UNBALANCED BUILDINGS, AND BUILDINGS WITH LIGHT TOWERS, UNDER EARTHQUAKE FORCES

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SUMMARY

Torsionally unbalanced buildings, and buildings surmounted by relatively light towers, have special sensitive features which influence the seismic forces and movements in them. This sensitivity arises when a natural period of the building in translational movement is close to a torsional period in the one case, or to a period of the tower in the other case. Such buildings call for special consideration during dynamic analysis. When the building damping is low its behaviour may differ dramatically from that which occurs when the damping is high. The discussion is confined to elastic buildings but takes account of damping.

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1. LIST OF SYMBOLS

r	=	radius of inertia
$r(1+\Delta)$	=	radius of stiffness (Δ may be negative)
$r\delta$	=	offset from centre of gravity to centre of stiffness (may be taken as defining the positive direction, so that $\delta \geqslant 0$)
Y_a, Y_b	=	displacements of equivalent weights $W/2$ and $W/2$
Y_c, Y_d	=	displacements of ends of equivalent columns of stiffness $K/2$ and $K/2$
x	=	distance from C.G. to node
T	=	natural period
ε, p	=	auxiliary constants
g	=	acceleration due to gravity
F_1, F_2	=	inertia forces in modes 1 and 2
R_1, R_2	=	moments in modes 1 and 2
Y_c	=	displacement of the C.G.
θ	=	rotation
Y_s	=	displacement of the centre of stiffness
α	=	term relating tower period to building period
β	=	tower to building weight ratio
D	=	damping; fraction of critical.

2. INTRODUCTION

A natural period of a building in simple horizontal vibration may be the same as a natural period of torsional vibration. A small unbalance of such a building will couple these movements and the resulting two normal modes will have natural periods slightly greater than, and slightly less than, the natural period in translation only. In each of these normal modes the building will twist severely despite its small unbalance. If the seismic forces are calculated by combining the modal forces in the usual way then it is found that the torsional movements appear to increase these calculated forces considerably. However a more detailed analysis shews that such results actually occur only when the building damping is very low. This low damping occurs during small earthquakes and when building tests are made with the assistance of a vibrator. When the effective building damping has the high value to be expected during a severe earthquake then the effect of the small unbalance falls to about the value which would be expected on the basis of ordinary static analysis.

A similar case occurs when a building is surmounted by a relatively light tower and when a period of the building is equal to a period of the tower. Again a pair of normal modes arise in which one period is slightly greater than, and one period is slightly less than, the period of the building alone. In each of these normal modes the tower has very severe movements. The earthquake forces derived by the methods usually employed for elastic buildings are again very severe, while more detailed elastic analysis again shows that such large forces only occur when the building damping is very low. For the high damping to be expected during a severe earthquake the forces generated in the tower are typically several times the weight of the tower.

The usual method for assessing the seismic forces to be expected in an elastic building is as follows. The maximum forces and movements which the earthquake generates in each important normal mode are calculated separately, and then the total force or movement in any building component is obtained by combining the maximum contributions of each normal mode in some way. However, simple addition cannot be used since the maximum contribution of each normal mode occurs at a different time during the earthquake.

A very simple approximate method of combining two or more normal modes is to take the square root of the sum of the squares of the individual modal maxima ("R.S.S. value"). This convenient method breaks down when the modes concerned are of very short period, when algebraic addition must be used. The R.S.S. method breaks down also for the cases of the slightly unbalanced building and the tower-surmounted building, which were described above.

For the unbalanced building, with high modal damping, the maximum effects in the two close-period normal modes need to be added algebraically giving small net rotations.

The components of the tower are subjected to a pair of opposing modal forces of almost equal magnitude. Algebraic addition would now give small forces; these are too low. The true combined forces, which may be several times the weight of the tower, are still much smaller than the forces which would be obtained if the R.S.S. method was used.

When the unbalance of a building is increased, or when the weight of a tower is increased (without change of period), then pairs of natural periods, originally close in value, move further apart. When the natural periods are sufficiently far apart the earthquake-generated effects in a pair of normal modes may be added by the R.S.S. method, even when the building damping is high. When two normal modes are somewhat closer in period the combined earthquake-generated forces have values between those for close periods and those given by the R.S.S. method. Reduced modal damping has an effect similar to increased period difference, so that the method of modal combination is controlled by the ratio of period difference to damping.

3. ONE-STOREY TORSIONAL BUILDING

The important features of coupled translational and torsional vibrations are illustrated by the behaviour of a one-storey unbalanced building.

Fig. 1(a) represents a building which is unbalanced under horizontal inertia forces; the centre of stiffness is displaced from the centre of gravity. The dynamic character of a rigid floor is retained if its distributed mass is replaced by two equal masses at equal distances, the radius of inertia, from the centre of gravity. Similarly the system of supporting members may be replaced by a pair of columns of equal shear stiffness at an appropriate distance, "the radius of stiffness", on either side of a "centre of stiffness". Fig.1(b) illustrates positive values of all the quantities, which are defined in the list of symbols, section (1).

