CONSIDERATION OF THE ROTATION OF THE FOUNDATION IN THE ANALYSIS OF A STEEL CHIMNEY 130 m. HIGH, SUBJECT TO SEISMIC MOVEMENT.

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1.- Introduction

1.1. The Scientifical and Technological Research Department of the Catholic University of Chile (DICTUC) has developped in the past time a series of studies whose principal object in this first stage, has been to investigate the design of earthquake resistant structures and to analize the damages suffered by structures affected by the great 1960 chilean earthquake.

For the conduction of this investigation a Committee was organized whose members were: Civ. Eng. Prof. César Barros L.; Civ. Eng. Prof. Luis Crisosto; Civ. Eng. Prof. Arturo Morales; Civ. Eng. Prof. Hernán Ayarza; Civ. Eng. Prof. Jorge Troncoso. This Committee was responsible for the direction and planning of the different studies that had to be developped by civil engineers and graduates of our University.

During the study of the effects on structures observed in the chilean earthquake of 1960, we got the opportunity to analize the interaction between foundation of structures, ground conditions and the structure of the buildings specially at the locality of Rio Negro. We found that the mass of foundation in relation to the mass of the structure could have an important influence on the response to earthquake forces. A brief and general analysis of such influence was presented elsewhere (1) Later we have to analyse the response to earthquake of the Chimney for the Copper Foundry "Las Ventanas" whose principal characteristics are indicated in fig. 1. We decided to investigate this influence.

This is interesting, due to the fact that this type of structure requires a foundation with a mass commonly more than ten times greater than the mass of the rest of the structure. Under these conditions, if due to seismic action, the soil permits movement of the foundation, so that it vibrates together with the upper structure, the effect of this foundation mass on the upper structure, proportionately much smaller, must produce a resulting vibration very different to the one resulting if we suppose the upper structure embedded at its base.

(1) Análisis del comportamiento de las construcciones escolares en la zona Sur durante los sismos del 21 y 22 de Mayo de 1960 y, en especial, de las estructuras de Rio Negro. First Chilean Sessions on Seismology and Earthquake Engineering hold at Santiago, Chile, July 1963. By Civil Engineer Prof. César Barros L.

Furthermore, a soil not made of rock will always admit small movements of the foundation, which will be greater when the soil's compressibility is greater. We will assimilate these movements to a rotation of the foundation around an axis in the same plane as its base, and passing through its center of gravity.

The influence of the foundation's rotation on the distribution of stresses along the length of a steel chimney 130 m. high, when it is shaken by an earthquake, will be determined here.

Acknowledgment.

The present work has been conducted by Prof. César Barros L., Structural Assesor of EMPRESA NACIONAL DE MINERIA Copper Foundry Las Ventanas and developed by Hans Rossenfeld del Campo civil engineer.

2.- Hypothesis on the foundation's rotation.

2.1. It is necessary to determine the foundation's rotation as a function of the torque at its base. For this purpose we make the following suppositions:

a) We suppose that the soil is a perfectly elastic medium, with a compressibility coefficient K_s + 5,0 Kg/cm^3, in which the deformations are directly proportional to the stresses.

b) We suppose that a trapezoidal stress distribution pattern is maintained in the soil at all times. (see fig. 2).

Therefore, we will have:

Foundation rotation $\theta = \frac{\Delta}{L} \qquad \Delta = \frac{\sigma_f}{K_s} \qquad \sigma_f = \frac{M}{W} = \frac{M}{I_b/L} = \frac{M \cdot L}{I_b}$

Where = moment of inertia of the foundation base around axis 0.

$$\therefore \; \Delta = \frac{ML}{K_s I_b} \qquad \theta = \frac{M}{K_s I_b} \qquad (1)$$

3.- Dynamic analysis.

3.1 - Scheme of the system under consideration.

We consider three elements in our structural system:

a) An upper structure, which is the chimney proper, in which we shall consider that the inertia by rotation and the deformation due to shear stress are practically nonexistant.

b) A perfectly rigid foundation, not subject to lateral support.

c) A bed of springs supporting the foundation and transmits the loads to the firm soil (see figs. 2 and 3); at the same time, it transmits the soil's movement to the foundation. This bed of springs represents the soil immediately adjacent to the foundation's base, which we have supposed perfectly elastic, with a compressibility coefficient K_S defined above.

It is considered that the external load indicated in fig. 3 acts on the system. That is to say, a variable load $q(x,t)$ per unit length along the height of the chimney, and on the foundation a force $Q_S(t)$ acting on the centre of mass, and a torque $M_S(t)$.

