DYNAMIC RESPONSE OF MULTI-LEVEL GUYED TOWERS TO EARTHQUAKE CONSIDERING NON-LINEARITY OF THE ELASTIC SUPPORTS

By Dr.-Ing. Rudolph Szilard*

ABSTRACT

The elastic response of a structure is determined considering the actual earthquake displacements which are expressed in Fourier series. The pre-stressed guy cables provide non-linear elastic supports which, because of the relative motion of the structure and soil, are time-dependent. The differential equations of motion are given in matrix form using lumped masses. Navier's solution transforms the differential equations into algebraic equations, the solution of which yields the forced part of the vibration. The non-linearity of the elastic supports is approximated by polygons. The flexibility matrix of the structure is determined by inversion of the stiffness matrix of the "disassembled" structure. The eigenvalues and eigenvectors of a product matrix yield the natural frequencies and modes of the free vibration. A numerical example illustrates the use of the method.

1. Introduction.

In designing conventional-type structures for earthquake resistance, modern earthquake codes(1)** provide effective means to guard against structural damage. In the case of non-conventional structures, however, the use of detailed dynamic analysis is necessary in order to predict the dynamic response of the structure to earth motion, inasmuch as the prescribed lateral forces(1) do not represent the actual forces which act during an earthquake.

Multi-level guyed towers are by nature extremely tall and flexible (Figure 1); thus, earthquake resistance designs for such structures must be based on the actual dynamic characteristics of the earth's motion, coupled with the actual dynamic behavior of the structure, including the effect of the supporting soil, considering the relative displacement which takes place between foundation and soil, as well as the time-dependency and non-linearity of the elastic supports, all of which add to the overall complexity of the problem.

The difficulty of such an analysis can be largely overcome by a matrix approach coupled with extensive use of high-speed electronic computers. The effect of the complex interaction of the soil and structure can be approximated by introducing weightless elastic springs representing the soil.

* Senior Research Engineer in the Denver Research Institute and Lecturer in the Graduate School at the University of Denver.

** Figures in parenthesis refer to the Bibliography.

It is of interest to note that in the case of guyed TV towers, the fundamental mode of free vibration is not necessarily the critical factor and, therefore, higher modes should also be considered (Figure 4).

This method can be extended readily into investigations in the plastic region of the material by introducing plastic hinges at the points and of the time where and when the elastic limit of the material is exceeded.

2. Consideration of Actual Ground Motion.

Since motion of the ground actually excites the vibration in the structure, it is mandatory that earthquake resistance design of such structures be based upon the most probable dynamic characteristics of earthquakes in the region being considered. From the complex ground motion, only the longitudinal and the transverse waves are of particular importance for earthquake resistance analysis. Since stresses produced by the vertical oscillation of tall structures are only a fraction of those produced by the gravity loads (and by pre-stressing in case of guyed towers), the following analysis will concentrate on a determination of the response of multi-level guyed towers due to the horizontal component of the earth motions produced by earthquakes, with the understanding that, fundamentally, the same analysis should be performed to determine the vertical free and forced oscillations.

Studying the available records of earthquakes for a region in question, and applying the mathematical theory of probability(2)(3), an acceleration curve of the most probable earthquakes can be obtained. The use of high-speed electronic computers also makes entirely feasible the computing of the dynamic response of a guyed tower for several different recorded horizontal earthquake accelerations or displacements, respectively.

Figure 4a shows a typical horizontal displacement curve which was used in the numerical example and obtained from Reference 4.

In order to utilize the advantages of Navier's "forced solution" of the differential equation of motion, the forcing function is expressed in trigonometric series (sine or cosine):

$$y_I = \sum_m Y_{I\,m} \quad \sin \ pt \tag{1}$$

and

$$x_I = \sum_m X_{I\,m} \quad \sin \ pt,$$

where $p = \frac{m 2\pi}{T}$, $m = 1, 2, 3 \ldots$

T = period of Fourier-expansion,

and

Y_{Im}, X_{Im} = constants of the Fourier-expansion.

