EARTHQUAKE ANALYSIS OF SPACE STRUCTURES BY DIGITAL COMPUTERS

By Semih S. Tezcan[1]

ABSTRACT

The matrix formulation for the free vibration and the steady-state elastic response of a multi-degree freedom space framework subject to ground motions in three mutually perpendicular directions is presented for both damped and undamped cases. Assuming representative ground displacements or oscillatory rocking, a set of equivalent seismic loads acting at the lumped mass points are determined so that they simulate the effect of a strong motion earthquake. The accuracy is highly improved by considering a three dimensional mass distribution and taking into account the complete spatial stiffness properties of a structural member. The seismic loads are also evaluated by spectral analysis and compared with those of the proposed method.

METHOD OF ANALYSIS

1. General considerations.- The motion of a ground particle during an earthquake is generally in three directions and, as reported by Derleres(1), the actual strength of the structure is tested when all three components of the ground motion have been developed. For the purpose of evaluating the response of a space structure to a three directional vibration, a building frame relative to an orthogonal x y z coordinate system is considered. The total mass of the system is assumed to be lumped at selected points, which are regarded as the joints of the structure. Six deformations, namely three translations and three rotations, are specified at each joint by means of numbered arrows on a diagram. So that the sequence of the equations generated by the computer becomes exactly the same as required by the relations to be presented below, the numbering of the deformations should be done in the following order: First, the translations of the joints are numbered, proceeding from the top to the bottom along the x, then along the y and z directions. Next, the ground displacements are also numbered in the order of x, y and z. Finally, the joint rotations are assigned numbers in random order as they constitute the last set of equations and are subsequently eliminated at an intermediate stage in the calculations. As an illustration, a simple three-dimensional cubic structure has been numbered as outlined and shown in Fig. 1.

Sign convention: The joint translations and forces are assumed to be positive along the positive directions of the respective coordinate axes, while the positive directions of the joint rotations and moments are determined in accordance with the right hand screw rule.

2. Response of the damped structure.- The equations of motion of an elastic space structure vibrating in three directions with N lumped masses are

$$\underset{3N\times 3N}{\left[\diagdown m \diagdown\right]} \underset{3N}{\left\{\ddot{D}\right\}} = -\underset{3N}{\left\{F\right\}} - \underset{3N}{\left\{V\right\}} \tag{1}$$

---

1 Dept. of Civil Engrg., Univ. of British Columbia, Vancouver, Canada.

in which $\lceil m \rfloor$ is the diagonal mass matrix, $\{D\}$ is the column vector of joint translations, while $\{F\}$ and $\{V\}$ are the column vectors of inertia and damping forces respectively. Curly brackets are used to indicate a column matrix and the sizes of the individual matrices are shown below each matrix for clarity. Note that, for structures vibrating in only one or two directions, the matrix sizes reduce from 3N to N and 2N, respectively. The joint inertia forces $\{F\}$ and joint moments(2) $\{M\}$ are related to the joint deformations by means of the following stiffness equation:

$$\underset{(6N+3)}{\left\{\begin{array}{c} F \\ F_g \\ \hline M \end{array}\right\}} = \underset{(6N+3)(6N+3)}{\left[\begin{array}{cc|c} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ \hline S_{31} & S_{32} & S_{33} \end{array}\right]} \underset{(6N+3)}{\left\{\begin{array}{c} D \\ D_g \\ \hline \theta \end{array}\right\}} \begin{array}{l} \}\, 3N \\ \}\, 3 \\ \}\, 3N \end{array} \qquad (2)$$

in which, $\{D\}$ and $\{\theta\}$ are the translations and rotations of the joints, $\{F\}$ and $\{M\}$ are the point loads and moments respectively, while $\{F_g\}$ and $\{D_g\}$ denote the ground inertia forces and ground displacements. The partitioned matrix $[S]$ is the general stiffness matrix of the system. Note that it is not necessary to subscript or relate the joint deformations or the joint loads with their respective coordinate axes because they are readily identified by their numbers. This technique of distinguishing one plane of vibration from another by means of numbers is extremely convenient for the dynamical analysis of space structures.

For the purpose of simplicity, if the rigid body rotations of the lumped masses are neglected, all the joint inertia moments $\{M\}$ may then be regarded as zero(3). Upon partitioning Eq. 2 and eliminating $\{\theta\}$

$$\underset{(3N+3)}{\left\{\begin{array}{c} F \\ \hline F_g \end{array}\right\}} = \underset{(3N+3)(3N+3)}{\left[\begin{array}{c|c} K_{11} & K_{12} \\ \hline K_{21} & K_{22} \end{array}\right]} \underset{(3N+3)}{\left\{\begin{array}{c} D \\ \hline D_g \end{array}\right\}} \qquad (3)$$

in which

$$\underset{(3N+3)(3N+3)}{\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}} = [K] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} - \underset{(3N+3)(3N)}{\begin{bmatrix} S_{13} \\ S_{23} \end{bmatrix}} \underset{3N\times 3N}{[S_{33}]^{-1}} \underset{3N(3N+3)}{[S_{31}\ S_{32}]} \qquad (4)$$

From the first line of Eq. 3, the joint inertia forces $\{F\}$ are

$$\underset{3N}{\{F\}} = \underset{3N\times 3N}{[K_{11}]} \underset{3N}{\{D\}} + \underset{3N\times 3}{[K_{12}]} \underset{3}{\{D_g\}} \qquad (5)$$

The damping forces $\{V\}$ are related to the derivatives of the joint translations by the general damping matrix $[C]$ as follows:

$$\underset{(3N+3)}{\left\{\begin{array}{c} V \\ \hline V_g \end{array}\right\}} = \underset{(3N+3)(3N+3)}{\left[\begin{array}{c|c} C_{11} & C_{12} \\ \hline C_{21} & C_{22} \end{array}\right]} \underset{(3N+3)}{\left\{\begin{array}{c} \dot{D} \\ \hline \dot{D}_g \end{array}\right\}} \qquad (6)$$

From the first line, the required damping forces $\{V\}$ are obtained as

$$\underset{3N}{\{V\}} = \underset{3N\times 3N}{[C_{11}]}\underset{3N}{\{\dot{D}\}} + \underset{3N\times 3}{[C_{12}]}\underset{3}{\{\dot{D}_g\}} \tag{7}$$

Using Eqs. 5 and 7 and rearranging the terms, Eq. 1 becomes

$$\underset{3N\times 3N}{[m]}\underset{3N}{\{\ddot{D}\}} + \underset{3N\times 3N}{[C_{11}]}\underset{3N}{\{\dot{D}\}} + \underset{3N\times 3N}{[K_{11}]}\underset{3N}{\{D\}} = -\underset{3N\times 3}{[K_{12}]}\underset{3}{\{D_g\}} - \underset{3N\times 3}{[C_{12}]}\underset{3}{\{\dot{D}_g\}} \tag{8}$$

