-by-step procedure. Within each short time increment, the structure is assumed to behave in a linear elastic manner. The elastic properties may be changed, however, from one interval to the next, thus the non-linear response is obtained as a sequence of linear responses of successively differing systems.

The analysis procedure involves the repeated application of the following steps for each successive time interval:

First: the stiffness of the structure appropriate to the time interval is evaluated, based on the moments existing in the members at the beginning of the time interval.

Second: changes in displacements of the elastic structure are computed, assuming the accelerations to vary linearly during the interval.

Third: these incremental displacements are added to the deformation state existing at the beginning of the interval, to obtain total member deformations.

Finally: based on these member deformations, member forces are computed from which stiffness coefficients appropriate to the next time interval may be determined.

Assumptions and Limitations

The program is designed to analyze any regular rectangular building frame, or combination of frames up to 30 stories high and 15 bays wide. Shear walls may be incorporated arbitrarily into the frame by the expedient of treating them as columns of finite width. Flexural and shear distortions are considered in all members, but axial deformations are neglected for simplicity. To provide a form of bi-linear moment resistance, each member is assumed to consist of two components in parallel: a basic elasto-plastic beam which develops a plastic hinge at either end when that end moment exceeds a specified yield value, M_y, combined with a beam which remains fully elastic. A typical member is shown in Fig.1a. It will be noted that the fully elastic component is rotated at each end through the total joint angle, Θ, while the elasto-plastic component deforms elastically only through the angle, ϕ. The additional joint rotation, α, indicated in these components represents the plastic hinge deformation, which is assumed to have the ideal plastic hinge property depicted in Fig.1b. It should be recognized that the total member moment continues to increase beyond the yield value, however, due to the contribution of the elastic component. In the present study, the fully elastic component contributed 5 percent of the (initial) total member stiffness.

Evaluation of Member Stiffnesses

To obtain the stiffness of the complete frame, it is necessary first to evaluate the stiffness of each of its constituent girders and columns. Because a non-linear moment curvature relationship has been assumed for each member, its stiffness properties may be expressed in matrix form only for the linear behaviour assumed to apply during each time increment. In general, the incremental moment-rotation relationship for each member may be expressed in the following form:

$$\begin{Bmatrix} \Delta M^i \\ \Delta M^j \end{Bmatrix} = \begin{bmatrix} S_a & S_b \\ S_b & S_c \end{bmatrix} \begin{Bmatrix} \Delta\theta^i \\ \Delta\theta^j \end{Bmatrix} \tag{1}$$

in which the stiffness coefficients, S, include contributions from both the elastic and the elasto-plastic member components.

The fully elastic component stiffness is given by

$$\begin{Bmatrix} \Delta m^i \\ \Delta m^j \end{Bmatrix} = p \begin{bmatrix} k_a & k_b \\ k_b & k_a \end{bmatrix} \begin{Bmatrix} \Delta\theta^i \\ \Delta\theta^j \end{Bmatrix} \tag{2}$$

in which

$$k_a = \frac{2EI}{L}\frac{(2+\beta)}{(1+2\beta)}$$

$$k_b = \frac{2EI}{L}\frac{(1-\beta)}{(1+2\beta)}$$

$\beta = \dfrac{6EI}{L^2 A' G}$ (shear flexibility factor)

A' = effective shear area

p = bilinear factor (5% assumed here)

m = moment in elastic component

The elasto-plastic stiffness contribution depends on the yield condition of the member, which depends in turn on whether the yield moments at the ends of the member have been exceeded. Four different member yield conditions may be defined, for which elasto-plastic component stiffness coefficients may be expressed as follows:

(a) No hinges: $|\mathcal{M}^i| < q M_y > |\mathcal{M}^j|$; $(\alpha^i = \alpha^j = 0)$

$$\begin{Bmatrix} \Delta \mathcal{M}^i \\ \Delta \mathcal{M}^j \end{Bmatrix} = q \begin{bmatrix} k_a & k_b \\ k_b & k_a \end{bmatrix} \begin{Bmatrix} \Delta\theta^i \\ \Delta\theta^j \end{Bmatrix} \tag{3a}$$

where $\mathcal{M}$ = moment in elasto-plastic component
$q = 1 - p$

(b) Hinge at "i": $|\mathcal{M}^i| \geq qM_y > |\mathcal{M}^j|$; $(\Delta\mathcal{M}^i = \Delta\alpha^j = 0)$

$$\begin{Bmatrix}\Delta\mathcal{M}^i\\ \Delta\mathcal{M}^j\end{Bmatrix} = \frac{q}{k_a}\begin{bmatrix}0 & 0\\ 0 & (k_a^2 - k_b^2)\end{bmatrix}\begin{Bmatrix}\Delta\theta^i\\ \Delta\theta^j\end{Bmatrix} \quad (3b)$$

(c) Hinge at "j": $|\mathcal{M}^i| < qM_y \leq |\mathcal{M}^j|$; $(\Delta\alpha^i = \Delta\mathcal{M}^j = 0)$

$$\begin{Bmatrix}\Delta\mathcal{M}^i\\ \Delta\mathcal{M}^j\end{Bmatrix} = \frac{q}{k_a}\begin{bmatrix}(k_a^2 - k_b^2) & 0\\ 0 & 0\end{bmatrix}\begin{Bmatrix}\Delta\theta^i\\ \Delta\theta^j\end{Bmatrix} \quad (3c)$$

(d) Hinge at "i" and "j": $|\mathcal{M}^i| \geq qM_y \leq |\mathcal{M}^j|$; $(\Delta\mathcal{M}^i = \Delta\mathcal{M}^j = 0)$

$$\begin{Bmatrix}\Delta\mathcal{M}^i\\ \Delta\mathcal{M}^j\end{Bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\begin{Bmatrix}\Delta\theta^i\\ \Delta\theta^j\end{Bmatrix} \quad (3d)$$