In the case of soft soil and an extremely tall structure, which requires a considerable distance between cable anchorages in order to provide stability against overturning, there is a phase difference between the horizontal motion of the foundation of the tower and that of the foundation of the guys, which results in relatively complex forcing functions at the points of elastic supports. Knowing the seismic velocity of the soil in question(5)(6), this time difference can be easily determined. Another peculiarity of earthquake excitation transmitted through the guys to the structure is that only motions "away" from the tower are transmitted since cables are not able to transmit compression. Thus, the oversimplified earthquake excitation at any elastic support may have the form shown in Figure 2c. Since any type of curve can be expressed by Fourier series using a sufficient number of terms, this fact does not create any problem in the actual analysis.

3. Preliminary Design.

The preliminary design of a multi-level guyed tower should be based on wind loads(7). In order to determine the critical direction of an earthquake, the using of approximate methods given in earthquake codes(1) is satisfactory. In order to facilitate the preliminary design, one should make extensive use of the applicable formulae(6)(8) for continuous beams on linear elastic supports. It is conceivable that a different earthquake direction (Figure 1) will be critical for the tower structure as well as for the cable support depending on the geometrical arrangement and relative stiffness of the structural elements. Preliminary investigation in the case of the numerical example (Paragraph 7), however, has indicated that, for that particular structure, an earthquake in the direction of one of the cables (direction I, Figure 1) produces the most critical condition for the cable as well as for the tower structure.

4. Mathematical Model of the Structure.

In order to handle the complex problem rationally, a somewhat simplified mathematical model is introduced which consists primarily of a continuous beam with elastic supports having "n" lumped masses "M_j" (Figure 2).

The interaction between soil and structure is approximated by a weightless spring which is obtained by multiplying the dynamic bedding coefficient "K" (pci) with the projected horizontal area of the foundation in the direction of the critical earthquake:

$$C_{soil} = K \cdot A. \tag{2}$$

A better approximation can be obtained by introducing the Coulomb damping of the soil "η" (5). Since the mass of the soil participating in the vibration is unknown, the use of a weightless spring is recommended. The non-linearity of the stress-strain relationship of certain soil types can be handled in a similar manner as that described for the cable supports. Pre-stressed cables made of high-strength wires usually exhibit a non-linear force-displacement relationship (Figure 3a), which can be effectively approximated by a polygon. Thus, the horizontal force exerted by the cable support, when the "i"-th point moves in the (+)y direction, can be expressed by:

$$^{(+)}R_{i(y)} = R_{io} + \frac{R_{i1} - R_{io}}{y_{i1}} \; y, \tag{3}$$

if $\quad o < y < y_{i1}$,

and

$$^{(+)}R_i\,(y) = R_{i1} + \frac{R_{i2} - R_{i1}}{y_{i2} - y_{i1}} \; y, \tag{4}$$

if $\quad y_{i1} < y < y_{i2}$, etc.

By definition, the horizontal spring constant provided by the cables is:

$$^{(+)}C_i = R_{io} + \frac{R_{i1} - R_{io}}{y_{i1}} \tag{5}$$

if $\quad o < y < y_{i1}$,

and

$$^{(+)}C_i = R_{i1} + \frac{R_{i2} - R_{i1}}{y_{i2} - y_{i1}} \tag{6}$$

if $\quad y_{i1} < y < y_{i2}$, etc.,

assuming fixed supports for the spring. The values of "R_i" can be determined following Reference 7.

In feeding this information into the memory of the computer, equation (6) is automatically used when the horizontal displacements at point "i" are larger than "y_{i1}", etc. Naturally, the force-displacement relationship might be different when the structure moves in the negative direction, resulting in different spring constants in the positive and negative directions of motion.

Since not only the structure but the anchorages of the cables move, a time-dependency in the elastic support is introduced. This time-dependency of the cable support can be quite easily considered by fixing the supports of the springs and by introducing a time-dependent force at the cable support. The magnitude of this force is:

$$^{(+)}F_i(t) = {}^{(+)}\rho_{ii} \sum_m^{II} Y_{Cm} \;\sin\; pt \tag{7}$$

in the positive "y" direction and

$$^{(-)}F_i(t) = {}^{(-)}\rho_{ii} \sum_m^{II} Y_{Am} \;\sin\; pt. \tag{8}$$

in the negative "y" direction.