A convenient solution of the differential equations can be arrived at by assuming a trial displacement expression of the form

$$D = Re\left[(a+ib)e^{i\omega t}\right] \tag{9}$$

for all the joint translations and ground displacements. Re indicates that only the real part of the expression inside the brackets will be used. Differentiating Eq. 9 and substituting into Eq. 8

$$\underset{3N\times 3N}{\left[\left[K_{11}-\omega^2[m]\right]+i\omega\left[C_{11}\right]\right]}\underset{3N}{\{a+ib\}} = -\underset{3N\times 3}{\left[\left[K_{12}+i\omega\left[C_{12}\right]\right]\right]}\underset{3}{\{a_g+ib_g\}} \tag{10}$$

in which ω is the circular frequency and $\{a_g + ib_g\}$ is the column vector of the complex amplitude of the ground motions. These complex equations can be conveniently expressed in real form simply by separating the real and imaginary parts of each quantity(4) as follows:

$$\underset{6N\times 6N}{\begin{bmatrix} [K_{11}-\omega^2[m]] & -\omega[C_{11}] \\ \omega[C_{11}] & [K_{11}-\omega^2[m]] \end{bmatrix}}\underset{6N}{\begin{Bmatrix}\{a\}\\ \{b\}\end{Bmatrix}} = -\underset{6N\times 6}{\begin{bmatrix} [K_{12}] & -\omega[C_{12}] \\ \omega[C_{12}] & [K_{12}] \end{bmatrix}}\underset{6}{\begin{Bmatrix}\{a_g\}\\ \{b_g\}\end{Bmatrix}} \tag{11}$$

The amplitudes of the system are obtained from the root mean squares of the real and imaginary parts of the displacements. However, to evaluate the final stress resultants of each individual member, the remaining rotation components $\{\theta\}$ are also required, which are calculated from the bottom line of Eq. 2 in the form

$$\underset{3N}{\{\theta\}} = -\underset{3N\times 3N}{[S_{33}]^{-1}}\underset{3N\times(3N+3)}{[S_{31}\quad S_{32}]}\underset{3N+3}{\begin{Bmatrix}D\\ D_g\end{Bmatrix}} \tag{12}$$

If the damping coefficients of the ground displacements $[C_{12}]$ as well as the imaginary part of the ground amplitudes $\{b_g\}$ were taken as zero, Eq. 11 could be simplified to

$$\underset{6N}{\begin{Bmatrix}\{a\}\\ \{b\}\end{Bmatrix}} = -\underset{6N\times 6N}{\begin{bmatrix} [K_{11}-\omega^2[m]] & -\omega[C_{11}] \\ \omega[C_{11}] & [K_{11}-\omega^2[m]] \end{bmatrix}^{-1}}\underset{6N\times 6}{\begin{bmatrix}K_{12}\\ 0\end{bmatrix}}\underset{3}{\{a_g\}} \tag{13}$$

which, after partitioning, yields

$$\underset{3N}{\{a\}} = -\underset{3N\times 3N}{\left[K^{*}\right]}\ \underset{3N\times 3}{\left[K_{12}\right]}\ \underset{3}{\{a_g\}} \qquad \text{Real part} \tag{14}$$

$$\underset{3N}{\{b\}} = \omega \underset{3N\times 3N}{\left[K^{*}\right]} \underset{3N\times 3N}{\left[C_{11}\right]} \underset{3N\times 3N}{\left[K_{11} - \omega^2 [m]\right]^{-1}} \underset{3N\times 3}{\left[K_{12}\right]} \underset{3}{\{a_g\}} \qquad \text{Imaginary part} \tag{15}$$

in which

$$\underset{3N\times 3N}{\left[K^{*}\right]} = \left[\underset{3N\times 3N}{\left[K_{11} - \omega^2 [m]\right]} + \omega^2 \underset{3N\times 3N}{\left[C_{11}\right]} \underset{3N\times 3N}{\left[K_{11} - \omega^2 [m]\right]^{-1}} \underset{3N\times 3N}{\left[C_{11}\right]}\right]^{-1} \tag{16}$$

3. Assumed damping matrix.- The damping characteristics of a space structure are far more complex than the unknowns involved in the general response of a building. Despite the extensive analytical and experimental research in the field of structural damping(5), the nature and extent of the damping that should be considered cannot be predicted. There is no doubt, however, that the enormous energy transmitted by an earthquake to a structure is dissipated very rapidly in varying degrees by the following types of damping: viscous damping within the structure itself, coupling of the soil and the building during the transmission of the motion, dynamic and static hysteretic damping both in the structural materials and the surrounding soil, damping due to plastic deformations beyond yielding and interfacial slip at the contact surfaces and support junctions. For example, it has been shown by Jennings(6) that for a period of 1 sec., a critical damping coefficient of .02 and an acceleration of .5g, one quarter of the energy is dissipated by viscous damping and the remaining three quarters by yielding. Nevertheless, damping has a small influence insofar as the maximum earthquake stresses are concerned and, confirming Salvadori's(7) conclusion, it can be stated that the essential feature of damping is to absorb the energy and the higher modes of vibration rapidly.

In order to approximate the effect of damping the viscous damping matrix $[C_{11}]$ of Eq. 13 is considered to be composed of the absolute or relative viscous damping matrices $[c]$ of each individual member, which are given by

$$[c]_{\text{Absolute}} = \begin{bmatrix} c_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_3 \end{bmatrix} \quad \text{or} \quad [c]_{\text{Relative}} = \begin{bmatrix} c_1 & 0 & 0 & -c_1 & 0 & 0 \\ 0 & c_2 & 0 & 0 & -c_2 & 0 \\ 0 & 0 & c_3 & 0 & 0 & -c_3 \\ -c_1 & 0 & 0 & c_1 & 0 & 0 \\ 0 & -c_2 & 0 & 0 & c_2 & 0 \\ 0 & 0 & -c_3 & 0 & 0 & c_3 \end{bmatrix} \qquad \begin{matrix}(17)\\(18)\end{matrix}$$

The above matrix representations are particularly useful in assembling the general damping matrix conveniently in the computer in a manner similar to that applied for the stiffness matrix of the structure, as explained in Sec. 13. The physical representation of these matrices for the absolute and relative damping of a space structure is illustrated in Fig. 2. The general viscous damping coefficient c_{ij}, for the i^{th} mode and j^{th} mass, in any particular direction, is assumed to be of the form

$$c_{ij} = 2\,\beta\, m_j\, \omega_i \tag{19}$$

in which β is the percentage of critical damping and ω_i is the i^{th} fundamental circular frequency of the system when vibrating in the direction concerned[2] . The damping coefficient c_3 in Eqs. 17 and 18 represents the ability of girders or columns to absorb vibrational energy during their length changes. This is a very significant factor in the damping of a structure and requires experimental research in order to assess its numerical values.