Nomenclature.

- x = height of a point of the chimney, measured from its base
- S = centre of mass of the foundation.
- h_S = height of the foundations's centre of mass, measured from its base.
- $\mu(x)$ = mass per unit length of the chimney.
- m_F = foundation mass.
- $I(x)$ = moment of inertia of the chimney's sections.
- I_b = moment of inertia of the foundation's base.
- I_S = moment of inertia of the foundation's mass around a horizontal axis passing through "S".
- E = Coefficient of elasticity of steel $= 21 \cdot 10^6$ TON/M²
- K_S = Compressibility coefficient of the soil, defined in 2.
- t = time.
- $q(x,t)$ = variable external force per unit length, acting horizontally on the chimney.
- $Q_S(t)$ = variable external force acting horizontally on the foundation.
- $M_S(t)$ = variable external moment acting on the foundation.
- $y(x,t)$ = horizontal movements of the chimney's sections at each instant.
- $y_S(t)$ = horizontal movements of the foundation's centre of mass at each instant.
- $\theta(t)$ = rotation angle of the foundation at each instant.

3.2 - Theorical development.

3.21 - Equillibrium equation.- We can state the following dynamic equillibrium equation for a differential element of the chimney:

$$\mu(x)\frac{\partial^2 y(x,t)}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 y(x,t)}{\partial x^2}\right] = q(x,t) \qquad (2)$$

3.2.2- Generalized coordinates and normal coordinates of the system.- The displacements of the system's mode shapes or normal modes can be expressed in generalized coordinates. On this way, for the displacements of the n^{th} mode shape we have:

$$\text{CHIMNEY: } y_n(x,t) = Y_n(t)\phi_n(x) \;, \quad \text{FOUNDATION: } y_{sn}(t) = Y_n(t)\phi_{sn} \;; \quad \theta_n(t) = Y_n(t)\alpha_n \qquad (3)$$

where $Y_n(t)$ is the generalized coordinate, representing the vibration's amplitude at each instant, at an arbitrary reference point.

$\phi_n(x)$ and ϕ_{sn} are displacements relative to the reference point, constituting the normal form of vibration.

$\alpha_n = \phi_{sn}/h_s$ is the foundation's relative rotation.

The system's normal cordinates are the generalized coordinates which represent the amplitudes of the system's different mode shapes.

3.2.3- Orthogonality of the system's mode shapes.- Let us consider two mode shapes ϕ_n and ϕ_m . As their movements are harmonic, the forces of inertia can be expressed as a function of the displacements. In this way, for the system's n^{th} mode we have:

forces of inertia in the chimney, per unit length: $\mu(x)\,\omega_n^2\,\phi_n(x)$
inertial force of the foundation: $m_F\,\omega_n^2\,\phi_{sn}$
inertial moment of the foundation: $I_s\,\omega_n^2\,\alpha_n$

where ω_n is the n^{th} mode shape's angular frequency.

According to Betty's law, the work performed by the n^{th} mode's inertial forces, due to the displacements of the m^{th} mode, must be equal to that performed by the forces of the m^{th} mode due to the displacements of the n^{th} mode.

That is to say: $$\int_0^L[\mu(x)\omega_n^2\,\phi_n(x)]\,\phi_m(x)\,dx + m_F\omega_n^2\,\phi_{sn}\,\phi_{sm} + I_s\,\omega_n^2\,\alpha_n\alpha_m =$$
$$= \int_0^L[\mu(x)\,\omega_m^2\,\phi_m(x)]\,\phi_n(x)\,dx + m_F\,\omega_m^2\,\phi_{sm}\,\phi_{sn} + I_s\,\omega_m^2\,\alpha_m\,\alpha_n$$

and therefore, for $n \neq m$:
$$\int_0^L \mu(x)\phi_n(x)\,\phi_m(x)\,dx + m_F\,\phi_{sn}\,\phi_{sm} + I_s\,\alpha_n\alpha_m = 0 \qquad (4)$$
which is the orthogonality condition for our system's mode shapes.