3.1 MODE SHAPES AND NATURAL PERIODS

The shapes of the two normal modes and their natural periods may be obtained from the equations of steady-state motion. Equating translational forces:

$$\frac{W}{2g}\left(\frac{2\pi}{T}\right)^2\left(Y_a + Y_b\right) = \frac{K}{2}\left(Y_c + Y_d\right). \qquad (1)$$

Taking moments about a node:

$$\frac{W}{2g}\left(\frac{2\pi}{T}\right)^2\left(Y_a^2 + Y_b^2\right) = \frac{K}{2}\left(Y_c^2 + Y_d^2\right). \qquad (2)$$

Geometrical relationships may now be used to express the displacements Y in terms of distances to the node; and eliminating T we obtain

$$x^2 - 2r\frac{\varepsilon}{\delta}x - r^2 = 0, \qquad (3)$$

where $\varepsilon = \Delta + \frac{1}{2}\left(\Delta^2 + \delta^2\right)$ (so that $\varepsilon = \Delta$ approximately for small Δ and δ). Solving equation (3) we obtain the position of the two nodes (which define the two mode shapes):

$$x_1 = \frac{r\left(\sqrt{\varepsilon^2 + \delta^2} + \varepsilon\right)}{\delta} \qquad \text{(Mode 1)}, \qquad (4)$$

$$x_2 = -\frac{r\left(\sqrt{\varepsilon^2 + \delta^2} - \varepsilon\right)}{\delta} \qquad \text{(Mode 2)}. \qquad (5)$$

Note the useful relationship

$$x_1 x_2 = -r^2, \qquad (6)$$

which may be obtained either by multiplication, or directly from equation (3). It shows that the nodes (of the two modes) lie on opposite sides of the centre of gravity, one within and one beyond the radius of inertia. Fig.(2) gives for positive Δ the node positions in the form r/x_1 which $= -x_2/r$.

From equation (1) we may also obtain the natural periods in the form:

$$\left(\frac{2\pi}{T}\right)^2 = \frac{gK}{W}\left(1 - \frac{r\delta}{x}\right) \qquad \text{(either mode)}, \qquad (7)$$

whence substituting x from equations (4) or (5) we obtain:

$$\left(\frac{2\pi}{T_1}\right)^2 = \frac{gK}{W}\left[1 + \varepsilon - \sqrt{\varepsilon^2 + \delta^2}\right], \qquad (8)$$

$$\left(\frac{2\pi}{T_2}\right)^2 = \frac{gK}{W}\left[1 + \varepsilon + \sqrt{\varepsilon^2 + \delta^2}\right], \qquad (9)$$

(so that $T_1 > T_2$ and the modes are correctly identified), (Fig. (3)).

For a balanced building $\delta = 0$, the two modes become purely translational (node effectively at infinity) and purely torsional (node at the C.G., $x = 0$). Which is "Mode 1" (longer period), depends on the sign of ε or Δ : if $\Delta > 0$ Mode 1 becomes purely translational, if $\Delta < 0$ torsional. For the translational mode (1 or 2) the square bracket in the period equation (8) or (9) becomes 1; for the torsional mode, $(1 + 2\varepsilon)$. Thus approximately

$$\frac{\text{torsional period}}{\text{translational period}} = 1 - \Delta \qquad (\Delta \text{ small}, \delta = 0). \qquad (10)$$

It may be shown more directly that the ratio is $1/(1 + \Delta)$ exactly.

As the unbalance δ increases from 0, the translational mode acquires some rotational character, and vice versa. Thus one node moves from the C.G. towards one of the concentrated masses: the other node moves from infinity (of the opposite sense) towards the other mass: equation (6) holds always.

As the same time the period difference grows larger (whichever the sign of Δ):

$$\frac{T_2}{T_1} = 1 - \sqrt{\Delta^2 + \delta^2} \qquad \text{approximately.} \qquad (11)$$

3.2 MODE FORCES AND MOVEMENTS UNDER "ONE-g" STATIC ACCELERATION

When calculating the seismic forces and movements in a normal mode of a building, a convenient first step is to calculate the forces and movements of the normal mode under standard conditions, a static acceleration of one "g", (1). These forces and displacements can be expressed in terms of the mode shape and period.

When a system of n weights is under a static acceleration of g, in the 'y' direction, and when a mode shape is given by the relative movements $Y_1, Y_2 \ldots\ldots Y_n$, then the inertia force is given by

$$F_r = W_r Y_r \frac{\sum_1^n W_i \ Y_i}{\sum_1^n W_i \ Y_i^2} , \qquad (i, r = \text{weight No.}). \quad (12)$$

For either mode of the present building, we can deduce from equations (4) or (5) the relative displacements of the two weights, and substitute in equation (12) to get the forces contributed by the two weights. They can be resolved into a force at the C.G. plus a moment about the C.G. for each mode:

Mode 1: Force $$F_1 = \frac{W}{2}\left[1 + \frac{\varepsilon}{\sqrt{\varepsilon^2 + \delta^2}}\right], \qquad (13)$$

Moment $$R_1 = \frac{W}{2} \frac{r\delta}{\sqrt{\varepsilon^2 + \delta^2}}; \qquad (14)$$

Mode 2: $$F_2 = \frac{W}{2}\left[1 - \frac{\varepsilon}{\sqrt{\varepsilon^2 + \delta^2}}\right] = \left(W - F_1\right), \qquad (15)$$

$$R_2 = -R_1 . \qquad (16)$$

Equations (13) and (14) are illustrated by Figs. (4) and (5). It can be seen that large modal torsional forces, R, can occur for very small unbalance, $r\delta$, provided that Δ is comparably small, that is provided the torsional and translational periods are sufficiently close; equation (11).