4. Response of the undamped structure.- The equations for the undamped response of a structure to ground excitations are obtained from the previously derived expressions for the damped case by substituting zero for the damping effects. Putting $[C_{11}] = 0$, Eq. 16 reduces to

$$\underset{3N \times 3N}{\left[K^*\right]} = \underset{3N \times 3N}{\left[K_{11} - \omega^2 [m]\right]^{-1}} \qquad (20)$$

Further, the imaginary parts $\{b\}$ of the amplitudes given by Eq. 17 cancel and the real parts $\{a\}$, of Eq. 14 become

$$\underset{3N}{\{a\}} = -\underset{3N \times 3N}{\left[K_{11} - \omega^2 [m]\right]^{-1}} \underset{3N \times 3}{\left[K_{12}\right]} \underset{3}{\{a_g\}} \qquad (21)$$

5. Seismic loads and member stresses.- Once the joint translations have been obtained for the damped or undamped cases and the joint rotations have been calculated from Eq. 12, there is little left in computing the earthquake stress resultants at the ends of each individual member. This is accomplished by taking the matrix product of the member stiffness matrices and the corresponding values of the column vector of joint deformations. In contrast to the customary approach, it is not necessary to determine the seismic loads of the structure because the stress resultants are the final stage that would interest the engineer. However, for the purpose of comparison with other methods, the values of the earthquake forces acting at the joints may be required instead of the member stress resultants. If so, the earthquake forces designated by $\{F\}$ are obtained from the matrix product of the reduced stiffness matrix $[K]$ and the above calculated displacements $\{D\}$ in accordance with Eq. 3. It should be noted that the sum of the earthquake forces in any particular direction of vibration should be equal to the ground force F_g in the corresponding direction. This equilibrium is a valuable check on the correctness of the computations.

6. Numerical examples.- To investigate the effect of various mass and stiffness idealizations and also to compare the results of the proposed procedure with those of other methods, several building frames 5, 10, 15 and 20 story high, as shown in Figs. 3 and 4, were selected as example structures. Using the computer program described in Sec. 13, the seismic joint loads and story shears of these structures were computed following a steady state approach and modal analysis(9) as well as according to the New Zealand, SEAOC(9) and Canadian building codes. Some typical comparative results are illustrated in Figs. 5, 6, 8 and 9. The modal forces were obtained from the maximum probable shears using the idealized displacement spectrum curves of the 1940 El Centro earthquake.

To further demonstrate the generality of the matrix procedure, a vertical vibration problem has been exemplified by means of the statically indeterminate truss shown in Fig. 7.

---

2 In steady-state response ω_1 is taken to be the first fundamental frequency as suggested by Jennings and Newmark(8)

7. Equivalent ground motion.- The chief interest of an engineer lies not in complex vibration problems, but rather in designing his structure so that it can withstand the strongest probable earthquake without serious damage. The object of the dynamical analysis following a steady state approach as presented herein, is to furnish the structural engineer with a system of equivalent joint loads which will produce approximately the same effect as that of an actual earthquake.

Referring to Eqs. 3 and 13, it is seen that both the joint displacements and joint loads are explicit functions of the ground period T_g and the ground amplitudes a_g. In order to arrive at realistic values of seismic loads, which can be used in place of code requirements, a set of equivalent ground motion characteristics T_g and a_g were obtained from a series of computer analyses for various types of frames and plotted in Fig. 10. For an assumed ground period and a given building height, the corresponding ground amplitude is read from the horizontal scale.

8. Rocking and tilting of the structure.- Besides the three directional translation of the ground considered so far, the foundations of a building may also be directly subject to rotations in all three directions during an earthquake. Moreover, even if such ground rotations or twisting phenomena do not occur, the rocking of a building on its foundations in the vertical plane will invariably be brought about by small elastic deformations of the soil under the non-uniform pressures developed by oscillatory lateral seismic loads. The extent of the rocking, which increases the natural periods, will depend on the elastic properties of the soil. Such an increase in the period is undesirable because it results in a relative weakening in the response of the building.

The mathematical formulation of the dynamical behaviour of a structure subject to three directional ground rotations remains exactly the same as described above except that the symbols used for designating the ground translations and forces are replaced by new symbols for the ground rotations and moments, as follows:

Ground forces	F_g	are replaced by	M_g	Ground moments
Ground translations	D_g	do	θ_g	Ground rotations
Amplitudes of translation	a_g	do	α_g	Amplitudes of rotation

Apart from oscillatory ground rotations, it is possible during an earthquake for a building to be permanently tilted at the base as a result of the soil's yielding. Designating the angle of tilt by $\{\theta_g\}$ and substituting zero for the joint forces $\{F\}$ and moments $\{M\}$ the remaining joint deformations are obtained from Eq. 3 as

$$\underset{6N}{\begin{Bmatrix} D \\ \theta \end{Bmatrix}} = - \underset{6N\times 6N}{\begin{bmatrix} S_{11} & S_{13} \\ S_{31} & S_{33} \end{bmatrix}^{-1}} \underset{6N}{\begin{bmatrix} S_{12} \\ S_{32} \end{bmatrix}} \underset{3}{\{\theta_g\}} \qquad (22)$$

9. Buildings with a box system.- A box system, as defined by the SEAOC(9), is a structural system without a complete vertical load-carrying space frame, which resists seismic loads by means of shear walls alone. The dynamical analysis of such a box system remains exactly the same as described above for a framed structure, except that, due to the lack of a complete framework, the

reduced stiffness matrix $[K]$ of Eq. 3 is generated in a different manner, as recommended below:

(i) Each shear wall, i, is taken individually and regarded as an independent cantilever structure with lumped masses and joint deformations numbered at each floor level. The corresponding stiffness matrix of the wall $[S]_i$ of Eq. 2, is then generated from the stiffness matrices of the individual wall segments between floor levels taking due account of shear deformations (see Eq. 42) and openings in the walls. Subsequently, Eq. 4 is employed to yield the reduced matrix of the wall $[K]_i$.

(ii) Finally, assuming the lateral displacements of each wall at any particular floor level to be equal, i.e. regarding the floor system to be inextensible, the total reduced matrix of the box system is obtained from the algebraic sum of the reduced matrices $[K]_i$ of the individual walls as

$$[K] = \sum_{i=1}^{W} [K]_i \qquad \text{(W = Number of walls)} \tag{23}$$

According to the approximate method recommended by most textbooks(9)(10) the story shears are distributed in proportion to the relative stiffnesses of each wall. This independent distribution at any particular level without regard to the interaction between adjacent stories may yield erroneous values especially if the wall stiffnesses vary non-uniformly as a result of irregular door or window openings. The internal shears of the walls of a nineteen story building were evaluated by the method recommended herein as well as by the approximate method and it was found that the errors of the approximate method reached as high as 70 to 90%.

NATURAL PERIODS AND MODAL DISPLACEMENTS

The empirical expressions proposed by numerous authors and codes to evaluate the natural periods of buildings are all attractively simple and, in most cases, highly informative. However, they cannot compete with the accuracy and generality of a frequency determinant approach because one or two parameters alone, related to the overall dimensions, are not sufficiently representative of the complex dynamic properties of a structure. On the other hand, the frequency determinant is believed to provide more reliable natural period values, especially if a three-dimensional mass distribution is considered and the complete spatial stiffness properties of the structure are taken into account. In order to investigate the relative accuracy of various methods, the first mode natural periods of the example frames, No. 1, 2, 3, 4 and 5, have been evaluated by each and the comparative results are shown in Fig. 11, which clearly demonstrates that substantial errors may occur when using the cantilever rod approach or the empirical formulas.