In these expressions:

$$^{(+)}\rho_{ii}^{II} = {}^{(+)}C_i \qquad (9)$$

and

$$^{(-)}\rho_{ii}^{II} = {}^{(-)}C_i, \qquad (10)$$

by the definition of the stiffness factors as discussed in the following paragraph. The other terms of equations (7) and (8) are obtained from the expression of the earth-disturbances, equation (1). In a similar way, the forcing function at the tower foundation (point B, Figure 1) can be written as:

$$F_1(t) = \rho_{11} \sum_m Y_{Bm} \sin pt, \qquad (11)$$

where

$$\rho_{11} = C_{soil} .$$

The vibration of point "i" of the mathematical model has an effect on the vibration of point "j", and vice versa. This coupling effect is expressed by the flexibility matrix $[\delta_{ij}]$, the elements "δ_{ij}" of which express the motion of point "i" due to the unit force acting at point "j". In the following, the motion and force are used in general terms. Since the determination of the flexibility matrix, in the case under consideration, is extremely cumbersome, the stiffness matrix $[\rho_{ij}]$ is determined; between the stiffness and flexibility matrices the following relationship exists:

$$[\delta_{ij}] = [\rho_{ij}]^{-1} . \qquad (12)$$

5. Determination of the Stiffness Matrix.

By definition(11)(12)(13), the stiffness coefficient is the force created by moving the spring with unit displacement. By dealing with a stiffness rather than a flexibility matrix, a great advantage is derived from the fact that its elements can be determined using a "disassembled" structure.

Between the lumped masses, the structure is disassembled yielding fixed beams with or without elastic supports. Then, a unit motion (one at a time) is introduced at the end; the resulting forces are determined yielding the elements of the matrix of the disassembled structure. For instance, between points F & G (Figure 2a and 2b), the stiffness matrix is:

$$[\rho_{ij}]^{FG} = \begin{array}{c} \\ 7 \\ 8 \\ 9 \\ 10 \end{array} \begin{array}{c} \begin{array}{cccc} 7 & 8 & 9 & 10 \end{array} \\ \left[\begin{array}{cccc} \frac{12}{\ell^2} & \frac{6}{\ell} & -\frac{12}{\ell^2} & \frac{6}{\ell} \\ \frac{6}{\ell} & 4 & -\frac{6}{\ell} & 2 \\ -\frac{12}{\ell^2} & -\frac{6}{\ell} & \frac{12}{\ell^2}+\frac{\rho G \ell}{EI} & -\frac{6}{\ell} \\ \frac{6}{\ell} & 2 & -\frac{6}{\ell} & 4 \end{array} \right] \end{array} \times \frac{EI}{\ell} . \qquad (13)$$

The numbers outside of the matrix indicate the motions which can actually take place on the structure (Figure 2b). The simple sign convention, valid for displacements and forces, is indicated in Figure 2. The modulus of elasticity of the material is represented by "E"; "I" represents the moment of inertia of the section and "ℓ" represents the distance between points "F" and "G".

The term "ρG" represents the force produced by unit horizontal displacement of the elastic support and can be obtained from equations (5) and/or (6), respectively.

Naturally, if the elastic support is considered with its full value in $[\rho_{ij}]^{FG}$, then it should not be considered in $[\rho_{ij}]^{GH}$. Or, it might be considered with half value in each partial matrices. The stiffness matrix of the total structure is easily obtained by simple algebraic addition of the elements having the same subscripts. A schematic way to compile the stiffness matrix is shown in Figure 3. Great care should be exercised in numbering the actually possible motions, (Figure 2b) which determine the number of elements in the stiffness matrix. A matrix inversion, done by the computer, yields the flexibility matrix of the structure, equation (12). Since the procedure for obtaining the stiffness matrices of the individual elements is extremely simple, a large number of lumped masses can be considered with care, thereby increasing the accuracy of the computation. Both the stiffness matrix and flexibility matrix are always symmetrical.

