ANALYTICAL AND EXPERIMENTAL STUDIES OF VIBRATION IN TWO BUILDINGS

By E. del Valle and J. Prince [1]

Abstract. Dynamic characteristics of two buildings were determined both analytically and experimentally. The buildings are of the bearing wall type, eight and 22 stories tall. Analytical studies included effects of bending and shear deformations of the structure as well as motion of the base. The experimental studies comprised three different types of tests.

Good agreement was obtained between experimental and analytical results.

Introduction

A housing project is near completion in Mexico City. It consists of more than 140 buildings ranging from 4 to 24 stories.

Dynamic studies were made of several representative buildings. This paper reports studies corresponding to two of them which will be referred to as buildings B (eight stories) and M (22 stories). These were selected attending to their unusual characteristics: both buildings depend largely on walls to transmit vertical loads as well as to resist lateral forces. Of additional interest was the influence of pile foundations on the actual modes and periods of vibration; evaluation of this effect involved certain simplifying assumptions.

General description

Building B. It is eight stories tall. Plan dimensions are 9.15 m by 63.15 m; total height from ground floor to roof level is 20.95 m. Longitudinally the building is divided in five sections, alternating three low and two high with a drop of half the story height between consecutive sections (fig 1).

The floor system in the high sections consists of concrete joists in both directions, 30 cm deep, with filler blocks; in the low sections the floor is a 10 cm concrete slab. Vertical loads are transmitted to the foundation through brick and concrete bearing walls and concrete columns. The foundation consists of cylindrical shells and deep beams; the weight of the building is in part taken by friction piles.

Seismic and wind loads in the transverse direction are resisted

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mainly by bearing walls and by panels with concrete diagonals. In the longitudinal direction stiffness between ground floor and first floor is afforded by frames and walls. From the first level up, spandrel beams, which are half the story height in depth, contribute strongly to stiffness. This effect is more important in the rear façade, where the elevation of the bottom of beams in high sections coincide with the top of beams of the adjoining low sections.

Concrete in columns, diagonals and elevator shafts from ground floor to fifth level had a nominal strength of 210 kg/cm^2. Between levels five and eight, as well as in beams and slabs of all floors, concrete had a nominal strength of 140 kg/cm^2.

Building M. This building is 22 stories tall. Plan dimensions are 22.80 m by 18.60 m; total height is 61 m (fig 2). The floors consist again of two-way concrete joists, 30 cm deep, with filler blocks. Vertical loads are transmitted to the foundation through concrete bearing walls. The foundation is similar to that of building B, with cylindrical shells and friction piles.

Lateral stiffness in the E-W direction is supplied by concrete walls which may be regarded as cantilevers built-in at the foundation. From the tenth level up some wall panels, marked with dotted lines in fig 2, are replaced with crossed diagonals. In the N-S direction both concrete walls and rigid frames contribute to stiffness.

Experimental Studies

Three different types of tests were performed for the experimental determination of natural periods of vibration:

a) Measurement of vibrations excited by wind and traffic by means of a modified portable Sprengnether seismograph. The characteristics of this instrument are the same as those described in ref. 1.

b) Recording of vibrations excited by the sudden release of a lateral force. The test procedure and apparatus have been described elsewhere (2).

c) Forced vibration tests. A description of the shaker used is found in ref. 3. Only building B was subjected to this type of test. Although resonance curves obtained were of doubtful quality, these tests permitted the identification of a mode consisting of horizontal displacements at the center, 180° out of phase with respect to the displacements at both ends at the same elevation. Similar modes have been reported in the literature (4).

Experimental results are presented in Tables 1 and 2.

Analytical Studies

Assumptions. In the computation of periods and natural modes of vibration the following assumptions were made:

Loads. Vertical loads were assumed concentrated at the floor levels. The weights were obtained from the architectural and structural drawings. Live loads were not considered.

Modulus of elasticity. It was first assumed that the modulus of elasticity of concrete was that given by the ACI formula, 1963. The value thus obtained was too high in comparison with laboratory determinations. After some study the formulas used were:

$E = 17{,}500\ (f'_c)^{0.4}$ for normal weight concrete and

$E = 11{,}600\ (f'_c)^{0.4}$ for lightweight aggregate concrete ($1400\ kg/m^3$), where f'_c (compressive strength) and E (modulus of elasticity) are in kg/cm^2. The discrepancy with concretes from other parts of the world must be ascribed to properties of local aggregates.