The following matrix operations are recommended to convert the frequency determinant into a form convenient for computer application:

10. Damped natural periods.- Using a trial solution for the homogeneous part of Eq. 8, of the form

$$D = \mathrm{Re}(ce^{\lambda t}) \tag{24}$$

in which both c and λ are complex quantities, the frequency determinant of the damped structure is obtained as

$$\det \left| \lambda^2 [m] + \lambda [C_{11}] + [K_{11}] \right|_{3N\times 3N} = 0 \qquad (25)$$

The symmetrical form of the damping and stiffness matrices can be maintained by pre- and post-multiplying Eq. 25 by $[m]^{-\frac{1}{2}}$. Hence,

$$\det \left| \lambda^2 [U] + \lambda [A] + [B] \right|_{3N\times 3N} = 0 \qquad (26)$$

in which $[U]$ is a unit matrix,

$$[A]_{3N} = [m]^{-\frac{1}{2}} [C_{11}] [m]^{-\frac{1}{2}} \quad \text{and} \quad [B]_{3N} = [m]^{-\frac{1}{2}} [K_{11}] [m]^{-\frac{1}{2}} \qquad (27)\ (28)$$

A convenient method for obtaining the eigen values of the above second order quadratic determinant is to convert it into a double size first order system(11) in the form

$$\det \left| E - \lambda U \right| = 0 \quad \text{in which} \quad [E] = \begin{bmatrix} 0 & U \\ -B & -A \end{bmatrix}_{6N\times 6N} \qquad (29)\ (30)$$

The eigen values of Eq. 29 are the required natural circular frequencies while the eigen vectors are the modal displacements.

11. Undamped natural periods.- The frequency determinant of an undamped multi-degree freedom system is obtained from the steady state vibration equation of Eq. 21 after setting $\{a_g\} = 0$ and transferring the inverted matrix to the left hand side. This gives

$$\left[K_{11} - \omega^2 [m] \right]_{3N\times 3N} \{a\}_{3N} = 0 \qquad (31)$$

The coefficient determinant of the above equation is unsymmetrical. However, for convenient computer application, it is possible to convert it into an equivalent symmetrical form by transferring the mass matrix to the right-hand side and pre-multiplying both sides by $[m]^{-\frac{1}{2}}$. It follows that

$$\underbrace{[m]^{-\frac{1}{2}} [K_{11}] [m]^{-\frac{1}{2}}}_{H} \overbrace{}^{U} \underbrace{[m]^{\frac{1}{2}} \{a\}}_{X} = \omega_n^2 \underbrace{[m]^{-\frac{1}{2}} [m] \{a\}}_{\{X\}} \qquad (32)$$

or

$$[H] \{X\} = \omega_n^2 \{X\}_{3N} \qquad (33)$$

The undamped natural frequencies are the eigen values of the above substitute symmetrical system. However, as indicated in Eq. 32, the modal displacements $\{a\}$ are obtained from the eigen vectors $\{X\}$, of the substitute system by means of

$$\{a\} = [m]^{-\frac{1}{2}} \{X\}_{3N} \qquad (34)$$

As an illustration, the modal displacements of frame No. 4B were computed in the manner described above, taking the full stiffness properties of the frame into account, and the results are shown in Fig. 12.[3]

SOIL CONDITIONS

12. Amplified ground motion.- The magnitudes of the dynamical properties of ground motion depend largely on local geological conditions. If there is a layer of soft soil between the foundation of a building and an underlying layer of granitic rock, the seismic waves are greatly amplified during their travel from the rock base to the surface of the ground. As reported by Neumann(12), the acceleration in the filled harbour areas of San Francisco in 1906, was ten to fifteen times higher than the acceleration experienced in the adjoining hills. As will be seen from the subsequent discussion, the thickness and the modulus of elasticity of the underlying soft layer of soil as well as the period of the seismic waves are the main factors in determining the magnification of the ground displacements.

Assuming a perfectly elastic soil and neglecting the radiation, refraction and reflection of the waves, the vertical displacement equation of a volume of soil of unit cross-sectional area and height is analogous to the longitudinal vibration of elastic prismatic bars(13), and is given by

$$m\ddot{u} = E\frac{\delta^2 u}{\delta z^2} \quad \text{and} \quad u = z \sin \omega t \qquad \begin{matrix}(35)\\(36)\end{matrix}$$

wherein, m is the mass, E is the modulus of elasticity of the soil, and u represents vertical harmonic oscillations of variable amplitude z. Denoting the weight density of the soil by γ, Eq. 35 becomes

$$\ddot{z} + p^2 z = 0 \quad \text{in which} \quad p^2 = \frac{\gamma\omega^2}{Eg} \qquad \begin{matrix}(37)\\(38)\end{matrix}$$

Introducing a trial solution for Eq. 37 in the form

$$z = A \sin pz + B \cos pz \qquad (39)$$

and using the two boundary conditions shown in Fig. 13

i. $\sigma = 0$ at $z = 0$

ii. $u = u_o \sin \omega t$ at $z = h$ $\qquad$ ($\frac{\sigma}{E} = \frac{\delta u}{\delta z}$ by Hooke's law)

the integration constants are obtained as $A = 0$ and $B = \frac{u_o}{\cos ph}$

Finally, substituting the above constants into Eq. 39, the required ground displacement amplitude at any level z is obtained in terms of the amplitude of the rock u_o as

$$z = MF\, u_o \quad \text{where,} \quad MF = \frac{\cos pz}{\cos ph} \quad (\text{for } ph < \frac{\pi}{2}) \qquad (40)$$

---

3 To obtain extremely convenient Fortran programs for eigen value problems write, for symmetrical matrices to the Institute of Computer Science, Toronto, Canada, Program name HOW; for unsymmetrical matrices, to the IBM Program Distribution Center, P.O. Box 790, White Plains, N.Y., U.S.A., File 7090 - 1373 NUET 63.

The percentages of the magnification factor MF are plotted against the modulus of elasticity of the soil E for various values of soft soil layer thickness h and ground motion period T = 1 sec in Fig. 14. It is observed that the ground displacements are magnified by 35% on the surface of a fifty feet thick layer of loose sand.