3.2.4- Displacements expressed in normal coordinates.- The system's displacements under the action of external forces can be expressed in normal coordinates, as follows:
$$y(x,t) = \sum_{n=1}^{\infty} Y_n(t)\,\phi_n(x); \quad y_s(t) = \sum_{n=1}^{\infty} Y_n(t)\,\phi_{sn} \quad ; \quad \theta(t) = \sum_{n=1}^{\infty} Y(t)\alpha_n \qquad (5)$$

3.2.5- External load expressed in normal coordinates.- We shall express the external forces as a function of the forces of inertia associated with the different mode shapes. Therefore we have, for the load acting on the chimney, per unit length:
$$q(x,t) = \sum_{n=1} Y_n(t)\,\omega_n^2\,\phi_n(x)\cdot\mu(x)$$

If we now define as the system's generalized force, for the n^{th} mode shape:
$$P_n(t) = Y_n(t)\,\omega_n^2\,m_n$$
where m_n is a reference mass for the n^{th} mode, which corresponds to the concept of generalized mass of the system for said mode shape.

We can then write: $$q(x,t) = \sum_{n=1}^{\infty} \frac{P_n(t)}{m_n}\,\phi_n(x)\,\mu(x) \qquad (6)$$

In the same way, for the external force applied to the foundation:
$$Q_s(t) = \sum_{n=1}^{\infty} Y_n(t)\,\omega_n^2\,\phi_{sn}\,m_F = \sum_{n=1}^{\infty} \frac{P_n(t)}{m_n}\,\phi_{sn}\,m_F \qquad (7)$$
and for the external moment applied to the foundation:
$$M_s(t) = \sum_{n=1}^{\infty} Y_n(t)\,\omega_n^2\,\alpha_n\,I_s = \sum_{n=1}^{\infty} \frac{P_n(t)}{m_n}\,\alpha_n\,I_s \qquad (8)$$

3.2.6- Equilibrium equation in normal coordinates.- Replacing in equation (2) the displacements $y(x,t)$ and the external load $q(x,t)$ by their expressions in normal coordinates, we finally come, by separation, to the well known set of normal equations for the system:
$$\ddot{Y}_n(t) + \omega_n^2\,Y_n(t) = \frac{P_n(t)}{m_n} \qquad \text{FOR } n = 1, 2, 3 \ldots\ldots \infty \qquad (9)$$
one for each amplitude Y_n , and in a manner entirely similar to that which determines the forced vibration of a system with one degree of liberty, which reduces the analysis to that of such systems.

3.2.7- Determination of the system's generalized forces and masses.- Equations (6) (7) and (8) show us two ways of expressing the external loads. For these expressions to be equivalent, they must give as a result an equal amount of work for an arbitrary displacement of the system; putting down this equality, we can determine the system's generalized mass and force for each mode shape.

Let us suppose that the system is displaced according to the m^{th} mode shape, with $Y_m = 1$; then the following must hold:

$$\int_0^L q(x,t)\,\phi_m(x)dx + Q_s(t)\phi_{sm} + M_s(t)\alpha_m = \int_0^L \Big[\sum_{n=1}^{\infty}\Big(\frac{P_n(t)}{m_n}\phi_n(x)\mu(x)\Big)\Big]\phi_m(x)dx + \Big[\sum_{n=1}^{\infty}\frac{P_n(t)}{m_n}\phi_{sn}\,m_F\Big]\phi_{sm} +$$
$$+ \Big[\sum_{n=1}^{\infty}\frac{P_n(t)}{m_n}\alpha_n I_s\Big]\alpha_m = \sum_{n=1}^{\infty}\frac{P_n(t)}{m_n}\Big[\int_0^L \phi_n(x)\phi_m(x)\,\mu(x)dx + m_F\,\phi_{sn}\phi_{sm} + I_s\alpha_n\alpha_m\Big]$$

Due to the orthogonality condition:

$$\because \quad = \frac{P_m(t)}{m_m}\Big[\int_0^L \phi_m^2(x)\,\mu(x)dx + m_F\,\phi_{sm}^2 + I_s\alpha_m^2\Big]$$

If we define the following as the system's generalized mass for the m^{th} mode shape: $m_m = \int_0^L \phi_m^2(x)\mu(x)dx + m_F\,\phi_{sm}^2 + I_s\alpha_m^2$

We have for the system's generalized force for the same mode:

$$P_m(t) = \int_0^L q(x,t)\,\phi_m(x)\,dx + Q_s(t)\,\phi_{sm} + M_s(t)\,\alpha_m$$

These expressions suppose that the external load indicated in figure 3 is acting.