The shear modulus of elasticity, G, of brick masonry walls was initially taken as $14500\ kg/cm^2$. This value was obtained in laboratory tests of wall panels built with the same kind of brick (5). Lower values were assumed at a later stage since actual constructive clearances and defects seem to reduce stiffness considerably.

Stiffnesses. The assumptions made regarding stiffness of resistant elements were as follows:

a) Rigid frames. Moments of inertia of cross sections were computed for the total concrete area neglecting steel reinforcement. Slab contribution to the moment of inertia of the beams was considered. Lengths were measured to axis of members. Story stiffnesses of frames were computed by means of Wilbur's formulas (6) assuming the columns fixed at the foundation. Flexure of the building was not included in the computation of stiffness of rigid frames.

b) Panels with concrete diagonals. The stiffness of these elements was computed by conventional methods, using transformed area for diagonals in compression and steel area for diagonals in tension. Axial deformations of panel framing members were neglected.

c) Concrete walls. Shearing stiffness between any two consecutive floors was computed assuming the walls as cantilevers fixed at the lower floor, taking into account shearing and flexural deformations. For computations including bending deformations of the building, concrete walls were considered as cantilever beams fixed at the foundation.

d) Brick masonry walls. Story stiffness was computed first, assuming that the panels deform in pure shear and second, using Smith's theory (7). Moments of inertia used in the computation of bending deformations of the building were calculated as described in b) above.

Period Computations

Translation and rotation of the base were neglected in a first stage, and considered later on. Only final computations will be described in detail.

Building B. Three modes of vibration were computed for the transverse direction and two for the longitudinal direction. Higher modes were not computed as it was not possible to excite them experimentally.

Transverse direction. In the computation of periods for this direction, shearing and flexural deformations of the building were considered. The moment of inertia was obtained adding those of concrete walls and of brick walls or concrete diagonal panels. The total shearing stiffness was the sum of those corresponding to elements of the low and high sections computed separately.

Masses were lumped at the floor levels of high sections; half the mass of every floor in the low sections was concentrated at the upper and lower levels of adjoining high sections.

In a preliminary calculation modes were computed considering simultaneously shearing and flexural deformations on the assumption of fixed foundation. Rotation and translation of the base in the fundamental mode were considered as independent additional degrees of freedom. Computation of second and third modes involved coupling between base motion and deformations of the structure. A similar computation for the first mode yielded practically the same results as the Southwell-Dunkerley approximation (8,9).

The natural frequency in rotation of the base was computed assuming the building as an infinitely rigid block resting on the ground and on the piles. The soil is a very compressible clay with a high water content. Its properties are described elsewhere (10). Resistance of the ground was estimated using Barkan's coefficient of elastic non-uniform compression of soil (11). The elastic constant of piles under axial loads due to rotation of the base was derived from results of load tests of similar piles in the immediate vecinity. The effect of ground resistance was negligible in comparison with that of the piles.

Base frequency in translation involved superposition of restrictions provided by all surfaces in direct contact with soil and by the piles. Shearing stresses at the bottom and retaining walls were estimated after Barkan (11). Due allowance was made of the variation of soil properties with depth. Under the assumption of piles built-in at the foundation beams, their lateral resistance was obtained, first, from formulas proposed by Brooms (12) and Vesić (13) and, second, following Barkan's criteria (11).

Longitudinal direction. For the computation of periods the rear façade mentioned was considered as a concrete wall with rectangular holes. Flexure of the building was neglected, as the height to length ratio is about one third.

Computations of stiffnesses of frames and walls was similar to that for the transverse direction.

Rotation of the base was neglected due to the high moment of inertia of the pile group. Translation of the base was estimated as above.

Building M. This building also shows different types of resisting elements in each direction. In the E-W direction, it was regarded as a cantilever beam fixed at ground level with shearing and flexural deformations. The moment of inertia of this beam was taken as the sum of those corresponding to the concrete walls. Shearing deformations were computed with a total area equal to the sum of areas of walls parallel to the E-W axis.

First and second modes were computed including effects of base translation and rotation with the hypotheses described in the computations for building B.

In the N-S direction stiffnesses included the restriction provided by the beams fixed into the walls.

The first two modes were computed to compare with experimental results. Rotation and translation of the base were also included.