COMPUTER APPLICATION

13. General procedure.- The matrix formulation described above for the dynamical analysis of space structures is particularly suitable for high speed digital computers. The multi purpose Fortran programs[4] developed by the writer for the IBM 7090 and 1620 start with the fundamental data which consist of the geometric and elastic properties of the constituent members, the weights of the lumped masses and the properties of the ground motion; then proceed with the necessary calculations required by the above derived equations and yield the steady state earthquake forces as well as the natural periods and the maximum probable modal analysis shears and forces for both damped and undamped cases. All the computations are performed in complete automation and absolutely no calculations are required from the engineer. Though it was possible to handle space structures with up to several thousands degrees of freedom, only some important results of 100 to 180 degrees of freedom analyses have been presented as numerical examples because of space limitations. The key phases of these programs may be summarized as follows:

1. Generate the general stiffness matrix of Eq. 2 from the stiffness matrices of the individual members by the code number approach(14) Code numbers are indispensable in defining the interconnection of the members and the joints of a structure and are so versatile that they can be used to generate any portion of the stiffness matrix in any order and at any stage in the program. This is particularly useful when handling an extremely large number of degrees of freedom by the method of substructures. The code numbers of two members of the cubic structure of Fig. 1 are written beside the respective members for the purpose of illustration. It is of interest that the code numbers are not furnished as part of the input data, but rather are set up by the computer itself from information provided about the support restraints and joint numbers.

The 12 by 12 stiffness matrix $[k]$ of an inclined individual member as shown in Fig. 15, is

$$[k] = \begin{bmatrix} [11] & [21] & -[11] & [21] \\ [21] & [22] & -[21]^T & [42] \\ -[11] & -[21]^T & [11] & -[21] \\ [21] & [42] & -[21] & [22] \end{bmatrix} \qquad (41)$$

The contents of the 3 by 3 block matrices $[11]$, $[22]$, $[21]$ and $[42]$ are given in Fig. 16. Note that the stiffness matrix includes the length changes and torsional rigidities of the members. Moreover, the effect of shear deformations on the stiffness influence coefficients has also been duly considered by means of a factor ε defined by

4 These programs are obtainable from the writer.

$$\varepsilon = \frac{1}{1 + k\,\frac{12EI}{L^2 AG}} \qquad (42)$$

in which, $[k]$ is a numerical constant related to the shape of the cross-section and may be taken as 1.2.

2. Obtain the reduced stiffness matrix $[K]$ in accordance with Eq. 4.

3. Generate the damping matrix $[C]$ of Eq. 6 from the damping matrices of each individual member using the above mentioned code number approach.

4. Determine the earthquake displacements $\{D\}$ either from Eqs. 14 and 15 for a damped system or from Eq. 21 for an undamped system. These displacements are subsequently used to evaluate the stress resultants of each individual member as explained in Sec. 5.

5. Obtain the earthquake forces acting on each lumped mass and, hence, the story shears, from the matrix product of the reduced stiffness matrix $[K]$ and the joint displacements $\{D\}$ as in Eq. 3.

6. Determine the damped or undamped natural periods from Eqs. 29 and 33.

7. Evaluate the spectral maximum probable shears and forces following the modal analysis approach described by Blume, Newmark and Corning(9).

CONCLUSIONS

1. The matrix approach presented herein appears to be very generally applicable and provides a realistic idea of the true dynamic behaviour of a structure. It is noteworthy that the same procedure is valid even for the vibration analysis of plates and shells by making use of the framework analogy concept(4).

2. The accuracy in the assessment of the basic dynamical properties of a building is substantially increased because a three-dimensional mass distribution is considered and the complete rigidity of a space member against joint rotations, length changes, torsional and shear deformations is duly taken into account. The common assumptions and limitations of the customary cantilever rod analysis namely an infinitely stiff floor, a single concentrated mass in place of the total mass of a story, a single spring constant representing the stiffness of an entire story, horizontal oscillations restricted to one plane etc., are completely avoided.

3. The object of the analysis is shifted from the classical approach of evaluating the seismic shears to the direct determination of the seismic deformations and member stress resultants. However, the characteristics of the representative ground motion are so selected that the resulting equivalent seismic loads, if required, can be used in place of the current design loads.

4. In view of the versatility of the matrix approach, structural engineers are strongly encouraged to use digital computers to determine the seismic design requirements of any structure since the entire problem is reduced merely to the clerical job of preparing the basic data in a standard form.

ACKNOWLEDGMENTS

The writer wishes to express his gratitude to his research assistant Ravindar K. Kinra for his assistance in the preparation of the paper and the computer work, to Prof. J.D. Anderson for his valuable suggestions, to Mr. I. Farkas of the University of Toronto for his help in running the programs and to the National Research Council of Canada for financial support.

REFERENCES

1. Derleres, M., "Earthquake as Dynamic Phenomenon and Earthquake Resistant Structures", 2nd World Conf. on Earthquake Eng., Vol. II, Japan 1960.

2. Tezcan, S.S., "Moment Equations for Computer Analysis of Frames", Journal of the Structural Division, ASCE, Vol. 90, Proc. Paper 3926, June, 1964.

3. Rubinstein, M.F. and Hurty, W.C., "Effect of Joint Rotation on Dynamics of Structures", Journal of the Engineering Mechanics Division, ASCE, Vol. 87, No. EM6, Dec. 1961.

4. Pestel, E.C. and Leckie, F.A., "Matrix Methods in Elasto Mechanics", McGraw-Hill Book Company, Inc., 1963, p. 101 and 346.

5. "Structural Damping", Colloquim papers published by The American Society of Mechanical Engineers, December, 1959.

6. Jennings, P.C., "Response of Simple Yielding Structures to Earthquake Excitation", California Institute of Technology, June, 1963, p. 169.

7. Salvadori, M.G., "Earthquake Stresses in Shear Buildings" ASCE, Transactions, Paper No. 2666, March 1953.

8. Jennings, R.L. and Newmark, N.M., "Elastic Response of Multi-story Shear Beam Type Structures Subjected to Strong Ground Motion", Proc. 2nd World Conf. on Earthquake Eng. Vol. II, Japan 1960.

9. Blume, A.J., Newmark, N.M., Corning, L.H., "Design of Multistory Reinforced Concrete Buildings for Earthquake Motions", Portland Cement Assn., 1961.

10. "Analysis of Small Reinforced Concrete Buildings for Earthquake Forces", Portland Cement Association, 1955, p. 30.

11. Frazer, R.A., Duncan, W.J. and Collar, A.R. "Elementary Matrices", Cambridge University Press, London, 1938, p. 164.

12. Neumann, F. "Some Generalized Concepts of Earthquake Motion", Symposium on Earthquake and Blast Effects on Structures, Earthquake Engineering Research Institute, Los Angeles, June 1952, p. 12.

13. Thomson, W.T., "Mechanical Vibrations", Prentice-Hall, Inc., 1956.

14. Tezcan, S.S., Discussion of "Simplified Formulation of Stiffness Matrices" by P.M. Wright, Journal of the Structural Division, ASCE, Vol. 89, No. ST6, Proc. Paper 3743, December, 1963, pp. 445-449.

15. Ifrim, M., "Dynamic Analysis of Tall Structures Subjected to Earthquake Motion", 2nd World Conf. on Earthquake Eng., Vol. II, Japan 1960.

16. Housner, G.W. and Brady, A.G., "Natural Periods of Vibration of Buildings", Journal of the Engineering Mechanics Division, ASCE, Vol. 89, Dec. 1961.