The total member stiffness is given by the combination of Eqs. 2 and 3, i.e. $\{\Delta M\} = \{\Delta m\} + \{\Delta\mathcal{M}\}$. Thus the stiffness coefficients of Eq.1 may be expressed as follows for the four yield conditions:

	S_a	S_b	S_c
(a) No Hinges:	k_a	k_b	k_a
(b) Hinge at "i"	pk_a	pk_b	$k_a - q\frac{k_b^2}{k_a}$
(c) Hinge at "j"	$k_a - q\frac{k_b^2}{k_a}$	pk_b	pk_a
(d) Hinges at "i" and "j"	pk_a	pk_b	pk_a

Frame Stiffness

When the stiffness of each member has been determined, the stiffness of the complete frame may be obtained by well-known techniques of matrix structural analysis. The procedure used in the present investigation is described in Reference 13 and will not be discussed in detail here. The result of the frame stiffness analysis is a stiffness matrix made up of submatrices arranged in tri-diagonal form. Each submatrix represents the relationship between the vector of all forces developed at one floor level as the result of a corresponding displacement vector imposed at that or at an adjacent floor level. The complete frame stiffness matrix $[K]$ is defined by the following expression:

$$[K]\{\Delta r\} = \{\Delta R\} \quad (4)$$

in which Δr = change in all joint displacements

ΔR = change in corresponding forces

Computation of Displacements

In matrix form, the dynamic equilibrium of a linear multiple degree of freedom system may be expressed as follows:

$$[M]\{\ddot{r}\} + [C]\{\dot{r}\} + [K]\{r\} = \{R\} \quad (5a)$$

To be applicable to the non-linear system considered in this study, however, the equation must be modified to represent the linear conditions which are assumed to exist only in each limited time interement:

$$[M]\{\Delta\ddot{r}\} + [C]\{\Delta\dot{r}\} + [K]\{\Delta r\} = \{\Delta R\} \qquad (5b)$$

Thus, Eq.5b represents equilibrium of the <u>changes</u> of forces which occur within the time interval; it corresponds to Eq.4, but with the addition of terms associated with inertia forces (due to changes in the acceleration vector, $\{\Delta\ddot{r}\}$). and damping forces (due to changes in the velocity vector, $\{\Delta\dot{r}\}$). It should be noted that in this equation, the mass matrix $[M]$ has non-zero terms only at the positions on the major diagonal which correspond with lateral story displacements, because the mass is assumed to be concentrated at these levels.

Eq.5b may be solved easily for the change in the displacement vector, $\{\Delta r\}$, if it is assumed that the acceleration varies linearly during the time interval. A similar procedure is discussed in detail in Reference 14, so only a brief description will be presented here. Assuming that the acceleration vector varies linearly, its change during the time interval is given by

$$\{\Delta\ddot{r}\} = \frac{6}{\Delta t^2}\{\Delta r\} + \{A\} \qquad (6a)$$

in which Δt represents the length of the time interval, and

$$\{A\} = -\frac{6}{\Delta t}\{\dot{r}\}_{t_o} - 3\{\ddot{r}\}_{t_o} \qquad (6b)$$

where the subscript "t_o" is used to denote conditions existing at the beginning of the time interval. Similarly, the change in the velocity vector is given by

$$\{\Delta\dot{r}\} = \frac{3}{\Delta t}\{\Delta r\} + \{B\} \qquad (7a)$$

in which

$$\{B\} = -3\{\dot{r}\}_{t_o} - \frac{\Delta t}{2}\{\ddot{r}\}_{t_o} \qquad (7b)$$

A second assumption was made in deriving the displacement relationships for this system: that the damping matrix is proportional to the mass matrix, i.e.

$$[C] = \lambda[M] \qquad (8)$$

where λ is the proportionality constant. This assumption is not essential to the present analysis, but is not unreasonable and tends to simplify the equations. Introducing Esq.8, 7a and 6a into Eq.5b permits this dynamic response equation to be written in the following pseudo-static form:

$$[K^*]\{\Delta r\} = \{\Delta R^*\} \qquad (9)$$

in which

$$[K^*] = [K] + \left(\frac{6}{\Delta t^2} + \frac{3}{\Delta t}\lambda\right)[M] \tag{10a}$$

$$\{\Delta R^*\} = \{\Delta R\} - [M]\{A\} - \lambda[M]\{B\} \tag{10b}$$

Because of the tri-diagonal form of $[K^*]$, Eq.8 may be solved by means of recursion equations (as explained for static analysis in Reference 13) to determine the change in the displacement vector Δr which takes place during the time interval. Finally this change in displacement may be introduced into Eqs.6a and 7a to determine the corresponding changes in the acceleration and velocity vectors. Total displacements, velocities and accelerations are obtained, of course, by merely adding these incremental vectors to the quantities existing at the beginning of the time interval.

Evaluation of Member Deformations

After the joint rotations and story displacements of the frame have been determined, the analysis of the corresponding member deformations would be a simple matter if the system were linearly elastic; in such cases, a unique transformation matrix may be derived which expresses the member deformations in terms of the structure displacements. Analysis of the member deformations in the present case is greatly complicated by the bi-linear member properties which have been assumed. For this type of system only the changes of deformations occurring during each time interval may be computed directly; the total deformation at any time must be obtained by superposing the incremental deformations which have been produced up to that time.