6. Differential Equations of Motion and the Use of Navier's "Forced Solution".

Knowing the flexibility matrix $[\delta_{ij}]$, and the forcing functions of equations (7), (8), and (11), the differential equation of motion of the multi-degree vibrating system in the "y" direction can be written in the matrix form:

$$\{y_i\} = -[\delta_{ij}][M_j]\{\ddot{y}_j\} + [\delta_{ij}]\{P_j(t)\}, \tag{14}$$

where $[M_j]$ represents the mass matrix which can be written in the following form:

$$[M_j] = \begin{bmatrix} \frac{W_1}{g} & 0 & 0 & . & . & 0 & 0 \\ 0 & H_2 & 0 & . & . & 0 & 0 \\ . & . & . & . & . & 0 & 0 \\ 0 & 0 & 0 & \frac{W_{i-1}}{g} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & H_i & 0 & 0 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{W_n}{g} \end{bmatrix} \tag{15}$$

where "W_n" is the weight lumped in point "n" , "g" is the gravitational acceleration and "H_i" represents the rotational moment of inertia of the "i"-th lumped mass. The mass matrix is always diagonal having only positive elements.

Certain elements in the force matrix $\{P_j\}$ (column matrix) are zero. The non-zero elements are given in equations (7), (8) and (11).

The solution of equation (14) is investigated in form of:

$$y_i = \sum_m Y_{im} \sin pt, \tag{16}$$

which for a specific "m" value yields a series of algebraic equations:

$$\{Y_{im}\} = p^2[\delta_{ij}][M_j]\{Y_{jm}\} + [\delta_{ij}]\{P_{jm}\}, \tag{17}$$

since the trigonometric term cancels out (Navier's solution). The only unknowns are $\{Y_{im}\}$, which are obtained using electronic computers with standard programs for solution of coupled algebraic equations.

The free vibration of the structure is obtained by making equation (14) homogeneous:

$$\{Y_i\} + [\delta_{ij}][M_j]\{\ddot{y}_j\} = 0, \tag{18}$$

The solution can be introduced as:

$$y_i = \sum_m Y^*_{im} \quad \sin(\omega t + \alpha_i) \tag{19}$$

where "α_i" is an arbitrary phase angle depending on initial condition, and "ω" represents the natural circular frequency of the structure.

Substituting equation (19) into equation (18) and introducing:

$$\lambda = \omega^2 \tag{20}$$

equation (18) can be expressed, after the performation of some matrix operations, in the following form:

$$[\rho_{ij}] \cdot [M_j]^{-1} - \lambda \mathcal{E} = 0 \tag{21}$$

where $\mathcal{E}$ is the unit matrix.

In this way, equation (21), the solution of the free vibration, i.e., the determination of the lowest frequencies (ω_1, $\omega_2 \ldots \omega_n$), and of the corresponding modes of vibration, is reduced to the problem of determination of eigenvalues and eigenvectors of the matrix product:

$$[\rho_{ij}] \; [M_j]^{-1} \tag{22}$$

for which, again, ready-made programs on electronic computers are available.

The total solution is obtained by adding the free vibration to the forced vibration. Knowing the deflections, the moments and forces in the tower and in the cable can be determined(12)(13).

7. Numerical Example.

Applying the method outlined above, a numerical example has been worked out for the structure shown in Figure 1 and for the earthquake direction I shown in Figure 1. It has been assumed that the foundation is rock having a bedding constant $K = 1000 \times 10^3$ pcf. Ground disturbances at points A, B, and C were assumed to be identical, which is allowable in the case of rock foundation. The units used in the numerical example are pounds, inches and seconds. The stiffness matrix is given in Table I.

The diagonal elements of the mass matrix $[M_j]$, equation (15), were: 2490, 316000, 12.42, 632000, 9.65, 489000, 6.89, 349000, 6.89, 349000, 6.89, 349000, 6.89, 349000, 6.89, 349000, 3.45, and 17500 (lbs. sec^2/in).

The elements of the column matrix of the force were:

$$\{P\} = \{45200 \times 10^4, 0, 0, 0, 7470, 0, 0, 0, 5150,$$

$$0, 0, 0, 8640, 0, 0, 0\} \times \sin \frac{2\pi}{15} 10 \quad \text{(pounds)},$$

representing the first term in the Fourier series. Because of extremely rapid convergence, only three terms of the Fourier series were used.