Individual building components may be designed in terms of the movement of the part of the floor to which they are attached. For either normal mode, under one-g static acceleration (1), the displacement and rotation of the C.G. are given by

$$Y_C = \left(\frac{T}{2\pi}\right)^2 \left(\frac{gF}{W}\right), \qquad (17)$$

$$\theta = \left(\frac{T}{2\pi}\right)^2 \left(\frac{gR}{r^2 W}\right). \qquad (18)$$

The R.H. terms are given by equations (8) (9) and (13) to (16), or the corresponding graphs Fig.(3) (4) and (5). The displacement y of a point on the floor at any distance x from the centre of gravity is now given, for mode 1 or 2, as

$$y = Y_c + \Theta x \qquad (19)$$

3.3 RESPONSE OF NORMAL MODES TO EARTHQUAKES

In order to obtain the earthquake-generated forces and movements in each of the two modes, for which the one-g static responses are given by equations (13) to (19), we must multiply these results by the earthquake response factor, that is, by the maximum response of a unit resonator to a "standard" earthquake, (Fig.6); Skinner (2). Corresponding responses are given by Housner (3). We now have the maximum earthquake-generated forces and movements in each of the normal modes and it remains to combine them to obtain the maximum total forces and movements.

3.4 COMBINING THE EARTHQUAKE RESPONSES OF THE TWO NORMAL MODES

At a given point in the building the forces or movements may have the same, or opposite, signs. As derived above only the maximum modal forces or movements are known. In order to find the way in which the two modes may be combined and the maximum combined response estimated, the following detailed analysis was performed on an electrical analogue.

A pair of resonators, with close natural periods and with damping 5% of critical, were subjected to the El Centro earthquake (1940, N.S. component). The resonators each had unit weight, and their instantaneous response values were added with the same sign ("1:1") or with opposite signs ("1:-1"). The maximum of the combination was recorded and graphed, Fig (7).

The Fig (7) shows that when two equal-weight resonators have the same sign and also have periods whose ratio T_1/T_2 is not greater than 1.1, then their individual earthquake responses may be combined (approximately) by algebraic addition. On the other hand when equal-weight resonators are of opposite sign, so that algebraic addition would give zero response, then the "minus" curves of Fig (7) show that such algebraic addition no longer applies. Physical arguments suggest that when the period ratio is small and the algebraic sum is not too small then algebraic addition will apply. It suggests further that an increase in damping will increase the range of cases in which the two modal responses may be combined by algebraic addition.

In practice the design of components in an unbalanced building, for example by the application of equation (19), does not result in very small differences of normal mode forces or movements. These small algebraic sums may occur in the design of tower-surmounted buildings, which must then be designed on the basis of information such as the "minus" curves of Fig (7).

(Further computation will be performed to amplify these points).

Algebraic addition will often be called for in cases of small torsional unbalance. If the periods are close the one-mode earthquake response factors, Fig (6), for each of the two modes will be almost equal. In this case we can add the two one-g static responses (algebraically) before multiplying by the response factor. For a single storey (as above) the result is simply the ordinary one-g static displacement or force, as obtained by purely static analysis.

The displacements may be expressed most simply as follows:

$$Y_s = \frac{W}{K} \quad = \text{displacement of centre of } \underline{\text{stiffness}} \qquad (20(a))$$

$$\Theta = \frac{\delta}{r(1+\Delta)^2} Y_s \quad = \text{rotation;} \qquad (20(b))$$

$$Y = Y_s - \Theta(x - r\delta) \quad = \text{displacement of a point at any distance } x \text{ from centre of } \underline{\text{gravity}}; \qquad (20(c))$$

The significance of the torsional effect may be measured by comparing the displacement due to torsion at some typical radius such as r, with the translational displacement. The ratio is

$$\frac{r\Theta}{Y_s} = \frac{\delta}{(1+\Delta)^2} = \delta \text{ approximately.}$$

(since Δ must be small to give close periods, equation (11)).

Thus although the modal equations would appear to indicate otherwise, a small imbalance measured by δ and a small Δ will give small torsional effect (because the period difference is then small and justified algebraic addition).

Note that where algebraic addition is not justified the estimated maximum displacements will not vary linearly along the building as do the values just given.

The physical process occurring may be understood by considering a single earthquake pulse applied to the two equal resonators of opposite sign. They each proceed to execute a damped sine wave, starting 180 degrees out of phase, with complete cancellation when added. At later times the phase difference is progressively reduced since the periods are different. This results in an increasing net movement which is limited however, by the progressive decay of the motion of both resonators. Hence the damping opposes the effect of the period difference. It is expected that the maximum buildup of motion in this equal-and-opposite case is controlled by the ratio Q of relative period difference to damping:

$$Q = \frac{(T_1 - T_2)/T_1}{D} . \qquad (21)$$

4. MULTISTOREY UNBALANCED BUILDINGS

A simple form of unbalanced multistorey building has normal modes and periods which can be derived directly from its normal modes, when balanced, and from the results for the unbalanced single-storey building. For this to be justified, this building must have successive storeys which are geometrically the same; the C.G. and the C.S. (centre of stiffness) of each floor lie in two vertical lines and the radii of inertia and of stiffness have two constant values. Further it must be limited to certain types, for example those shown in Fig (8).