In general, the deformation at each end of each member may include both an elastic rotation, ϕ, and a plastic hinge rotation, α, as shown in Fig.1a. These deformations result from two types of joint displacements: joint rotation, ω, and chord rotation, γ (caused by joint translation). As may be seen in Fig.2, the displacement - deformation relationship for joint i of any member may be expressed

$$\phi^i + \alpha^i = \omega^i - \gamma \tag{11}$$

The elastic rotation, ϕ, determines the yield condition of the member. To determine ϕ, it is necessary to establish the changes in the plastic hinge rotation, α, which occur during each time increment. The type of plastic deformation which occurs depends upon the yield condition of the member, and again four categories can be established, corresponding to the four stiffness conditions of Eq.3. Incremental yield rotations developed in each of these cases are as follows:

(a) No Hinges: $$\Delta\alpha^i = \Delta\alpha^j = 0 \tag{12a}$$

(b) Hinge at "i": $$\Delta\alpha^i = \Delta\omega^i - \Delta\gamma + \frac{k_b}{k_a}(\Delta\omega^j - \Delta\gamma);\quad \Delta\alpha^j = 0 \tag{12b}$$

(c) Hinge at "j": $\Delta\alpha^i = 0$; $\Delta\alpha^j = \Delta\omega^j - \Delta\gamma + \frac{k_b}{k_a}(\Delta\alpha^i - \Delta\gamma)$ (12c)

(d) Hinges at "i" and "j": $\Delta\alpha^i = \Delta\omega^i - \Delta\gamma$; $\Delta\alpha^j = \Delta\omega^j - \Delta\gamma$ (12d)

(It must be noted that for hinge rotations to develop, the incremental rotation must be in the same direction as the elastic rotation; otherwise incremental displacements will produce a reduction of elastic rotation and no incremental yield displacement).

By superposing the plastic rotations developed during each time increment, the total state of member deformation may be established by means of Eq.11. Moments in the elasto-plastic member component, which control the member yield condition, may then be computed from the following matrix relationship

$$\begin{Bmatrix} m^i \\ m^j \end{Bmatrix} = q \begin{bmatrix} k_a & k_b \\ k_b & k_a \end{bmatrix} \begin{Bmatrix} \phi^i \\ \phi^j \end{Bmatrix} \qquad (13)$$

PROGRAM OF INVESTIGATION

General Scope

The research work reported herein represents only a preliminary investigation into the non-linear behaviour of tall buildings subjected to earthquakes. It was intended to demonstrate the order of magnitude of the flexural ductility which may be required of the columns and girders in a typical building frame, but only a very limited range of variables was considered. A single 20 story rectangular frame geometry was employed; the structural variation consisted only in changes of member stiffnesses and yield moments as described below. These frames were both subjected to the same pattern of ground motion excitation: the first four seconds of the El Centro 1940 earthquake accelerogram (N-S component). However, the intensity of the excitation was varied by multiplying the accelerogram by an appropriate reduction or amplification factor.

The building frames were analyzed first for their elastic response to the full El Centro ground motion intensity. For this purpose a standard mode-superposition computer analysis program was used (described in Reference 12, Vol.1) considering the first six modes of vibration and assuming each mode to be 10 percent critically damped. The program automatically computed the maximum forces developed in each column and girder of the frame, as well as the maximum story displacements, shears and moments for the entire structure. By comparing the member moments computed by this program with the yield moment specified for each member it was a simple matter to determine the relative overstressing of the structure which would be produced by the El Centro earthquake, or conversely, the reduction in earthquake intensity required to avoid overstress. On this basis, it was found that an earthquake intensity about 36% of El Centro would cause incipient yielding.

Taking this 36% (elastic limit) intensity as the reference level, additional intensity levels, 68%, 100% and 132% of El Centro, were used in evaluating the non-linear response of the building frames. These all represent possible earthquake conditions to which a building in a seismically active region might be subjected, and the relative response of the frames provides some indication as to how the ductility requirement varies with earthquake intensity.

It is recognized that the first 4 seconds of the El Centro accelerogram is not equivalent to the complete earthquake excitation, even though it includes the maximum recorded ground accelerations. Even when responding in a purely elastic fashion, the building does not develop its maximum response within the first 4 seconds. When non-linear behaviour is considered, the duration of the excitation may be expected to have a still greater influence on the amplitude of deformations which are produced. Thus, the results presented here are not intended to represent the response to the actual El Centro earthquake, but rather to a similar earthquake having only a 4 second duration. This very curtailed accelerogram was adopted in order to conserve computer time: about 20 minutes of machine time was required to perform each of the 4 second earthquake analyses described herein, using an IBM 7094 computer. One analysis was made, however, using the first 8 seconds of the accelerogram in order to indicate how the duration of excitation might affect the results.

Building A: *Stiff Frame*

The first building analyzed was an open frame structure with general dimensions as shown in Fig.3. The basic member sizes, also tabulated in this figure, were patterned after the example building of Reference 6. On the basis of these dimensions, a static analysis of the frame was carried out by computer, using lateral loads as suggested by the SEAOC Code[11] combined with vertical dead load plus 100 psf floor loading. This analysis yielded a design moment for each girder and a design axial force plus moment for each column.

In a normal design process, these design forces would serve to proportion the reinforcing of the various members. For the purpose of the present investigation, however, no detail design was required; it was sufficient merely to establish a yield moment for each member to serve as input for the non-linear analysis program. The yield moment in each girder was arbitrarily fixed at twice the member design moment. The yield condition of the columns presented a more difficult problem, due to the interaction effect of the axial forces. These members were designed by the Ultimate Strength Method, for a factor of safety of 2 with respect to the design axial force combined with a moment. Then, assuming that the member was subjected to a static design axial force (without the factor of 2), the ultimate moment given by the interaction curves of Fig.5.25 in Reference 15 was taken as the member yield moment. This process resulted in rather large column yield: design moment ratios ranging from 5 in the upper stories to 10 in the lower stories and averaging about 7.

Building A-1: Flexible Frame

Because Building A was found to be quite stiff for an open frame building (its period of vibration was 1.60 seconds) a second frame was studied, of similar geometry but with proportionately reduced stiffness in each member. The stiffness of this Building A-1 was taken at one-third of Building A (selected to give approximately 1/4 in. drift per ten ft. of height when subjected to code lateral loads), which required that the member cross-sectional dimensions be made 76 percent of those in Building A. The period of vibration of Building A-1 is 2.77 seconds, thus it is a rather flexible frame.