The obtained modes of free vibration of the structure are shown in Figure 4a. The horizontal displacement of the structures corresponding to the forced vibration for "t" equals 10 seconds and is shown in Figure 4b.

SUMMARY

A matrix method has been developed for the rational solution of free and forced vibration of multi-level guyed towers considering non-linearity and time-dependency of the elastic cable supports. "Actual" earthquake motions have been introduced in the form of Fourier series and forcing functions. The method developed in this paper can easily be extended for the determination of dynamic response of a structure in the post-elastic range. The numerical example was worked out using a high-speed electronic computer.

ACKNOWLEDGEMENT

The writer expresses his appreciation to Mr. M. C. Bjorgum and Mr. J. R. Shaver, Graduate Students at the University of Denver, for working out the numerical example.

REFERENCES

1. Joint Committee of San Francisco, California Section, ASCE and Structural Eng. Assn. of Northern California, "Lateral Forces of Earthquake and Wind", ASCE Proceedings, Vol. 77, Separate No. 66, April 1951.

2. Housner, G. M., "Characteristics of Strong-Motion Earthquakes", Bulletin Seismological Soc. of America, Vol. 37, No. 1, 1947, pp. 19-31.

3. Rasenbluth, E. and J. I. Bustarmante, "Distribution of Structural Response to Earthquakes", Journal of the Eng. Mechanics Division, Proceedings of ASCE, June, 1962.

4. Housner, G. M. and P. C. Jenning, "Generation of Artificial Earthquakes", Journal of the Eng. Mechanics Division, Proceedings of ASCE, February, 1964.

5. Lorenz, H., "Grundbau-Dynamik", Springer Verlag, Berlin, 1960.

6. Palotás, L., "Mérnöki, Kezikönyv", Vol. III, Müszaki Könyvkiadó, Budapest, 1959.

7. Cohen, E. and H. Perrin, "Design of Multi-level Guyed Towers", Journal of Structural Division, Proceedings of ASCE, September 1957.

8. Koelle, H. H., "Handbook of Astronautical Engineering", McGraw-Hill Book Company, New York, N. Y., 1961.

9. Zurmühl, R., "Praktische Mathematik", 4th Ed., Springer-Verlag, Berlin, 1963.

10. Fluegge, W., "Handbook of Engineering Mechanics", McGraw-Hill Book Company, New York, N. Y., 1962.

11. McMinn, S. J., "Matrices for Structural Analysis", John Wiley and Sons, Inc., New York, N.Y., 1962.

12. Borg, S. F. and J. J. Genaro, "Advanced Structural Analysis", D. Van Nostrand Company, Princeton, N. J., 1960.

13. Kersten, R., "Das Reduktionsverfahren der Baustatik", Springer-Verlag, Berlin, 1962.

TABLE I

Stiffness Matrix $[\rho_{ij}]$

3.47×10^4 (lbs/in)

960.3	121.2	-0.269	121.2	0	0	0	0	0	0	0	0	0	0	0	0
121.2	72600	-121.2	36300	0	0	0	0	0	0	0	0	0	0	0	0
-0.269	-121.2	0.532	0	-0.269	121.2	0	0	0	0	0	0	0	0	0	0
121.2	36300	0	145200	-121.2	36300	0	0	0	0	0	0	0	0	0	0
0	0	-0.269	-121.2	0.4331	-54.53	-0.148	66.67	0	0	0	0	0	0	0	0
0	0	121.2	36300	-54.53	112600	-66.67	2000	0	0	0	0	0	0	0	0
0	0	0	0	-0.148	-66.67	0.2964	0	-0.1482	66.67	0	0	0	0	0	0
0	0	0	0	66.67	20000	0	8000	-66.67	20000	0	0	0	0	0	0
0	0	0	0	0	0	-0.1482	-66.67	0.3073	0	-0.1482	66.67	0	0	0	0
0	0	0	0	0	0	66.67	2000	0	80000	-66.67	2000	0	0	0	0
0	0	0	0	0	0	0	0	-0.1482	-66.67	0.2964	0	-0.1482	66.67	0	0
0	0	0	0	0	0	0	0	66.67	20000	0	80000	-66.67	20000	0	0
0	0	0	0	0	0	0	0	0	0	-0.1482	-66.67	0.3147	0	-0.1482	66.67
0	0	0	0	0	0	0	0	0	0	66.67	20000	0	80000	-66.67	20000
0	0	0	0	0	0	0	0	0	0	0	0	-0.1482	-66.67	0.1482	-66.67
0	0	0	0	0	0	0	0	0	0	0	0	66.67	20000	-66.67	40000