3.2.8- Action of a seismic movement on the system.- Let us now consider the action of an earthquake which gives the foundation's base an acceleration $\ddot{y}_b(t)$

For the external force we shall then have:

$$q(x,t) = -\mu(x)\,\ddot{y}_b(t) \qquad Q_s(t) = -m_F\,\ddot{y}_b(t) \qquad M_s(t) = 0$$

and for the expressions of the system's generalized force and mass, for the m^{th} mode shape $P_m(t) = -\ddot{y}_b(t)\Big[\int_0^L \mu(x)\,\phi_m(x)dx + m_F\,\phi_{sm}\Big]$

$$m_m = \int_0^L \mu(x)\,\phi_m^2(x)dx + m_F\,\phi_{sm}^2 + I_s\alpha_m^2 \qquad (10)$$

If we now define:

$$\beta_n = \frac{\int_0^L \mu(x)\,\phi_n(x)\,dx + m_F\,\phi_{sn}}{\int_0^L \mu(x)\,\phi_n^2(x)dx + m_F\,\phi_{sn}^2 + I_s\alpha_n^2}$$

and introduce in equilibrium equation (9) a new variable $Z_n(t) = Y_n(t)/\beta_n$ we shall finally have:

$$\ddot{Z}_n(t) + \omega_n^2 Z_n(t) = -\ddot{y}_b(t) \qquad (11)$$

for n = 1,2,3 .... ∞

The general solution for this differential equation is given by Duhamel's integral.

Replacing $Y_n(t) = \beta_n Z_n(t)$ in equations (5), we obtain for the system's displacements at each instant:

$$y(x,t) = \sum_{n=1}^{\infty}\beta_n\cdot Z_n(t)\,\phi_n(x) \qquad y_s(t) = \sum_{n=1}^{\infty}\beta_n\cdot Z_n(t)\,\phi_{sn} \qquad \theta(t) = \sum_{n=1}^{\infty}\beta_n\cdot Z_n(t)\,\alpha_n \qquad (12)$$

The factors β_n are determined by knowing the system's mode shapes.

Later we shall realize the great importance of the term $m_F\,\phi_{sn}$ in the expression for the generalized forces, when calculatin their numerical value for the higher modes. This term, which appears due to the foundation's rotation, will considerably increase the value of the factors β_n corresponding to the high mode shapes, thus increasing the chimney's internal stresses.

3.2.9- Shear forces and bending moments in the chimney.- Making use of the normal coordinates, we can determine the instantaneous shears and bending moments along the chimney, by superposing the effects of each mode shapes.

Thus: $V(x,t) = \sum_{n=1}^{\infty} Y_n(t) V_n(x) = \sum_{n=1}^{\infty} \beta_n Z_n(t) V_n(x)$

$$M(x,t) = \sum_{n=1}^{\infty} Y_n(t)\ M_n(x) = \sum_{n=1}^{\infty} \beta_n Z_n(t) M_n(x) \qquad (13)$$

where $V_n(x)$ and $M_n(x)$ are the shears and bending moments associated with the different mode shapes ϕ_n, for $Y_n(t) = 1$

These expressions give us the exact values of V and M at each section of the structure and at each instant. Nevertheless, in practice we only determine, for each section, the maximun possible values of V and M, which are definitely the ones that interest us for the design. For this, the maximun Z_n values are calculated, corresponding to the participation of each mode in the system's total vibration, by means of the "Spectrum", which corresponds to the seismic movement to be considered.

Thus we have that the functions: $V_n(x)\beta_n Z_{n\,max}$ and $M_n(x)\beta_n Z_{n\,max}$ give us the maximun values of the shears and bending moments produced in the chimney by the component of the n^{th} mode. If we now superpose these functions, determined for each one of the mode shapes, we obtain the maximun possible values of V and M along the chimney. That is to say:

$$V(x)_{max} = \sum_{n=1}^{\infty} V_n(x)\beta_n Z_{n\,max.} \qquad M(x)_{max.} = \sum_{n=1}^{\infty} M_n(x)\beta_n Z_{n\,max} \qquad (14)$$

By "Spectrum" we must understand the family of curves which determine, for a given seismic movement, the maximun speed S_v of a system with one degree of liberty, as a function of its natural period T, and of its damping, which serves as a parameter, and which is expressed as a fraction λ of its critical damping. To determine the maximun displacement we have: $Z_{max} = \frac{1}{\omega} S_v$ with ω being the system's angular frequency.

In practice to consider the damping λ_n of the system in each mode shape, we apply the Spectrum with the curve corresponding to said damping λ_n

3.3.- Determination of Mode Shapes and Natural periods.