When balanced this building has pairs of modes, one translational and the other torsional with the same vertical profile; and antinodes & nodes occur at the same height in the two modes. This is true under certain restrictions on the effects of floor flexibility. When a small unbalance is introduced the modes of an original pair interact to give a derived pair of modes which have the same shape, along a vertical line, as the original pair. Along the horizontal axis, x , at any level, the modes of the pair have nodes at the same positions as a single storey building of the same geometry (C.G., C.S. and the radii of inertia and stiffness); as given by equations (4) and (5). The periods of the resultant mode pair are derived from the original translational period in accordance with equations (8) and (9) which derive the modal periods from the translational mode period (reading $[2\pi / T_{trans}]^2$ for gK/W).

Algebraic addition of responses of the modes <u>of one pair</u> may be called for just as in the one-storey case of the last section 3.4. The sum of the one-g static responses is no longer the ordinary "total" static response. Instead, the state of displacement is given as follows: The displacement of the centre of stiffness of each floor is equal to the one-g static displacement of that floor in the purely-translational mode from which the pair was derived. The displacement of the other points on the floor is then given by equations (20b) (20c) for each floor.

If a building has a small unbalance which is not uniform throughout its height, it is to be expected that there will be an approximately equivalent uniformly-unbalanced building which has similar behaviour, at least for the first few normal modes.

4.1 BUILDING TYPES PRONE TO TORSIONAL OSCILLATIONS

Since a small unbalance (due, e.g. to foundation assymetry) may excite torsional oscillations in a building which has close translational and torsional natural periods, a search is made for such buildings.

A typical building of column and beam or column and slab construction, tends to have torsional periods which are somewhat shorter than the corresponding translational periods; Fig (8a) shows the uniform floor mass distribution which would need to be associated with a set of equal columns to give approximately equal translational and torsional periods. However a moderate change in the stiffness or mass distribution of a conventional frame building could result in close translational and torsional periods.

Another building type which tends towards equal translational and torsional periods is the multibay structure with separating shear walls Fig (8b). Such structures have advantages as blocks of flats since they provide sound insulation and fire protection. If the longitudinal stiffness is obtained from a central spine wall or other system which does not add to the torsional stiffness then the translational and torsional periods will be close.

When a building has its lateral stiffness provided by two towers, Fig.(8c) there will be a certain spacing of the towers which gives close translational and torsional periods. A spacing of about this amount may well be adopted for structural reasons.

4.2 SHEAR WALLS AND CENTRAL SPINE WALL - FIELD TESTS

A building vibrator has been used to permit measurements of the periods and mode shapes of a 100-foot high building of the type illustrated in Fig (8b). The building was of balanced design, but it was evident that there was some unbalance in the stiffness of the foundations. The horizontal profiles of the normal modes are shown on the left of Fig (9) (Eash line represents a floor). We obtained the following building parameters:

x_1 = -93 ft, x_2 = +43 ft, T_1 / T_2 = 1.08 (taking building length as 200 ft). Whence from equations (4) to (9):

r = 63 ft (slightly larger than estimated from plans);

$$\Delta \doteqdot +0.025, \text{ radius difference} = r\Delta \doteqdot 1\tfrac{1}{2} \text{ ft}$$

$$\delta \doteqdot -0.07, \text{ offset between centres} = r\delta \doteqdot 4\tfrac{1}{2} \text{ ft}$$

If we anticipate 5% of critical damping (even more so, 10%) in a major earthquake, then with the period difference of 8%, algebraic addition of modal responses is justified and we can estimate the significance of the torsional effect as in section (2.4) using the total static deflection. We find that the north end of the building, about 110 ft from the centre of stiffness, is liable to about 12% more displacement than the centre; i.e. the torsional increase is not severe.

5. BUILDING SURMOUNTED BY TOWER

As described in the introduction, a building with a relatively light tower on top has many features in common with a torsionally unbalanced building. Close tower and building periods correspond to close translational and torsional periods. Also a small ratio of tower to building weight corresponds to a small unbalance and hence a small coupling between the translational and torsional movements. However, since the forces in the tower are separate from those in the building they tend to be given by the combination of two opposite and almost equal forces when the modes have close periods. Then neither "RSS" nor algebraic combination is appropriate: the special case distinguished in Sect. (3.4).

It is also possible for the tower to have a damping factor very different from that of the building; to take this into account, non-modal methods of analysis would be called for and these are beyond the scope of this paper.

Tower-surmounted buildings will be discussed briefly following the same lines as the fuller treatment of the unbalanced buildings.

5.1 SINGLE-STOREY BUILDING WITH SINGLE-STOREY TOWER

The building parameters are defined in Fig (10).

By solving the equations of steady-state motion we obtain the normal mode shapes and natural periods.

$$u = x_1/x_2 = -\rho \pm \sqrt{\rho^2+\beta} \qquad (22)$$

(satisfying $u^2 + 2\rho u - \beta = 0$),

where $\rho = (\alpha + \beta)/2$,

$\alpha = \dfrac{K/W}{k/w} - 1$ (approximately twice the relative difference of period between tower and base-building considered separately),

$\beta = w/W$

The product of the two values of u is $-\beta$

Also
$$\left(\frac{2\pi}{T}\right)^2 = \frac{gk}{w}\left[1 + \rho \mp \sqrt{\rho^2+\beta}\right], \qquad (23)$$

so that
$$(T_1/T_2) - 1 \doteqdot \sqrt{\rho^2 - \beta} \quad \text{for small } \alpha \text{ and } \beta. \qquad (24)$$

These periods are illustrated in Fig (12).