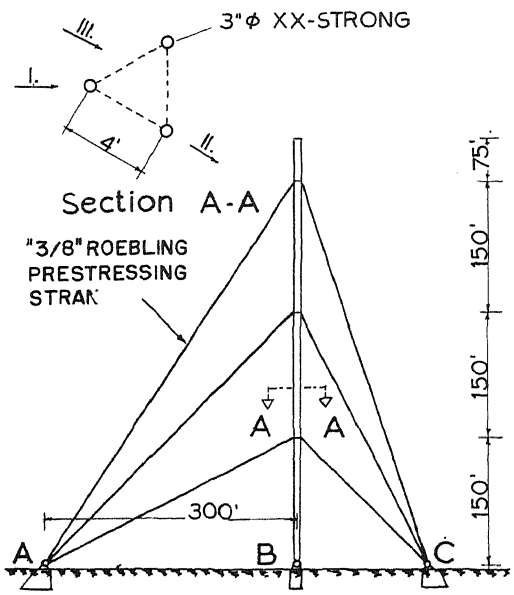

FIGURE 1.
Multilevel guyed tower

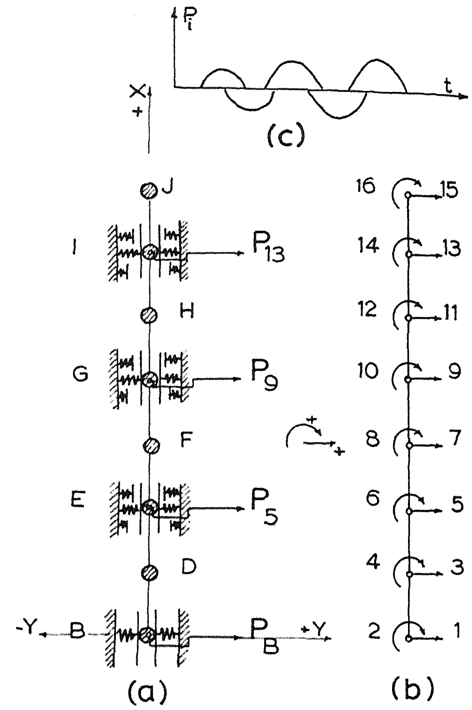

FIGURE 2.
Mathematical model

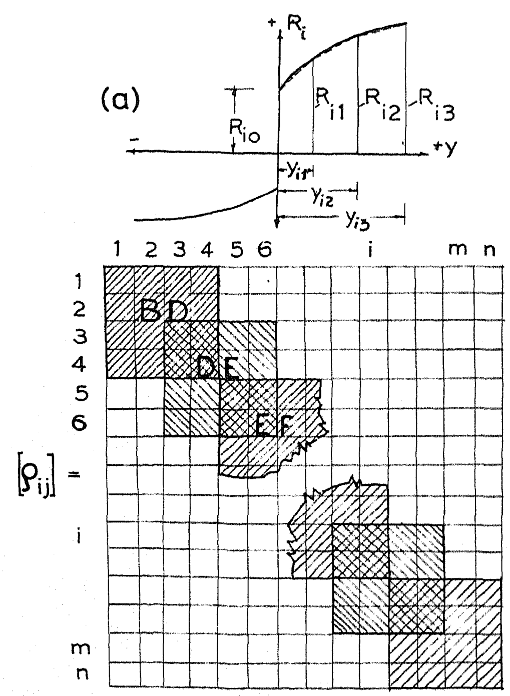

FIGURE 3.
Stiffness matrix $[\rho_{ij}]$

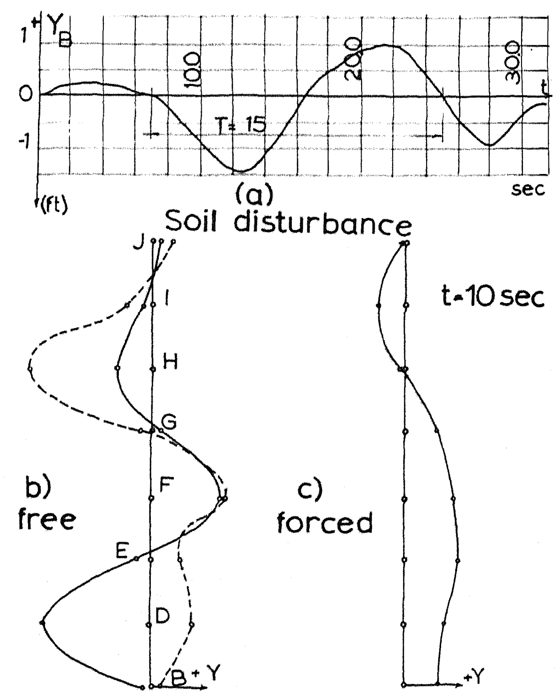

FIGURE 4.
Modes of vibrations

DYNAMIC RESPONSE OF MULTI-LEVEL GUYED TOWERS TO EARTHQUAKE CONSIDERING THE NON-LINEARITY OF THE ELASTIC SUPPORTS.

BY: R. SZILARD

QUESTION BY: M. WATABE - JAPAN

You treat elastic supports in horizontal direction only but how do you think of a normal force which subsequently occurs in the horizontal direction. Especially at the top of a guyed wire, we cannot discuss without considering normal force and some consideration of buckling problems.

QUESTION BY: T. KATAYAMA - JAPAN

The slope of the cables is quite steep. How do you consider the vertical forces? Also, for this slender tower the buckling problem will be very important in practical construction.

AUTHOR'S REPLY: These two questions are basically similar and have been partially answered in my paper. Equation (1) shows the general form of Fourier Series representation of the vertical component of the ground disturbance. Since the length of the papers were limited, more attention was given to the response of the structure to the governing horizontal ground motions with the understanding that basically the same approach can be taken if the consideration of the secondary vertical motions is desired.

In such a case the matrix of the stiffness factor of the individual elements must be expanded to consider the effect of the axial motion.

Concerning the matter of buckling, it can be generally stated that the derived matrix solution of the dynamic response of multilevel guyed towers can be used also to solve the dynamic stability problems of the structure. Needless to say, that the complete dynamic stability investigation of the structure is inherently complex, but again the use of computers can aid in obtaining the solution.