3.3.1- Fundamental mode.

Newmark's Method.- The system's mode shapes have been determined by Newmark's method of numerical integration. Figure 4 shows graphically one cycle in the method's convergency process, taking into account the foundation's rotation. As a reference point we have chosen the top of the chimney ($\phi(x) = 1$ for $x = 130\,m.$), which is supposed divided into 4 parts. $\phi^{(0)}$ is the supposed mode shapes with which the process starts. In this way we come to curve $\phi^{(1)}$, with which a new cycle starts, and like that, until we obtain $\phi^{(0)} = \phi^{(1)}$ by convergence. After that, the angular frequency ω and the natural period T are determined from the following expressions:

$$\phi_n^{(1)} = \delta_n \frac{\lambda^4 \omega^2}{144\, EI_e g} \quad \text{(for any given section) and} \quad T = \frac{2\pi}{\omega}$$

where g = acceleration due to gravity = $9{,}82\ m/seg^2$

Calculations.- The calculation was done for 10 divisions in the chimney. The results are given in figure 5.

3.3.2- Higher modes.

Purification process.- Let us analyze first the purification process, to permit convergence to the higher modes of our system, in the previous method.

Let $\bar{\phi}_2^{(0)}$, $\bar{\alpha}_2^{(0)}$ be an approximate form of our system's second mode, from which we want to extract the first mode's component. We can write down:

$$\bar{\phi}_2^{(0)} = \phi_2^{(0)} + a_{21}\phi_1 \qquad \therefore\ \phi_2^{(0)} = \bar{\phi}_2^{(0)} - a_{21}\phi_1$$
$$\bar{\alpha}_2^{(0)} = \alpha_2^{(0)} + a_{21}\alpha_1 \qquad \therefore\ \alpha_2^{(0)} = \bar{\alpha}_2^{(0)} - a_{21}\alpha_1 \qquad (15)$$

where $\phi_2^{(0)}$ = approximate form of the second mode shape purified of the first mode.

$a_{21}\phi_1$ = component of the first mode.

Now, as the purified second mode must comply with the condition of being orthogonal with the first mode, we have:

$$\int_0^L \phi_2^{(o)}(x)\,\phi_1(x)\,\mu(x)dx + m_F\,\phi_{s2}^{(o)}\phi_{s1} + I_s\alpha_2^{(o)}\alpha_1 = 0$$

and therefore, using equations (15), we must have:

$$\int_0^L \bar{\phi}_2^{(o)}(x)\phi_1(x)\cdot\mu(x)dx + m_F\,\bar{\phi}_{s2}^{(o)}\phi_{s1} + I_s\bar{\alpha}_2^{(o)}\alpha_1 - a_{21}\left[\int_0^L \phi_1^2(x)\mu(x)dx + m_F\,\phi_{s1}^2 + I_s\alpha_1^2\right] = 0$$

$$\therefore\ a_{21} = \frac{\int_0^L \bar{\phi}_2^{(o)}(x)\phi_1(x)\mu(x)dx + m_F\bar{\phi}_{s2}^{(o)}\phi_{s1} + I_s\bar{\alpha}_2^{(o)}\alpha_1}{\int_0^L \phi_1^2(x)\mu(x)dx + m_F\phi_{s1}^2 + I_s\alpha_1^2}$$

The integrals in this expression are also calculated by the numerical integration method. After the value of the coefficient a_{21} is found, each term ϕ_1 is multiplied by a_{21} and substracted from $\bar{\phi}_2^{(o)}$, as indicated in equations (15).

The process is similar for the purification of an approximate third mode curve, except that now the curve must be purified of the first and second mode. Therefore we must write down: $\bar{\phi}_3^{(o)} = \phi_3^{(o)} + a_{31}\phi_1 + a_{32}\phi_2 \quad \therefore\ \phi_3^{(o)} = \bar{\phi}_3^{(o)} - a_{31}\phi_1 - a_{32}\phi_2$

$$\bar{\alpha}_3^{(o)} = \alpha_3^{(o)} + a_{31}\alpha_1 + a_{32}\alpha_2 \quad \therefore\ \alpha_3^{(o)} = \bar{\alpha}_3^{(o)} - a_{31}\alpha_1 - a_{32}\alpha_2$$