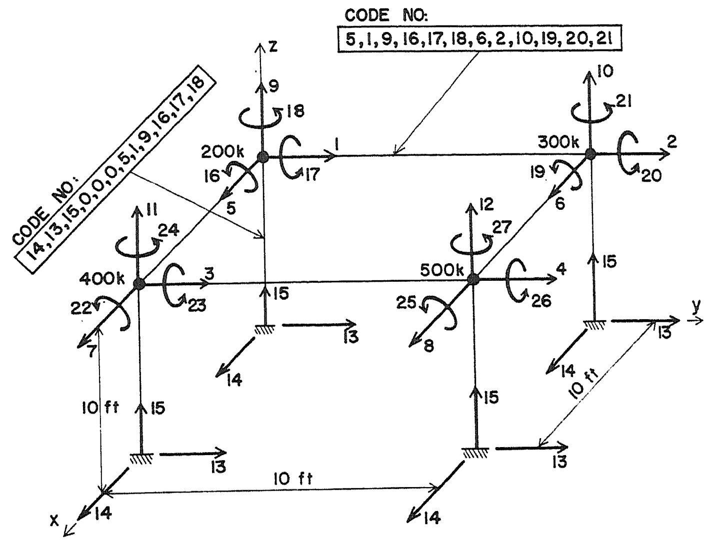


FIG. I THREE DIMENSIONAL CUBIC STRUCTURE

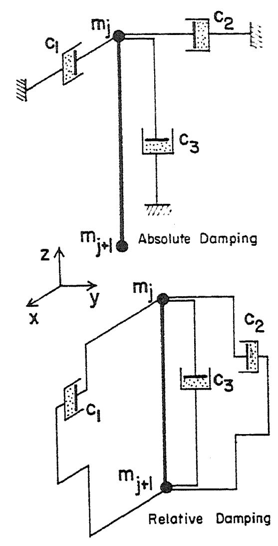


FIG.2 DAMPING MECHANISM

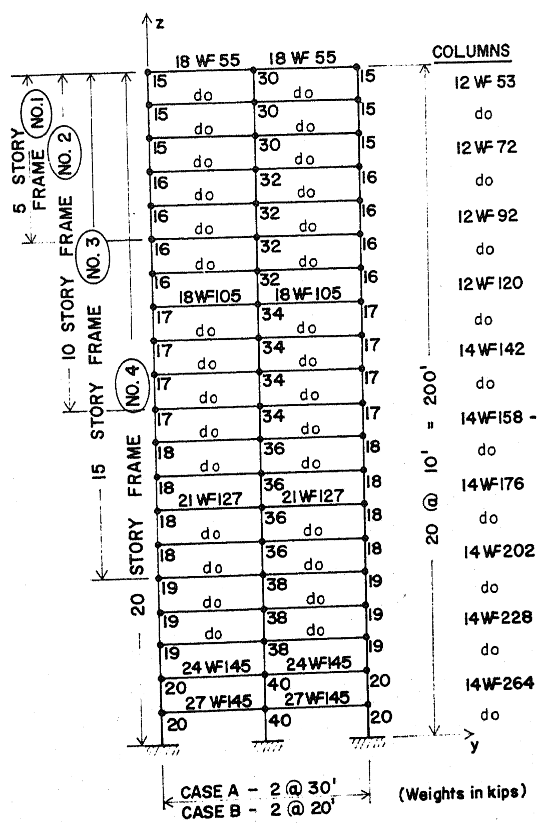


FIG.3 EXAMPLE FRAMES 1, 2, 3 AND 4

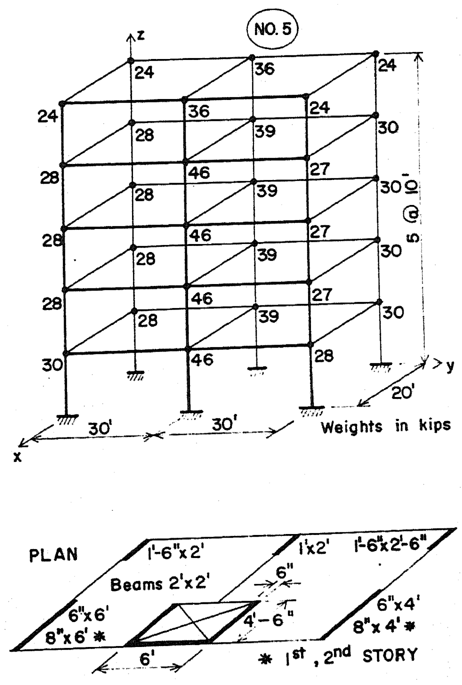


FIG. 4 SPACE FRAME OF A 5 STORY REINFORCED CONCRETE BUILDING - NO. 5

FRAME (NO. 3B) $T_g = \sqrt{2}$ sec, a_g = .9in

JOINT FORCES			SUM	β=0	β=.10 (absolute)	MODAL[9]
1.95	3.90	1.95	7.80	5.98	5.77	48.3
1.89	3.78	1.89	7.56	5.90	5.66	39.0
1.81	3.62	1.81	7.24	5.74	5.47	29.7
1.84	3.68	1.84	7.36	5.94	5.60	27.1
1.74	3.48	1.74	6.96	5.69	5.31	22.1
1.64	3.28	1.64	6.56	5.46	5.00	19.5
1.53	3.06	1.53	6.12	5.19	4.67	16.9
1.52	3.04	1.52	6.08	5.27	4.66	17.0
1.42	2.84	1.42	5.68	4.99	4.34	15.4
1.34	2.68	1.34	5.36	4.78	4.11	14.8
1.25	2.50	1.25	5.00	4.56	3.79	13.5
1.26	2.52	1.26	5.04	4.60	3.70	12.8
1.16	2.32	1.16	4.64	4.36	3.31	10.6
1.08	2.16	1.08	4.32	4.13	2.91	7.8
1.01	2.02	1.01	4.04	3.89	2.50	3.9
			89.86	76.56	66.87	298.5

FIG. 5 DAMPED AND UNDAMPED SEISMIC LOADS (in kips) COMPARED WITH MODAL ANALYSIS

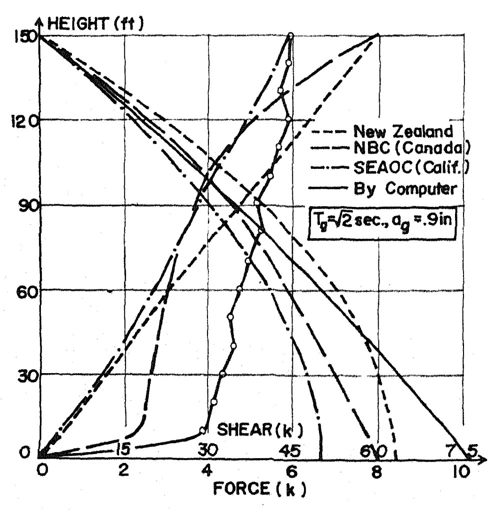
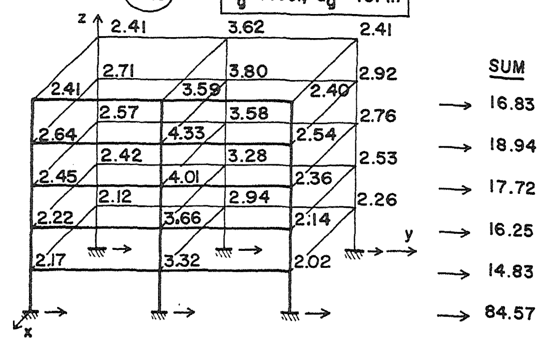