Because only a proportional change of member stiffness was made in Building A-1 and the same loads were assumed as for Building A, the member design forces are the same as were determined for that building. However, lower column yield moments were established in this frame (due to their reduced cross-sections), ranging from slightly over 2 in the upper stories to 6 in the lower stories, and averaging about 4.

RESULTS OF ANALYSES

Computer Output

The principal objective of the non-linear analysis program is the evaluation of the maximum inelastic flexural deformations produced in each member of the frame during the course of the earthquake. This deformation is represented by the plastic hinge angle, α, which is evaluated for each end of each member at the end of each time increment.

The maximum value of α, which is stored for each member in the computer and printed at the end of the analysis, represents the ductility requirement imposed on the member by the earthquake. In order that this ductility requirement may be interpreted readily, the angle α is compared with the maximum elastic rotation angle Φ_y which the member may develop. This elastic limit rotation angle is the angle developed when the member is subjected to its yield moment; this may be accomplished either by a simple beam test or by application of anti-symmetric yield moments as shown in Fig.4. For a uniform beam, the elastic yield rotation is given by

$$\Phi_y = \frac{M_y L}{12EI}(2+\beta) \qquad (14)$$

in which the symbols are as defined previously (Eq.2). The ductility factor, μ, then represents the ductility requirement, defined as follows:

$$\mu = \frac{\Phi_y + \alpha_{max.}}{\Phi_y} = 1 + \frac{\alpha_{max}}{\Phi_y} \qquad (15)$$

It will be noted that the ductility factor is the ratio of elastic yield plus plastic hinge rotation to the elastic yield rotation, and thus is consistent with previous usage of the term(3). The computer lists this quantity for each member of the frame; however, because of the manner in which it is computed, a value of $\mu = 1.00$ is listed for members which have not yielded. In these members, the maximum moment developed at either end represents the significant response parameter. The computer also lists this quantity, expressed as the ratio of maximum to yield moment, for each member. In addition it lists the maximum axial force developed in each column and the maximum lateral displacement of each story.

Lateral Displacements

The maximum lateral displacement produced at each story level of Buildings A and A-1 are presented graphically in Figs. 5 and 6 respectively. In each case, the displacement is seen to increase quite regularly with the ground motion intensity; however, in Building A a general shift of the larger yield amplitudes from the upper toward the lower stories also is evident. Although Building A-1 is three times more flexible, its displacement amplitudes are only about double those of Building A; this discrepancy is due to the fact that smaller forces are developed in the more flexible building. It should be noted that the elastic displacement response is only about 50% or 80%, respectively, of the non-linear response of each building to the full ground excitation.

Also shown in Fig. 5 are the maximum displacements produced in Building A by the first 8 seconds of the 100% intensity accelerogram. The additional 4 seconds of excitation may be seen to result in an average increase in displacement of about 25%; furthermore, the additional deflection clearly is due primarily to increased deformation in the lower stories.

Member Ductility Factors

The member ductility factors produced by the 100% earthquake intensity acting on the two buildings are presented in Figs. 7 and 8. It is of interest that the column yielding in both cases is confined to the upper stories; the lower story column response is fully elastic. In Building A, the girders tend to yield quite uniformly in the lower stories, with ductility factors of about 3.2. However, significantly greater girder yielding takes place in the upper stories, reaching a peak ductility factor of nearly 6.5. In Building A-1, on the other hand, girder ductility requirements are lower throughout, and it is the columns which show the sharp increase in ductility factor near the top. This contrasting behaviour results, of course, from the relatively reduced column yield moments in Building A-1. Fig. 7 also shows the increased girder ductility factors which were caused by the 8 second earthquake excitation. No change was observed in the column yield amplitudes, but very significant increases in the girder ductility factors were produced in the lower stories of the frame by the extended earthquake duration.

Figs. 9 and 10 show the variation of ductility factor with ground motion intensity for the interior girders of each frame. In Building A (Fig. 9) may be seen a shift of maximum yielding toward the lower stories with increasing earthquake intensity, which corresponds with the trend observed in Fig. 6. With the 132% intensity, girder ductility factors approaching 6 are indicated over most of the height of the frame. A similar trend may be noted for the more flexible frame in Fig. 10, but the maximum girder ductility requirement is only about half as great in this case.

The dashed lines in Figs. 9 and 10 indicate the maximum interior bay girder moments developed in the frame when responding elastically to the 100% intensity accelerogram. These results are expressed as the ratio of maximum to yield moment for each member. It is of interest that these elastic moment ratios average about 2 and 1 1/2 in Buildings A and A-1, respectively, while the non-linear ductility factors for the corresponding excitation intensity average about 3.2 and 2.2

Column Axial Forces

The maximum axial forces developed in the exterior columns of the two frames are shown in Figs. 11 and 12. Of particular interest in these figures is the very small influence of earthquake intensity on the axial forces. Nearly doubling the ground motion accelerations (from 68% to 132%) causes only about a 30% increase in axial forces at the base of Building A, and has negligible effect over the entire height of Building A-1. This relative independence of axial forces with respect to earthquake intensity is due to the fact that very little force increase can be developed in the frames once the plastic hinges are fully mobilized. Of course, significant increases may be observed as the intensity is increased from incipient yielding (36%) to 68%, because it is in this range that most of the plastic hinges are developed. Fig. 11 shows in addition that extending the duration of the excitation to 8 seconds also has very little effect on the column axial forces.

CONCLUSIONS

Although it is believed that the structures considered here are reasonably representative designs, and that the assumed earthquake motion is well within the realm of possibility, no broad conclusions should be drawn from this limited investigation. The results which have been presented demonstrate how a particular earthquake motion would affect a very specific class of structure, and it is impossible to infer from these results how other structural systems would respond to this ground motion, or how these structures would behave with other excitation.