Frame stiffnesses were recomputed using a modification of Maney-Goldberg's method (14), as Wilbur's formulas are not applicable since the beams are flexible in comparison with the columns. Stiffnesses for the first mode were approximately equal to those given by Wilbur's formulas; higher stiffnesses were obtained for the second mode.

Computation of natural modes and periods of vibration involved coupling between lateral deformations of walls and frames in each story. From the total story shear associated to the assumed mode configuration, y, the shear taken by the frames was deduced for a certain value of the circular frequency, p . The walls were then analyzed with the shear thus reduced, as cantilevers fixed at the foundation. It was found that the iterative procedure outlined above, which requires the assumption of both p and y, leads to divergence, oscillation or convergence depending on relative stiffnesses of frames and walls. To force or accelerate convergence the Aitken-Ryker (15) method of extrapolation was used.

Results

Analytical and experimental values of periods of vibration for buildings B and M are summarized in tables 1 and 2, respectively. Final mode configurations are presented in figs 3 and 4. These configurations show the relative participation of each mode in the building response when analyzed for the proposed design spectrum for the compressible zone of Mexico City (17).

For the relatively small displacements produced by pulling tests a damping coefficient of 1 to 2% of critical was established, associated with the fundamental mode, in both directions of building M. A reliable determination of damping for building B was not feasible.

Discussion of results

Building B. In table 1 periods of vibration computed with $G = 14\ 500\ kg/cm^2$ are underlined. These values are consistently lower than those obtained experimentally; this result was to be expected, as $G = 14\ 500\ kg/cm^2$ applies only to wall panels carefully built and tested in the laboratory. After a few trials it was found that a value of G of the order of $3000\ kg/cm^2$ gave a good approximation to the measured periods in both directions. Similar conclusions have been reported by Blume (9). All periods not underlined in table 1 correspond to this lower G.

Periods due to shear and flexure of the structure as well as to rotation of the base and period of base translation computed from Brooms and Vesić formulas, superimposed by means of the Southwell-Dunkerley approximation (8) yielded a result only 6% higher than the measured value for the transverse direction. On the other hand, similar considerations led to a difference with respect to the measured period of over 50% in the longitudinal direction.

The main restriction to base motion is provided by the piles. Barkan assumes that, for lateral loads, the resistance of piles is determined only by their characteristics and depth of fixation. With this hypothesis the period of base translation is reduced and superposition with other effects resulted in a fairly good agreement with experimental values. Table 1 shows that periods for the transverse direction are considerably larger than longitudinal periods. This is partly due to the negligible effect of flexure in the latter direction while, in the former, flexure predominates as may be seen in fig. 3.

Building M. (N-S direction). It was found that stiffness depended mainly on the frames and that frame stiffness was in turn governed by the relative rigidity of the beams. It is then likely that a slightly higher value, 0.7, of the ratio of concrete moduli would result in better agreement between computed and measured periods.

In the E-W direction this building deforms mainly in flexure. The simultaneous consideration of shear and flexure leads to a period of 1.90 sec, that is, an increment of 0.02 sec over the corresponding value for flexure.

Table 2 includes periods for the first two modes only because higher modes could not be excited or detected experimentally. Furthermore, a modal analysis demonstrated that the participation factors for the third and higher modes are negligible in comparison with those of the first two.

As may be appreciated in table 2 the influence of foundation displacements is small in both directions (6%). This applies to Brooms and Vesić's as well as to Barkan's criteria and is in agreement with results obtained by Thomson (16).

In all cases computations yielded periods which are higher than those observed. The ratio T_{comp}/T_{obs} is almost constant for the first mode and a slight change in the modulus of elasticity of concrete would result in a complete agreement between analytical and experimental values.

Conclusions

a) Lateral stiffness of brick panels framed by concrete members is well defined considering only shearing deformations. For brick it is necessary to assume a low modulus of rigidity, which may vary with construction clearances and amplitude of motions.

b) In buildings similar to those studied in which lateral stiffness depends mainly on walls, it is sufficiently accurate to take the moment of inertia of the building as the sum of the individual moments of inertia of walls about their principal axes.

c) In buildings in which lateral stiffness is supplied by both shear walls and rigid frames, coupling between these elements should be taken into account in period computations.

d) Base displacement and rotation were not important factors in the cases studied, due to pile stiffness to rotation and translation.