FIG. 8 SEISMIC SHEARS AND FORCES OF FRAME NO.3B

FRAME (NO.5) T_g = 1sec., a_g = .67in



(a) As a space frame (full stiffness)

JOINT FORCES			SUM	SUM*
4.16	6.26	4.16	14.58	13.70
4.73	7.18	4.81	16.72	15.83
4.54	6.89	4.62	16.05	15.40
4.28	6.49	4.35	15.12	14.72
4.15	6.06	4.15	14.36	14.20
			76.83	73.85

(b) As a plane frame (full stiffness)
*Assuming equal joint rotations at floor levels[3]

PROPOSED	MODAL[9]	SEAOC[9]	CANADA	NEW ZEALAND
11.90	144.6	20.89	22.40	23.11
13.99	148.3	19.69	21.60	18.49
13.91	121.6	14.77	18.28	13.87
13.80	88.1	9.85	15.84	9.24
13.88	49.2	5.00	14.19	4.62
67.48	551.8	70.20	92.31	69.33

(c) As a cantilever rod

FIG. 6 SEISMIC LOADS (in kips) OF PROPOSED METHOD COMPARED WITH MODAL & CODE VALUES

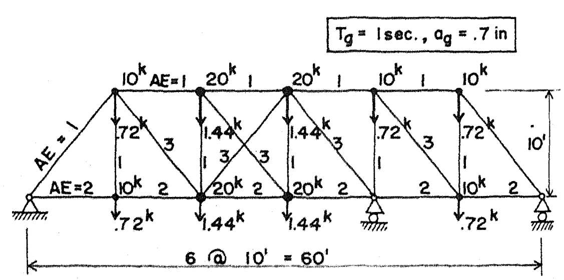


FIG. 7 SEISMIC FORCES DUE TO VERTICAL VIBRATION

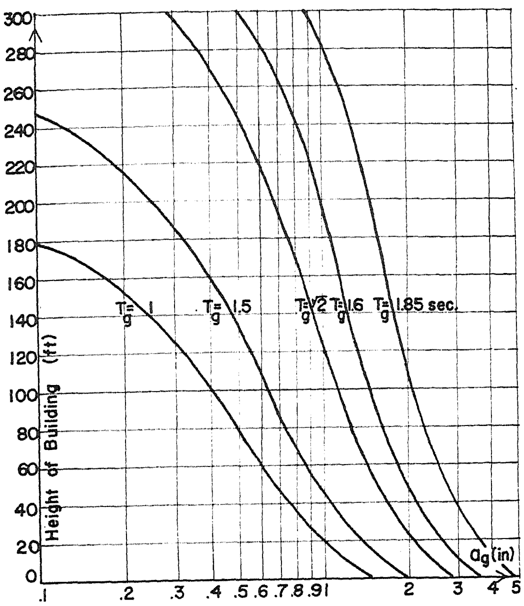


FIG. 10 PERIODS OF EQUIVALENT GROUND MOTION (T_g and a_g)

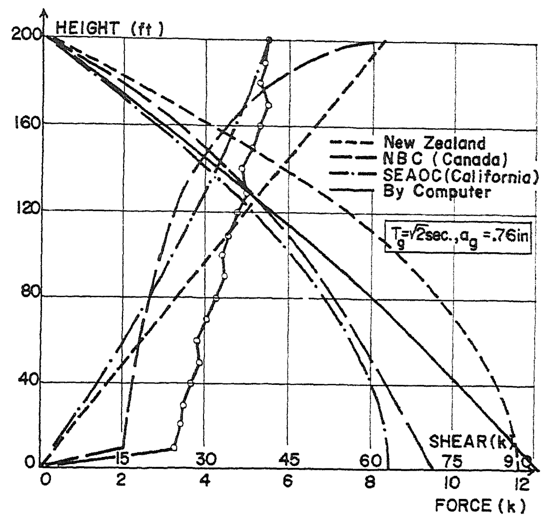


FIG. 9 SEISMIC SHEARS AND FORCES OF FRAME NO. 4B

FRAME NO.			TEZCAN FULL STIFFNESS	TEZCAN CANTILEVER ROD	SEAOC(9) $\frac{.05H}{\sqrt{D}}$	IFRIM(15) .08N TANIGUCHI	IFRIM(15) .0193H ULRICH	IFRIM(15) .095(N+1) ROSENBLUETH	HOUSNER-BRADY(16) $1.08\sqrt{N}-.86$	HOUSNER-BRADY(16) $.5\sqrt{N}-.4$
IN PLANE	1	A	.74	.46	.40	.40	.97	.57	1.55	.72
		B	.82	.46	.32	.40	.97	.57	1.55	.72
	2	A	1.26	.69	.79	.80	1.93	1.05	2.56	1.18
		B	1.41	.69	.65	.80	1.93	1.05	2.56	1.18
	3	A	1.66	.86	1.12	1.20	2.90	1.52	3.32	1.54
		B	1.83	.86	.97	1.20	2.90	1.52	3.32	1.54
	4	A	1.95	.99	1.58	1.60	3.86	2.00	3.97	1.84
		B	2.14	.99	1.29	1.60	3.86	2.00	3.97	1.84
	5	X	.38	.16	.56	.40	.97	.57	1.55	.72
		Y	.51	.17	.32	.40	.97	.57	1.55	.72
IN SPACE	5	X	.37	—	—	—	—	—	—	—
		Y	.52	—	—	—	—	—	—	—

FIG. 11 COMPARATIVE NATURAL PERIODS IN SECS. FOR FRAMES NO. 1, 2, 3, 4 AND 5.