Nevertheless, it is of interest to comment on a few aspects of these results:

(1) The maximum story displacements developed in the non-linear structures are significantly greater than the displacements produced in corresponding elastic structures by the same excitation. The discrepancy was found to be even greater when the 8 second duration accelerogram was used. These results indicate that there is little possibility of predicting the deformations of a multi-story non-linear structure from the results of an elastic analysis, in contrast to suggestions(6) resulting from previous studies of single story systems(3).*

(2) Ductile deformations tend to vary widely through the structure, in a manner which depends not only on the structural properties, but also on the intensity of the ground motions (and undoubtedly on its character, as well). The maximum:yield moment ratios obtained in an elastic analysis generally are much smaller than the corresponding member ductility ratios obtained in the non-linear analyses, and do not appear to provide a direct approach to estimating the ductility requirements.

(3) Providing for essentially elastic response in the columns, while absorbing the earthquake energy in plastic deformations of the girders appears to be an effective approach to the earthquake-resistant design of tall buildings. The present results indicate that the excessive column forces (and consequent danger of total collapse) can be avoided by such designs.

---

*This conclusion was based upon the analyses reported herein, for which no damping was considered. Subsequent studies have shown that damping generally causes a significant reduction in the displacements which are computed in the non-linear analysis; thus, this conclusion may not be valid when damping is considered.

ACKNOWLEDGEMENTS

The computer results reported herein were carried out under sponsorship of T.Y. Lin and Associates, Consulting Structural Engineers, Van Nuys, California. The paper was prepared during the residence of the senior author at the Cambridge University Engineering Laboratory. Only through the cooperation of both of these organizations was it possible to carry out the present phase of this investigation, and the authors gratefully acknowledge their contributions.

REFERENCES

1. Biot, M.A. "Analytical and Experimental Methods in Engineering Seismology" Trans. A.S.C.E. Vol.108, 1943.

2. Housner, G.W., Martel, R.R., and Alford, J.L. "Spectrum Analysis of Strong Motion Earthquakes" Bull. Seism. Soc. Amer. Vol.43, No.2, April 1953.

3. Veletsos, A.S., and Newmark, N.M. "Effect of Inelastic Behaviour on the Response of Simple Systems to Earthquake Motions" Proc. 2WCEE, Tokyo, Japan, 1960 pp.895-912.

4. Penzien, J. "Elasto-Plastic Response of Idealized Multi-story Structures Subjected to a Strong Motion Earthquake " ibid. pp.739-760.

5. Berg, G.V. and Thomaides, S.S. "Energy Consumption by Structures in Strong Motion Earthquakes" ibid. pp. 681-698.

6. Blume, J.A., Newmark, N.M. and Corning, L. "Design of Multistory Reinforced Concrete Buildings for Earthquake Motions" Published by the Portland Cement Association, Chicago, Illinois, 1961.

7. Clough, R.W. "On the Importance of Higher Modes of Vibration in the Earthquake Response of a Tall Building" Bull. Seism. Soc. Amer. Vol.45, No.4 Oct.1955.

8. Merchant, H.C. and Hudson, D.E. "Mode Superposition in Multi-degree of Freedom Systems using Earthquake Response Spectrum Data" Bull. Seism. Soc. Amer. Vol.52, No.2, April 1962, p.405.

9. Clough, R.W. "Earthquake Analysis by Response Spectrum Superposition" Bull. Seism. Soc. Amer. Vol.52, No.3, July 1962 p.647.

10. Blume, J.A. "Structural Dynamics in Earthquake Resistant Design" Proc. ASCE Vol.84, No.ST-4, July 1958.

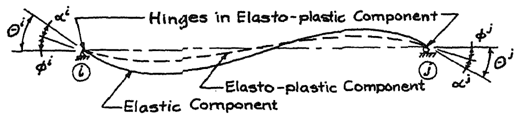
11. Binder, R.W. and Wheeler, W.T. "Building Code Provisions for Aseismic Design" Proc. 2WCEE Tokyo Japan, 1960, pp.1843-1876.

2. T.Y. Lin and Associates " A computer Program to Analyze the Dynamic Response of High Rise Buildings to Nuclear Blast Loading" Report to OCD Protective Structures Divsion (OCD-OS-63-44) October 1963. Vol.1: Elastic Analysis; Vol.2: Non-Linear Analysis.

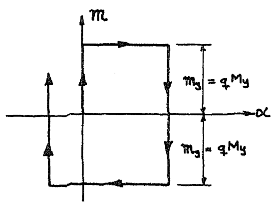
3. Clough, R.W., Wilson, E.L., and King, I.P. "Large Capacity Multi-story Frame Analysis Programs" Proc. ASCE, Vol.89, No.ST-4, August 1963 (p.179).

4. Wilson, E.L. and Clough, R.W. "Dynamic Response by Step-by-Step Matrix Analysis" Symposium on the Use of Computers in Civil Engineering, Lisbon, Portugal, October 1962.