In case of critical load, the structure offers no resistance to the disturbances, which means that the stiffness matrix is singular. In elastic stability investigation the stiffness matrix must include the effects of possible disturbances and consequently is generally larger than the one used to determine the dynamic response of the structure.

Finding the critical load is a matter of trial and error. The external loads are slowly increased by a safety factor β. As the load is increased, the elements of the stiffness matrix change. This procedure is repeated until the determinant of the stiffness matrix becomes zero. The corresponding β is the safety factor against buckling. This relatively tedious operation can be transferred to an eigenvalue problem of the stiffness matrix as it is described in the pertinent literature (10), (11). This method, however, does not cover the so called "snap-through" buckling case, which has limited importance in the problem at hand.

QUESTION BY: R.W. CLOUGH - U.S.A.

The type of structure you are describing is apparently a very lightweight tower, and it would appear that the mass of the cables might be a significant part of the system. It might be that the vibrations of the cables might be a very important part of the total response of this kind of system. You appear to have neglected the mass of the cable.

AUTHOR'S REPLY: In most of the practical cases the mass of the tower structure is considerably larger than that of the cables, which justifies the simplification taken in the paper. If this is not the case, or for any other reason the designer would like to know the order of magnitude of such secondary effects, the general method outlined in the paper allows not only the consideration of the effect of the masses of the cables, but also the effect of their coupled vibration since the cables can also be treated by a similar discrete element approach. The author feels that a good compromise in this respect is to use only the dynamically equivalent cable masses, which can be computed by equating the kinetic energy of the cable to that of an equivalent mass-spring system. Furthermore, the method outlined at this Conference by Dr. Rubinstein can also be considered.