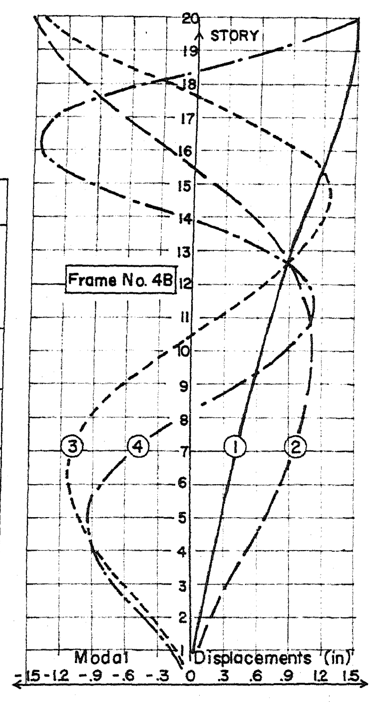


FIG. 12 DISPLACEMENTS AS A COMPLETE FRAME, FOR MODES 1,2,3 & 4

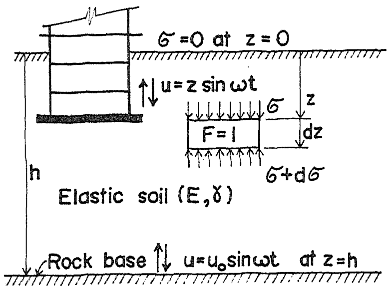


FIG. 13 VIBRATION OF ELASTIC SOIL

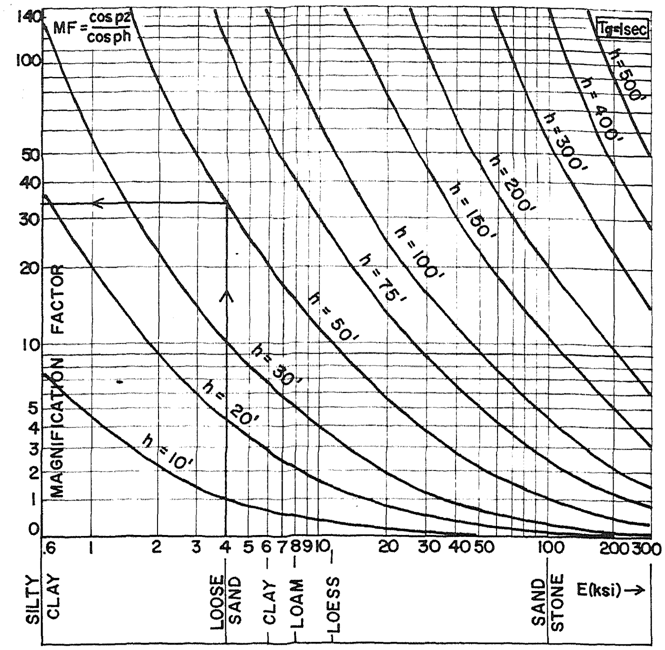


FIG. 14 AMPLITUDE MAGNIFICATION FACTORS FOR ELASTIC SOILS

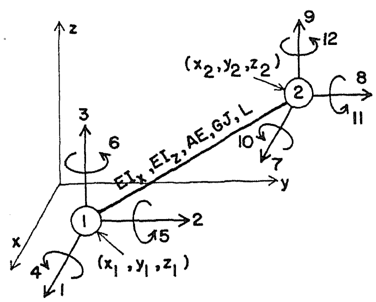


FIG. 15 INCLINED SPACE MEMBER

$[11] =$

$H l_x^2 +$ $S l^2$ $D l_z^2$	$H l_x m_x$ $S l m$ $D l_z m_z$	$S l n$ $D l_z n_z$
$H l_x m_x$ $S l m$ $D l_z m_z$	$H m_x^2$ $S m^2$ $D m_z^2$	$H m n$ $D m_z n_z$
$S l n$ $D l_z n_z$	$S m n$ $D m_z n_z$	$S n^2$ $D n_z^2$

$[21] =$

$C l_x l_z$ $-G l_x l_z$	$C l_x m_z$ $-G m_x l_z$	$C l_x n_z$
$C m_x l_z$ $-G l_x m_z$	$C m_x m_z$ $-G m_x m_z$	$C m_x n_z$
$-G l_x n_z$	$-G m_x n_z$	0

STIFFNESS COEFFICIENTS

$A = \frac{EI_x}{L}(3\varepsilon+1)$	$E = \frac{EI_z}{L}(3\varepsilon+1)$
$B = \frac{EI_x}{L}(3\varepsilon-1)$	$F = \frac{EI_z}{L}(3\varepsilon-1)$
$C = \frac{6EI_x}{L^3}\varepsilon$	$G = \frac{6EI_z}{L^3}\varepsilon$
$D = \frac{12EI_x}{L^3}\varepsilon$	$H = \frac{12EI_z}{L^3}\varepsilon$
$S = \frac{AE}{L}$	$T = \frac{GJ}{L}$

$[22] =$

$A l_x^2$ $T l^2$ $E l_z^2$	$A l_x m_x$ $T l m$ $E l_z m_z$	$T l n$ $E l_z n_z$
$A l_x m_x$ $T l m$ $E l_z m_z$	$A m_x^2$ $T m^2$ $E m_z^2$	$T m n$ $E m_z n_z$
$T l n$ $E l_z n_z$	$T m n$ $E m_z n_z$	$T n^2$ $E n_z^2$

$[42] =$

$B l_x^2$ $-T l^2$ $F l_z^2$	$B l_x m_x$ $-T l m$ $F l_z m_z$	$-T l n$ $F l_z n_z$
$B l_x m_x$ $-T l m$ $F l_z m_z$	$B m_x^2$ $-T m^2$ $F m_z^2$	$-T m n$ $F m_z n_z$
$-T l n$ $F l_z n_z$	$-T m n$ $F m_z n_z$	$-T n^2$ $F n_z^2$

DIRECTION COSINES

$l_x = \frac{m}{Q}$ $m_x = -\frac{l}{Q}$ $n_x \equiv 0$	$l_z = -\frac{ln}{Q}$ $m_z = -\frac{mn}{Q}$ $n_z = Q$
$l = \frac{x_2-x_1}{L}$ $m = \frac{Y_2-Y_1}{L}$ $n = \frac{z_2-z_1}{L}$	$Q = \sqrt{l^2+m^2}$

FIG. 16 TRANSFORMED STIFFNESS MATRIX OF AN INCLINED MEMBER (EQ. 41)

E R R A T A

EARTHQUAKE ANALYSIS OF SPACE STRUCTURES BY DIGITAL COMPUTERS

BY S. TEZCAN

PAGE 633: Equation 13; the last matrix should read

$$\begin{matrix} \{a_g\} \\ \{0\}_{6\times 1} \end{matrix}$$

PAGE 639: Equation 36; delete u=zsin t
replace by u=vsin t

Equation 36; line 2: for "...amplitude z", read "...amplitude v"

PAGE 639: Equation 37; should read $\ddot{v} + p^2 v = 0$

PAGE 639: Equation 39; should read $v = A \sin pz + B \cos pz$

PAGE 639: Equation 40; should read $v = MF\, u_o$ where, ...

PAGE 640: Equation 41 should read

$$[k] = \begin{bmatrix} [11] & [21]^T - [11]^T & [21]^T \\ [21] & [22] - [21] & [42]^T \\ -[11] & -[21]^T & [11] - [21]^T \\ [21] & [42] - [21] & [22] \end{bmatrix}$$

PAGE 646: **Fig.13;** **delete:** **u=zsin t**

replace by: u=vsin t

Fig.16; matrix [11] , 3rd column, 2nd row, first term in element; delete: H mn

replace by: S mn