5. Ferguson, Phil M. "Reinforced Concrete Fundamentals" John Wiley and Sons, New York, 1958.



a. Dual Component Bi-linear Member



b. Hinge Characteristic: Elasto-plastic Component

FIG. 1 TYPICAL FRAME MEMBER

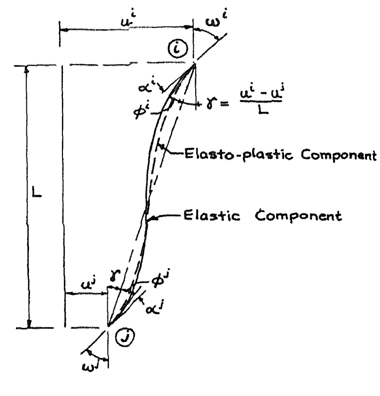


FIG. 2 DISPLACEMENT-DEFORMATION RELATIONSHIP

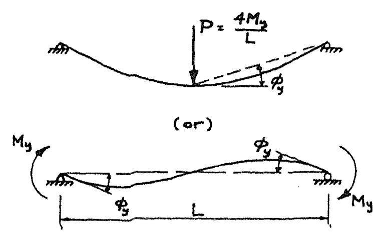


FIG. 4 ELASTIC LIMIT ROTATION

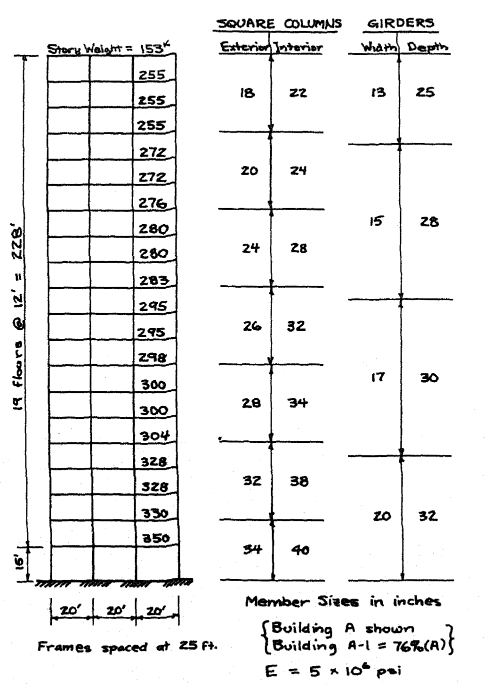


FIG. 3 EXAMPLE BUILDING: DIMENSIONS & PROPERTIES

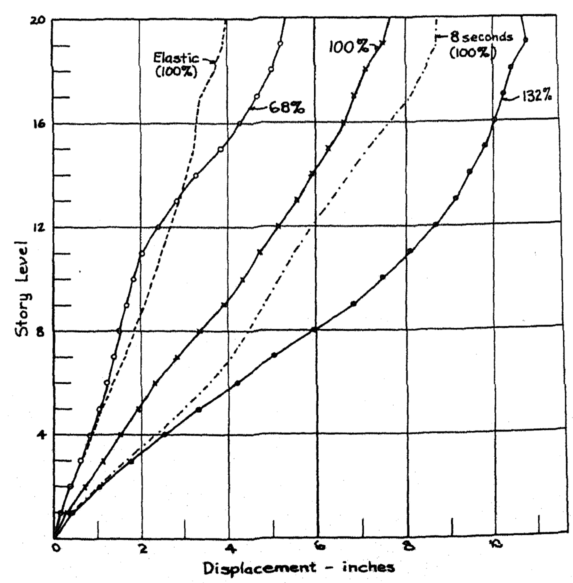


FIG. 5 MAXIMUM DISPLACEMENTS - BLDG. A

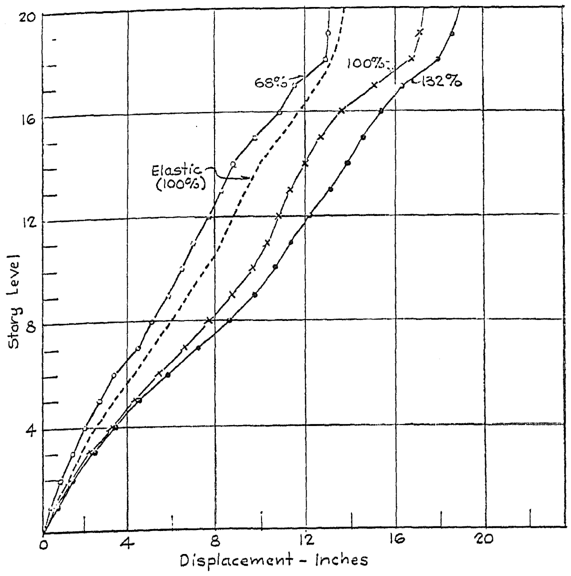


FIG. 6 MAXIMUM DISPLACEMENTS - BLDG. A-1

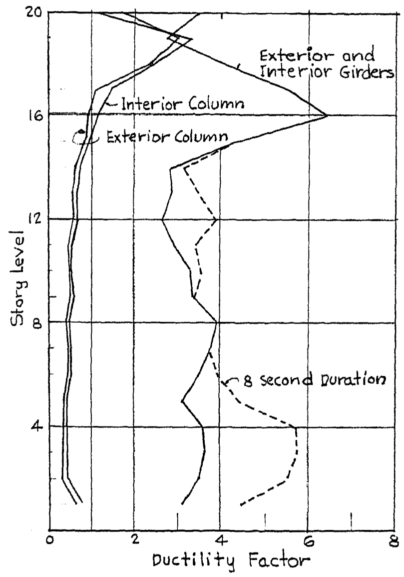


FIG. 7 MEMBER DUCTILITY - BLDG. A, 100% INTENSITY

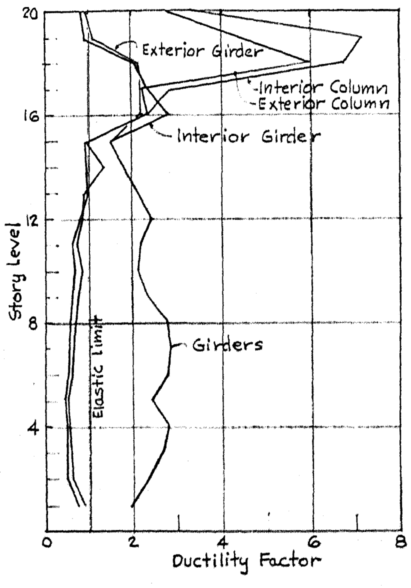


FIG. 8 MEMBER DUCTILITY - BLDG A-1, 100% INTENSITY

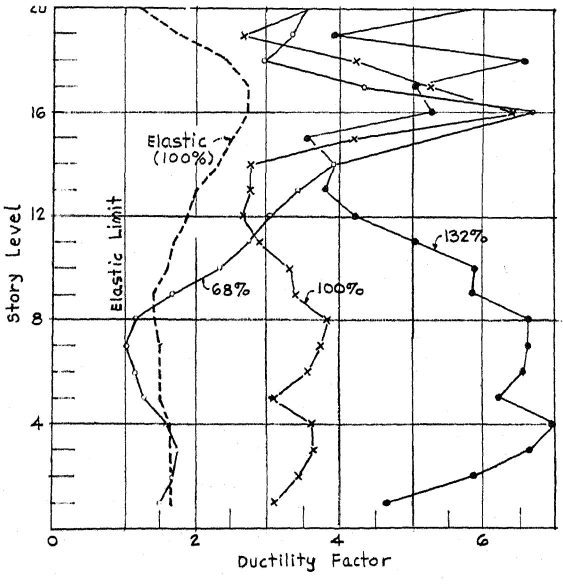


FIG. 9 INTERIOR GIRDER DUCTILITY - BLDG. A

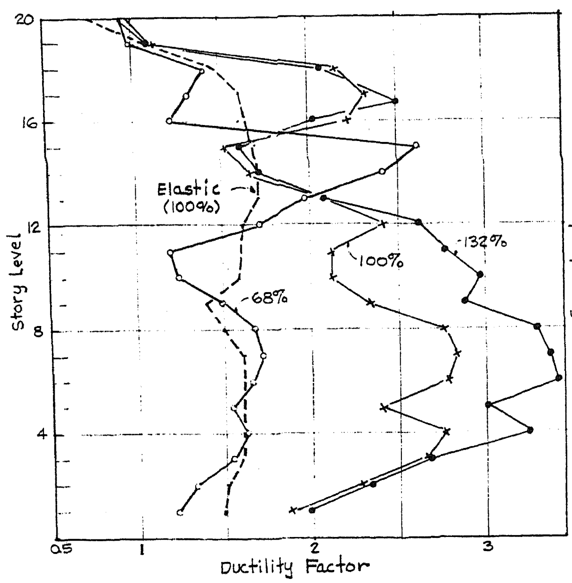


Fig. 10 Interior Girder Ductility - Bldg. A-1

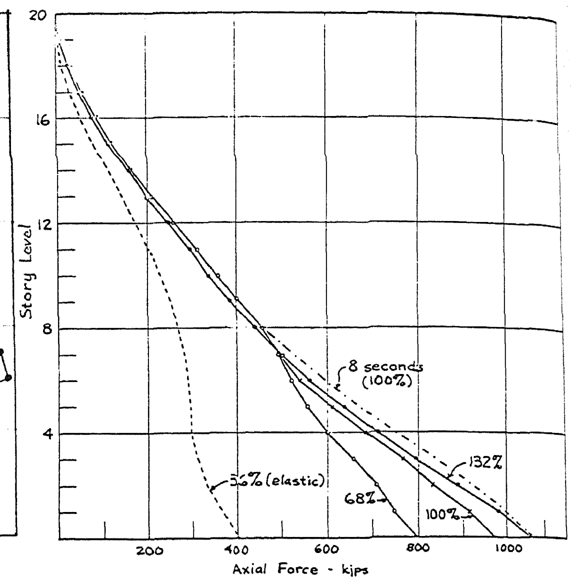


Fig. 11 Max. Force in Exterior Column - Bldg. A

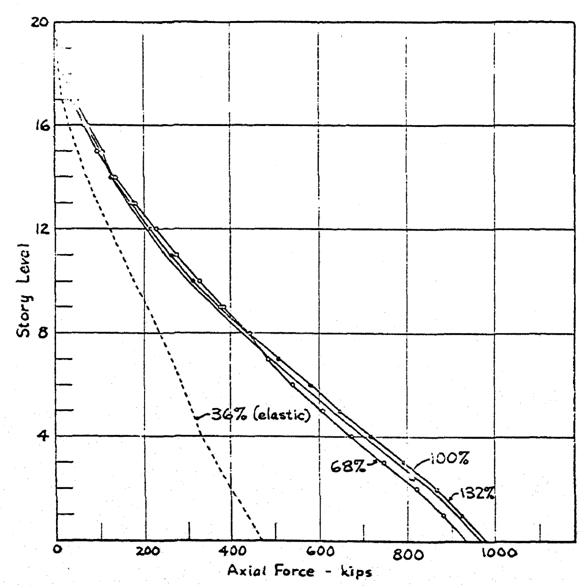


Fig. 12 Max. Force in Exterior Column - Bldg. A-1

INELASTIC EARTHQUAKE RESPONSE OF TALL BUILDINGS

BY R.W. CLOUGH, K.L. BENUSKA AND E.L. WILSON

QUESTION BY: T. HISADA - JAPAN

1. You consider two components for bi-linear moment resistance, where the contribution of the fully elastic component is taken as 5% of the initial total member stiffness. What is the reason for this value?

2. In the actual design of tall buildings in U.S.A., what value of ductility factor do you consider in general? If there is no criterion, what is your opinion of this matter? Furthermore, I would be very happy if I could hear the opinion of Mr. Chairman.

REPLY BY: K.L. BENUSKA

1. The contribution of the elastic component is taken as 5% of the initial total member stiffness during plastic deformation because it is a reasonable estimate of the "strain hardening" effects which are neglected in the normal ideally plastic idealization. This results in a member moment-flexural deformation relationship similar to the idealized story restoring force-displacement characteristic shown in Dr. Hisada's paper (Fig. 1b) presented in Session II. Dr. Hisada chose to use 10%, 0, and -5% as the portion of initial total story stiffness which might act during plastic deformation.