[The best (but still imperfect) analogy with corresponding torsional formulae is seen by letting α correspond to -2Δ of section (3.1), β correspond to δ^2, ρ to $-\epsilon$. If $\alpha = 0$ (base and tower periods equal) and β is small the mode shape parameters u are almost equal and opposite, like the modal distances of the torsional modes when $\Delta = 0$. On the other hand when $\beta = 0$ the -values by no means tend to 0 and ∞ as the analogy would lead us to expect.]

5.1.1 MODAL SHEARS IN TOWER UNDER ONE-*g* STATIC ACCELERATION

The modal shears in the tower may be obtained by substituting the modal shapes defined by equation (22) into equation (12). The tower shear forces for the two modes are

$$f = \frac{w}{2}\left[1 \pm \frac{1+\rho}{\sqrt{\rho^2+\beta}}\right] \qquad (25)$$

It is seen in Fig (11) that the shear forces are many times the tower weight when small relative period difference (α), occurs together with small relative tower weight β.

5.1 MODAL AND TOTAL EARTHQUAKE-GENERATED SHEARS

The earthquake-generated shear force in the tower, for each of the two normal modes, may be obtained by multiplying the shears of equations (25) by the earthquake response factor, Fig (6).

When the natural periods, as given by equation (23), are not too close the total earthquake-generated shear force may be obtained as the root-sum-of-squares of the values of the two modal shear forces. However for very close periods the shears are almost equal and must be added according to Fig (7) (for 5% damping). For these small period differences (under 5% or roughly under 10%) the curves of Fig (7) fall linearly with period difference. Hence the earthquake response of two equal resonators, which have one-g responses of plus and minus R and any such small period difference, is.

$$R_E = 40\,A\left(\frac{T_1}{T_2} - 1\right)R \qquad (26)$$

where A is the response factor from the particular curve "2-" of Fig (7).

When values from equations (24) and (25) are substituted in equation (26) the total earthquake-generated shear, for small period differences, is approximately

$$f_E = 20\,A\,w. \qquad (27)$$

This tower shear is therefore independent of the value of small period differences, and when A is substituted from Fig (7) typical tower shears are about 4 times the tower weight, when a building with 5% damping is subjected to the El Centro earthquake. This shear force probably falls to about 1.5 times the tower weight for 10% building damping.

It should be possible to extend the methods used for a single storey building with a single storey tower to multistorey cases, by using equivalent weights and stiffnesses.

The severe attack on light towers may be enhanced when it occurs with the forces in the base-building still relatively small since the damping of the building may then be small. The building damping tends to control the modal damping giving larger earthquake forces.

The above discussion applies to all small resonant building components, and in particular to cantilevers. The latter tend to be rapidly damaged when earthquake strains (due to building motion along the line of the cantilever) extend to plastic deformation, since the gravity force ensures that all plastic strains are in the same sense and strictly cumulative. This contrasts with a vertical tower where plastic strains in opposite directions partly cancel.

6. DISCUSSION

It is seen from the above that it is of advantage to avoid close torsional and translational periods. However the law of combination of normal modes tends to minimize the net result of extreme modal twisting particularly for the high damping expected during severe earthquakes. However, for the low damping encountered during vibrator tests on buildings, close periods and small unbalance will give large torsional effects which increase the difficulties of measurement and analysis.

Towers and cantilevers are attacked by close resonances of the building, which may be transverse or longitudinal. These two forms of building resonance may make it difficult to design a tower with no close periods.

This work should be extended by calculating more curves of the type given in Fig (7), based on mean values of several earthquakes. The curves should also be drawn for various sizes of the two resonators and for various damping values. The simple tower system should be investigated by another approach for the case where the tower damping is high, say 10%, and the building damping is low, say 2% to 5%.

7. REFERENCES

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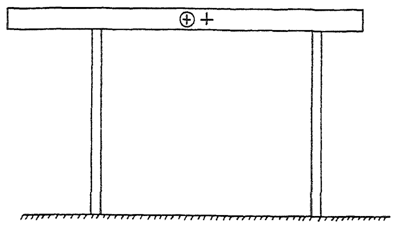

Fig. 1 (a) TORSIONALLY UNBALANCED BUILDING —— ELEVATION.

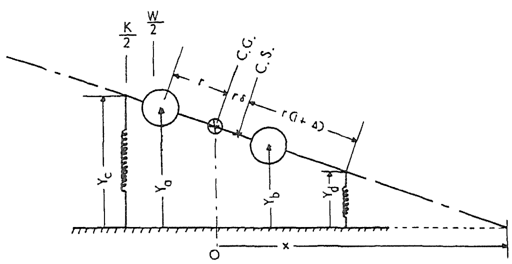

Fig. 1(b) PARAMETERS of EQUIVALENT UNBALANCED BUILDING —— PLAN VIEW.