Acknowledgements

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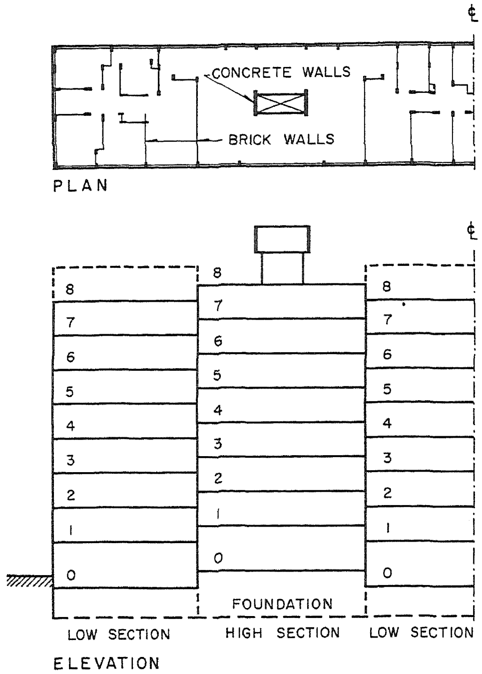
Table 1 Building B

Direction		Deformations considered	Criteria for analysis		Mode 1	Mode 2	Mode 3
Transverse	ANAL	F, S	Shear panels		0.80, 0.86	0.16, 0.23	0.08, 0.12
			Smith's theory		0.92	0.26	0.14
		F, S, R	Shear panels		0.95, 0.98	-	-
			Smith's theory		1.05	-	-
		F, S, R, T	Shear panels	Brooms, Vesić	1.06	-	-
				Barkan	0.96, 1.01	0.21, 0.26	0.10, 0.13
			Smith's Theory	Brooms, Vesić	1.13	-	-
				Barkan	1.06	-	-
	EXP	Pulling			0.96	-	-
		Forced vibration			1.00	0.25	0.12
Longitudinal	ANAL	S	Shear panels		0.32	0.10	-
		T	Brooms Vesić		0.36	-	-
			Barkan		0.16	-	-
		S, T	Brooms Vesić		0.48	-	-
			Barkan		0.36	0.12	-
	EXP	Wind			0.36		
		Forced vibration			0.34	0.13	

Table 2 Building M

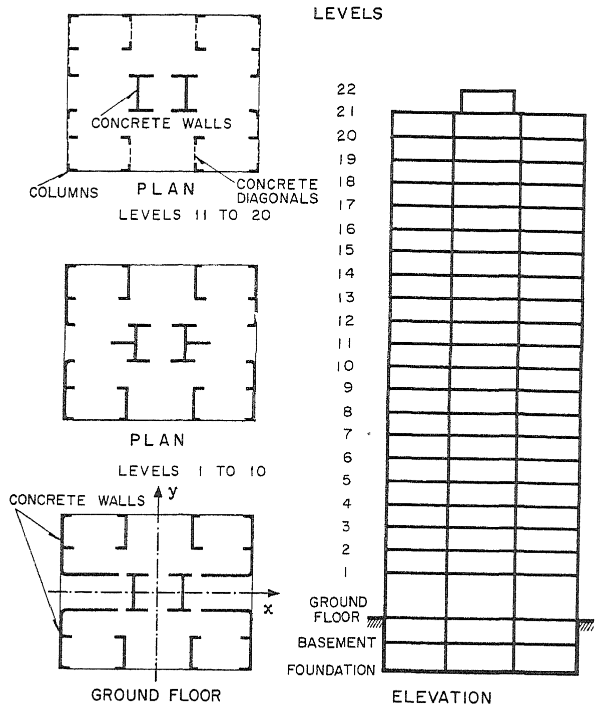
Direction		Deformations considered		Mode 1	Mode 2
N - S	ANAL	F, S		2.11	0.66
		F, S, R		2.19	-
		F, S, R, T	Brooms Vesić	2.25	-
			Barkan	2.20	-
	EXP	Wind		2.00	0.50
		Pulling		2.06	0.57
E - W	ANAL	F		1.88	0.33
		F, S		1.90	0.36
		F, S, R		1.99	0.37
		F, S, R, T	Brooms Vesić	2.02	0.37
			Barkan	2.00	0.37
	EXP	Wind		1.87	0.37
		Pulling		1.90	0.36