2. The earthquake recommendations of the Structural Engineers' Association of California anticipate overall structure ductility factors of 3 to 6, depending upon the type of structural system, when subjected to an El Centro 1940 intensity motion. The member ductility factors reported in this paper are consistent with this anticipation. However, the tendency for plastic deformation to concentrate in selected members rather than distribute uniformly throughout the structural system is of significance to actual building design.

COMMENT BY: K. MUTO - JAPAN

According to our experience of analysis, response of a 30-storey building depends on the values of damping coefficient and the intensity of earthquake waves. In the case of small damping the deformation or ductility factor is very unstable.

REPLY BY: R.L. BENUSKA

The moment resisting frames investigated during our continuing study have shown no instability even when considering no damping. Ductility requirements have increased with greater intensities of ground motion and/or low damping, and have decreased with small intensity of ground motion and/or high damping, in a uniform manner. Each of our analyses has included the 5% "strain hardening" effect.

COMMENT BY: N.M. NEWMARK - U.S.A.

In designing a building for a particular earthquake intensity, one has the option of providing for a certain force or degree of deformation in the elastic range, and then providing for greater intensities through the mechanism of yielding or energy absorption beyond the elastic range. Earthquake design codes more or less implicitly provide for resistance to a certain intensity of earthquake elastically, and resistance to stronger earthquakes through the mobilization of ductility in the structure. However, the presently available codes do not provide the same amount of ductility over the whole range of building periods that the designer might encounter.

If one designs according to the SEAOC code or the Uniform Building Code in the United States, the earthquake that can be resisted without yielding is roughly of an intensity one-fourth to one-fifth of the El Centro earthquake. In other words, for an earthquake of intensity similar to that of El Centro, the ductility factor must be sufficient to provide for a ductility factor of at least 4 to 5. However, with ductility factors of this order of magnitude one must be prepared for local irregularities or differences in the values in different members of the structure.

The actual magnitude of ductility factor required may be reduced because of the loss of energy transmitted to the building owing to the imperfect coupling with the ground, and other factors of a similar nature. In any event, if the maximum earthquake that might occur is no stronger than the El Centro earthquake, and if the design is made in accordance with the Uniform Building Code of the United States, and if one chooses the details in reinforced concrete, steel, or other materials properly so as to mobilize a ductility factor of the order of 4 to 6, then one generally can avoid collapse or failure. The building may have serious damage in such a strong earthquake, but it will not collapse unless the earthquake is somewhat stronger in intensity than the recorded El Centro earthquake. In most instances a more intense earthquake than El Centro could cause collapse of buildings designed under the conditions just described.

It was pointed out in the book, "Design of Multi-Story Reinforced Concrete Buildings for Earthquake Motions", by Blume, Corning, and myself, that there are differences in the ductility factors locally mobilized at the joints and connections in a building because of the distribution of stresses through the building. A uniform ductility factor from top to bottom also is not consistent with the inverted triangle of accelerations implied by the Uniform Building Code, but this is a reasonably good approximation.

I think the authors have made an excellent study and that we can learn a great deal from the results. Their paper indicates some of the things that must be looked at carefully in the design of any unusual structure. One of the points that should be considered is the difference in behaviour of a building with a very long period compared with one with a very short period. These differences are quite marked. In general it turns out that for long period systems, the maximum total displacement is reasonably constant, regardless of the amount of yielding, and when this is the case, one can make an estimate of what the ductility factor must be merely by comparing the design shears with those corresponding to an analysis for the particular earthquake considered.

However, for relatively short period structures, the maximum displacement is not constant but the shearing force is practically constant, being more or less determined by the mass and the maximum ground acceleration, and one can not use so readily the concept of the ductility factor in design because the maximum displacement may be very greatly increased under these conditions. For intermediate period structures, there is a tendency for the energy absorption to remain constant and the displacement does increase slightly, which involves a slight increase in ductility factor beyond the direct proportion between the analytical value and the design code value.

Of course the designer has the option, as pointed out in the book referred to, of selecting where he wants the ductility to be mobilized, or where the energy is to be absorbed. In most instances, he can avoid ductility or yielding in the columns merely by selecting the design of the girder so that yielding occurs in the girders. Nevertheless, the ductility factor that must be provided for in any earthquake code is related to the relative values of the design shears or moments computed by that code and those computed by spectrum analysis or other more exact analysis. The required ductility factor is at least as great as the ratio of these quantities, and may be considerably greater for very short period structures (in the range of periods less than about three-tenths of a second).

COMMENT BY: R.W. CLOUGH

Two points came up in the original presentation and the ensuing discussions on which I should like to make further comment.

The first point is in connection with the comments made by Dr. Housner in his introduction, concerning the relative amplitudes of displacements we computed compared with the elastic response. The calculations for this structure showed a significant increase of displacements inelastically as compared with our elastic computations. But, as Mr. Benuske pointed out, these inelastic computations were made with the undamped structure. At the time we began our computations we believed that the energy absorption in the inelastic process would be so much of an overriding factor that other damping characteristics would not be important. Subsequently we found that putting in viscous damping in addition to the inelastic mechanism does make a significant difference in response. The point was illustrated by the slide Mr. Benuske showed. We do not now think that the inelastic response will be significantly greater than the elastic.

The second point I wanted to comment on is in reference to Dr. Newmark's comments on the requirements of ductility being put into the structure. As he says, if one chooses to design members for a ductility factor of five or six, he can count on reasonably good performance for El Centro intensity earthquakes. The point I wanted to make is that in many cases it is very difficult to design into each member of the structure this kind of ductility requirement. Specifically, it may be quite difficult to provide any significant ductility in the columns in the lower portion of the structure, and one of the important points which our investigation demonstrated is that it is possible to force the ductility into other parts of the structure so that it will not be necessary to design every member for the same ductility requirements. In other words, our findings would indicate that it is possible to design columns for essentially elastic behaviour and by putting a greater energy absorption capacity into the girders to still have a stable structure under El Centro excitations or possibly even greater earthquakes.