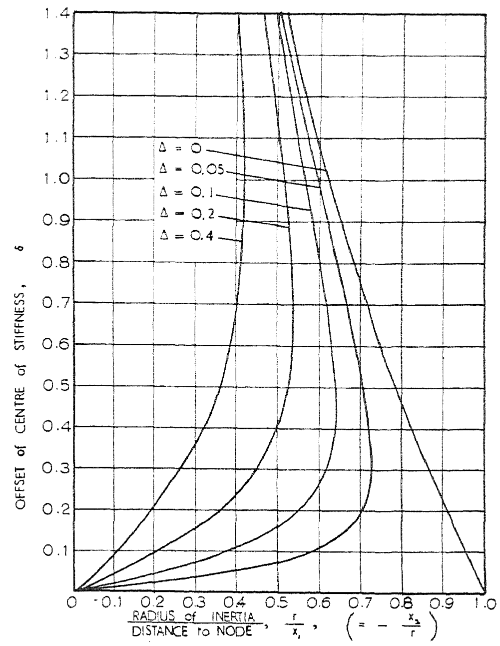

Fig. 2 NODE POSITIONS for MODES 1 and 2. (MODE SHAPE)

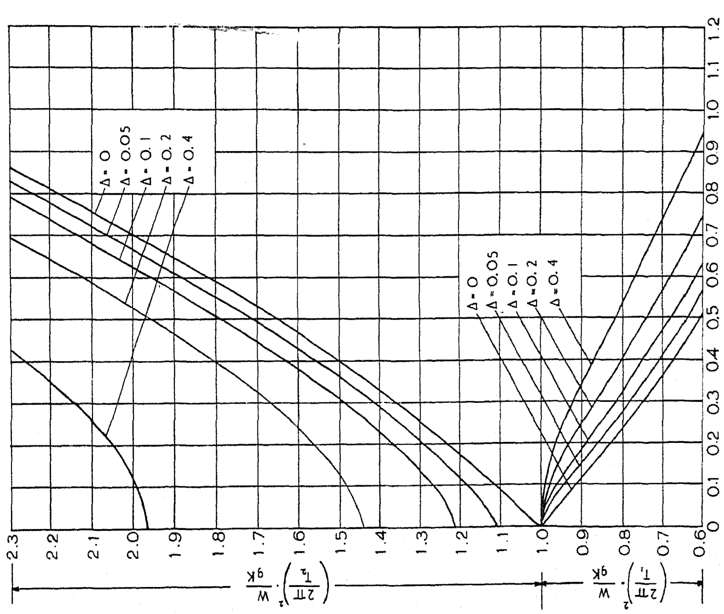

Fig. 3. PERIODS of MODES 1 and 2.

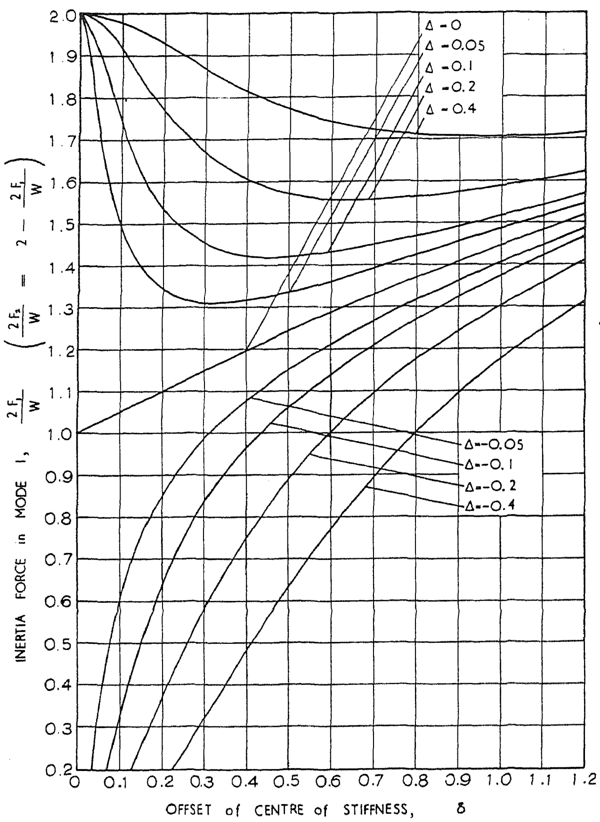

Fig 4. INERTIA FORCE at C.G. UNDER one – g, MODES 1 and 2.

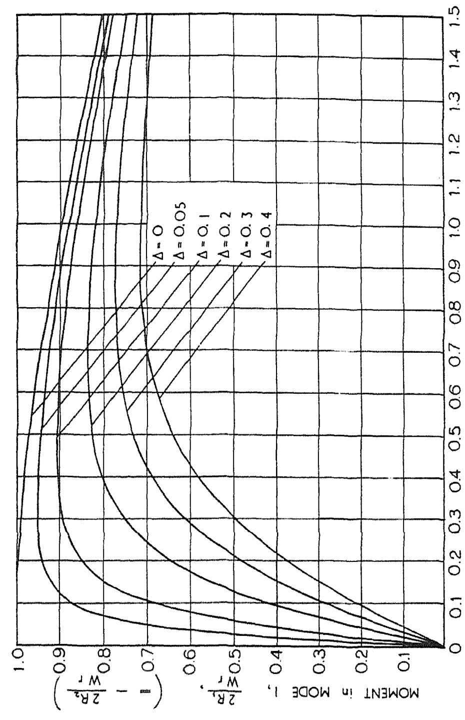

Fig 5. MOMENTS UNDER one – g, MODES 1 and 2.