F flexural deformation of structure
S shearing deformation of structure
R base rotation
T base translation



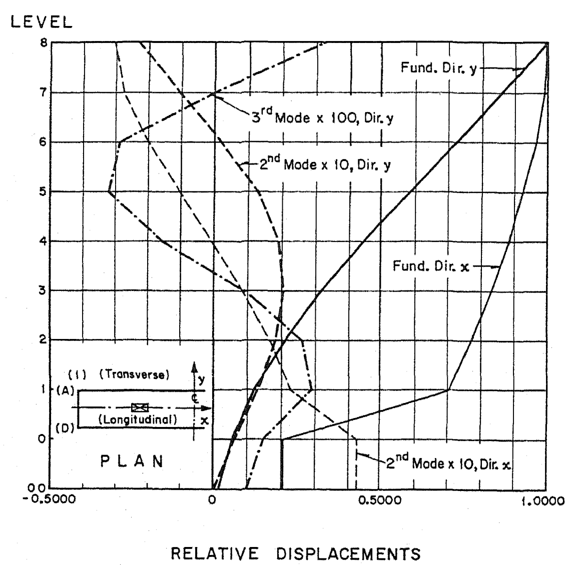
BUILDING B

FIG. 1



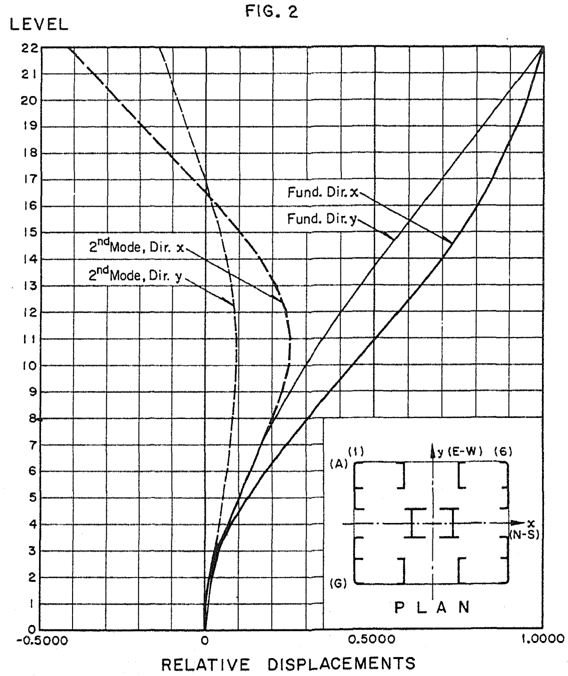
BUILDING M

FIG. 2



MODES OF VIBRATION OF BUILDING B
DIRECTION x (Longitudinal), y (Transverse)

FIG. 3



MODES OF VIBRATION OF BUILDING M
DIRECTION x (N-S), y (E-W)

FIG. 4

ANALYTICAL AND EXPERIMENTAL STUDIES OF VIBRATION IN TWO BUILDINGS

BY E. DEL VALLE AND J. PRINCE

QUESTION BY: C.F. CANDY - NEW ZEALAND

In the case of cantilever shear walls, would the authors consider it necessary to increase the design seismic coefficient due to lack of redundancy and hence of ductility?

AUTHORS' REPLY: We would consider an increase in this coefficient for two reasons. First, because in this type of structure, damping is smaller than in a more elaborate system, and second, for the lack of redundancy already mentioned. In the case of the particular building presented this was not done because the Mexican Code requires designing for at least 60% of the seismic forces resulting from a static analysis. In the present case the modal response computed as the square root of the sum of squares of the responses in the natural modes was below the 60% limit; it was therefore unnecessary to increase it beyond this value.

QUESTION BY: I.L. HOLMES - NEW ZEALAND

I would like to refer to Conclusion (b). While it was stated by the authors that each wall in Building M acted independently as a cantilever from the foundation it is hard to understand that there is not some interaction between walls, caused by the floor slabs. Was this not so and would this not increase the moment of inertia beyond the simple summation of Conclusion (b)?