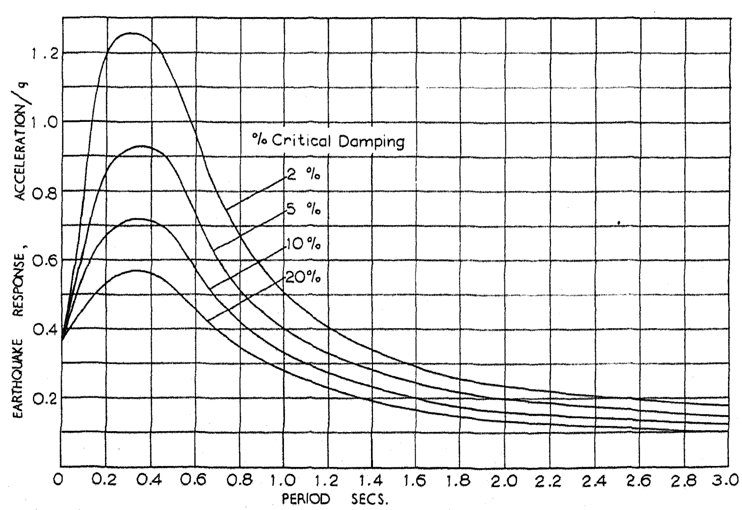

Fig 6. EARTHQUAKE RESPONSE of SIMPLE UNIT RESONATOR.

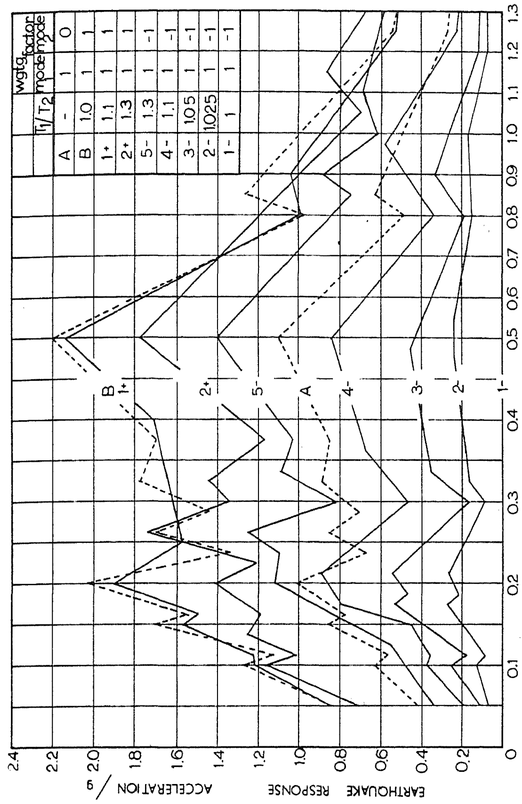

Fig 7. COMBINING TWO EQUAL MODES "1:1" and "1:-1".

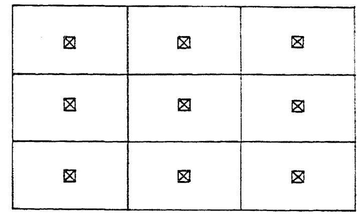

Fig 8(a). PLAN of FRAME BUILDING.

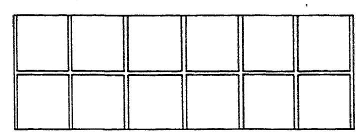

Fig 8(b). PLAN of SPINE WALL BUILDING.

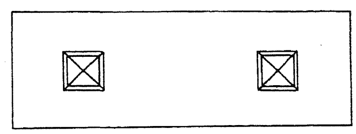

Fig 8(c). PLAN of TWIN-TOWER BUILDING.

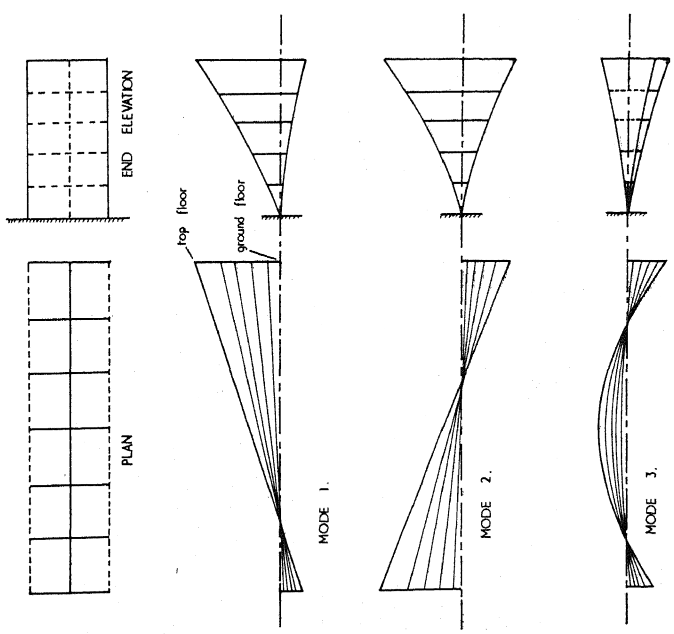

Fig 9. UNBALANCED SPINE-WALL BUILDING.