AUTHORS' REPLY: Conclusion (b) should not be interpreted in the sense that each wall acts independently. It is assumed that floor slabs act as rigid diaphragms in most cases (except in some long buildings or when the stiffness is concentrated in certain separated elements), forcing the walls to share the total shear force corresponding to each story and to deflect exactly alike (in the absence of torsional effects). If they were independent they would take a shear force corresponding to a certain tributary area, irrespective of their moments of inertia, and the analysis could not be done adding up the individual moments of inertia.

In the case of building M, there was no connection between walls in the EW direction that could be regarded as contributing to the stiffness, because of special distribution of walls and openings in the slabs. In the NS direction the interaction between slabs and walls was considered but not in the form as to increase the moment of inertia. It would be difficult to say how much this value would increase due to interaction between walls and slabs. We prefer to treat this problem as described in the analysis of the building for the NS direction.

QUESTION BY: I.L. HOLMES - NEW ZEALAND

It would help the understanding of Building M if the authors would explain the arrangement of apartments on each floor, the walling materials used when these are not load-bearing, and the reason for concrete diagonals rather than full bearing walls.

AUTHORS' REPLY: There were four apartments per floor in building M, symmetrically placed. Materials used in partitions were concrete blocks; there is only one such wall in the center of the building in the EW direction. The reason for using concrete diagonals instead of full bearing walls was a desire to reduce unnecessary weight, as the seismic forces above the 10th level, where the diagonals begin, could be easily taken by these elements thus saving approximately 25 metric tons per floor. Consider also the low cost of labor in Mexico.

QUESTION BY: O.A. GLOGAU - NEW ZEALAND

What are typical thicknesses for the concrete shear walls in the building discussed?

AUTHORS' REPLY: Typical thicknesses for the concrete shear walls of building M are 20 cm.

AUTHORS' ADDITIONAL COMMENTS

There are six buildings type K in the housing project referred to in the paper. They are 14 stories tall, and the plan dimensions are 52.80 x 12.05m. Lateral forces are carried by two frames in the longitudinal direction and by a combination of frames and bearing walls in the transverse direction. Partitions in both directions are separated from the structure along three edges by means of soft deformable material at the contact with slabs and columns. Periods of these buildings were measured with the results indicated in Table A (using the portable seismometer Sprengnether).

On July 6, 1964 an earthquake of moderate intensity (V-VI, MM) was felt in Mexico City. Due to this movement all type K buildings suffered minor damage, which consisted in cracking of plaster at the joints of partitions and slabs or columns, and cracking of stairway walls in the first two stories. There was no evidence of damage to the structure itself. Periods were measured again after the earthquake; an increase of nearly 50% was found in the longitudinal direction as shown in Table A (values given are for the largest and the smallest differences). Once the damage was repaired, period measurements were almost identical with those obtained before the earthquake. It is concluded that periods measured with the small displacements induced by wind or traffic may be of questionable value in buildings with partitions.

TABLE A

TYPE K BUILDINGS - MEASURED PERIODS (sec)				
	K - 1		K - 5	
	Long.	Transv.	Long.	Transv.
After Completion	1.04	1.68	1.07	1.54
After Earthquake	1.54	1.90	1.24	1.72
After Repairs	1.09	1.70	1.09	1.57

QUESTION BY: J. PRINCE - MEXICO

Since building periods are involved in both design and behaviour considerations, I would like to ask Mr. W.K. Cloud, who has had a great experience in this matter, to comment on period measurements of actual structures.

REPLY BY: W.K. CLOUD - U.S.A.

The measured period is controlled by the stiffest elements - which are not necessarily related to strength. Cracking of plaster etc. causes considerable change in building periods.

QUESTION BY: R.M. THOMPSON - NEW ZEALAND

What were the calculated periods as compared to experimental periods measured before and after earthquake?

AUTHORS' REPLY: Calculated periods for building K were longer than the experimentally measured values either before or after the earthquake, because partition walls were disregarded in the computations. It is difficult to estimate the effective stiffness of partition walls due to the presence of soft deformable material at the contact with columns and slabs.

QUESTION BY: N.A. MOWBRAY - NEW ZEALAND

Could the authors give us any idea of the damage to the building and the way it was repaired?

AUTHORS' REPLY: Damage to the building was confined only to walls in the staircase in the first two stories, and fracture of plastering between partition walls and slabs.

The stairway walls were repaired using a steel mesh and cement mortar with a thickness of 1" to 1½". Plastering was repaired without reinforcing, leaving a gap at the junction of walls and slabs or columns.