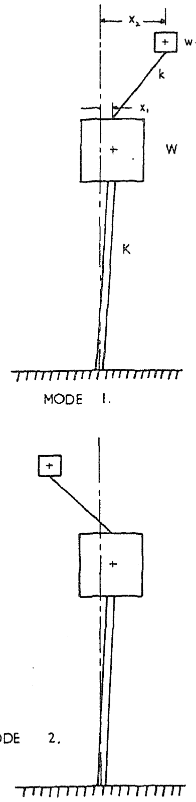

Fig 10. ONE-STOREY SHEAR BUILDING, SURMOUNTED BY ONE-STOREY SHEAR TOWER.

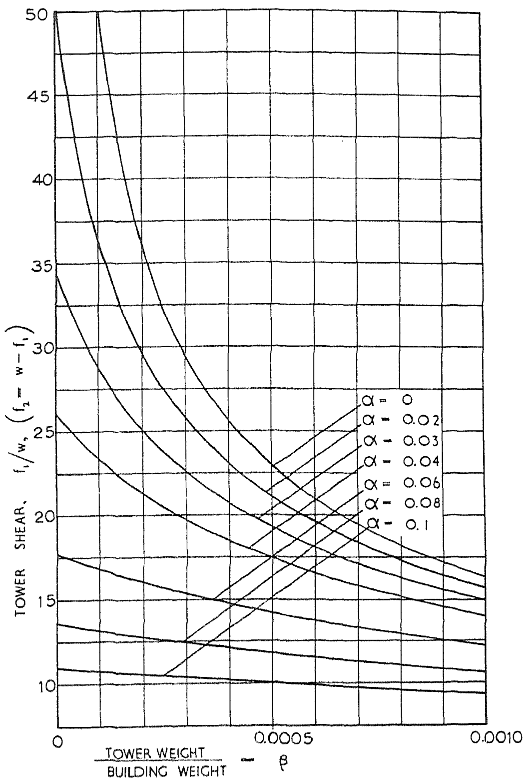

Fig 11. TOWER SHEAR FORCE under one — g, MODES 1 and 2

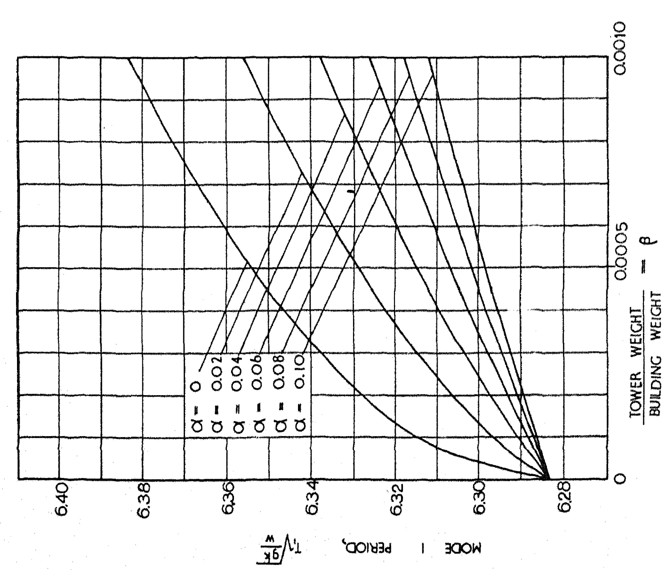

Fig 12. ONE STOREY BUILDING, SURMOUNTED by a TOWER; MODE 1 PERIOD.

UNBALANCED BUILDINGS, AND BUILDINGS WITH LIGHT TOWERS, UNDER EARTHQUAKE FORCES.

BY R.I. SKINNER, D.W.C. SKILTON AND D.A. LAWS

QUESTION BY: V.A. MURPHY - NEW ZEALAND.

Would Mr. Skinner explain further about the differences in acceleration between those measured at the base and those at the top of the building, particularly where depth of foundation rock varied a great deal.

AUTHORS' REPLY: The 100 foot shear-walled building of Fig. 9, was founded on a pad on weathered rock and clay with piles down to foundation rock. The depth to the rock varied from a few feet at one end to about 35 feet near the other end. Despite the varying depth to the foundation rock, the accelerations throughout the ground floor were only a few percent of the roof accelerations, for each of the normal modes excited by a vibrator.

QUESTION BY: R. SHEPHERD - NEW ZEALAND.

Would Mr. Skinner care to comment on the measurement techniques he used. Specifically what method had been used to determine the proportional critical damping.

AUTHORS' REPLY: Three building parameters were measured during tests with a building shaker; the natural periods, the shapes of the normal modes of vibration, and the damping of the normal modes. A normal mode was excited and displacement at a pair of points in the building recorded continuously on a central recorder. The frequency of the vibrator was slowly adjusted until a natural period was reached, as indicated by the amplitudes and relative phases of the movements at the two recording points. By moving one of the recorders to many stations we obtained a set of displacement ratios, which give the shape of the normal mode. The damping was obtained by suddenly removing the drive when a normal mode was being excited, and then measuring the logarithmic decrement of building movement.

QUESTION BY: O.A. GLOGAU - NEW ZEALAND.

What would be the magnitude of the effects of the *torsional* and *translational coupling* in an earthquake of El Centro intensity when damping is higher and the building goes into the ductile range. This is important to designers.

AUTHORS' REPLY: An increase in damping will not change the coupling between torsional and translational modes of resonance but will result in a smaller increase of forces due to the coupling. When considerable plastic deformation occurs, the unbalance in the stiffness of the building is almost certain to increase due to unavoidable variations in the yield point and subsequent flexibility of the building components and the fround.