After that, applying the conditions of orthogonality, which the purified third mode must comply with the first and second mode, we obtain:

$$a_{31} = \frac{\int_0^L \bar{\phi}_3^{(o)}(x)\phi_1(x)\mu(x)dx + m_F\bar{\phi}_{s3}^{(o)}\phi_{s1} + I_s\bar{\alpha}_3^{(o)}\alpha_1}{\int_0^L \phi_1^2(x)\mu(x)dx + m_F\phi_{s1}^2 + I_s\alpha_1^2}$$

$$a_{32} = \frac{\int_0^L \bar{\phi}_3^{(o)}(x)\phi_2(x)\mu(x)dx + m_F\bar{\phi}_{s3}^{(o)}\phi_{s2} + I_s\bar{\alpha}_3^{(o)}\alpha_2}{\int_0^L \phi_2^2(x)\mu(x)dx + m_F\phi_{s2}^2 + I_s\alpha_2^2}$$

The same method is used for the fourth and higher mode shapes:

Calculations.- The higher modes second, third and fourth were calculated. The results are given in figure 6.

3.4.- Calculation of the factors β_n

$$\beta_n = \frac{\int_0^L \mu(x)\phi_n(x)\,dx + m_F\,\phi_{sn}}{\int_0^L \mu(x)\phi_n^2(x)\,dx + m_F\phi_{sn}^2 + I_s\alpha_n^2} = \frac{P_n(t)/\ddot{y}_b(t)}{m_n}$$

In this expression, the numerator multiplied by the seismic acceleration $\ddot{y}_b(t)$ represents the system's generalized force, and the denominator represents the generalized mass of the same for the corresponding mode shape. These factors are of vital importance, as the chimney's shear forces and bending moments are directly proportional to them, according to equation (13) and (14).

The following is a summary of the numerical values:

	1st. mode	2nd. mode	3rd. mode	4 th. mode	Com. Fact.
$\int_0^L \mu(x)\phi_n(x)dx$	73.3344	−68.6479	42.0304	−16.8426	$\lambda/12g$
$m_F\,\phi_{sn}$	4.7950	−27.5000	78.1000	−210.1899	$\lambda/12g$
$\int_0^L \mu(x)\phi_n^2(x)dx + m_F\phi_{sn}^2 + I_s\alpha_n^2$	38.1858	45.8860	46.8218	71.4453	$\lambda/12g$

Therefore:

$$\beta_1 = \frac{73.3344 + 4.7950}{38.1858} = \frac{78.1294}{38.1858} = 2.0460$$

$$\beta_2 = \frac{-68.6479 - 27.5000}{45.8860} = \frac{-96.1479}{45.8860} = -2.0954$$

$$\beta_3 = \frac{42.0304 + 78.1000}{46.8218} = \frac{120.1304}{46.8218} = 2.5657$$

$$\beta_4 = \frac{-16.8426 - 210.1899}{71.4453} = \frac{-227.0325}{71.4453} = -3.1771$$

As we had already said in 3.2.8, we can here observe how the term $m_F \phi_{sn}$ in the expression for the generalized force increases in size with respect to the other term of the same $\int_0^L \mu \phi_n dx$, as the system's mode shape degree increases; with this, the influence of this term on the numerical value of the generalized force P_n , and therefore the value of factor β_n , is each time more important. In this way, this influence which comes to 6.1% in P_1 , rises to 28.6% in P_2 , 65,5% in P_3 and 92.5% in P_4 .

It is very interesting to stress this, as the presence of the term $m_F \phi_{sn}$ is due only to the foundation's rotation, the influence of which we are trying to determine in the structure's behavior and in the internal loads which are produced.

In the same way, the terms $m_F \phi_{sn}^2$ and $I_s \alpha_n^2$, which we find in the expression for the generalized mass, are due to the foundation's rotation, and its influence in its value also increases as the mode's degree increases; nevertheless, they are much smaller than the former. Thus we have that the joint influence of said terms is 0,04% in M_1, 1,02% in M_2, 8,1% in M_3 and 38,3% in M_4.

Now if we make use of the numerical value obtained, and we do not take into account the influence of the foundation's rotation on the values of $\int_0^L \mu \phi_n dx$ and $\int_0^L \mu \phi_n^2 dx$, which is relatively small (1), we can determine the values of β_n approximately, corresponding to the condition of perfect embedding of the chimney in its base. In this case we have:

$$\beta_n = \frac{\int_0^L \mu(x) \phi_n(x) dx}{\int_0^L \mu(x) \phi_n^2(x) dx}$$

and with this, the following approximate values: $\beta_1 = 1.92$ $\beta_3 = 0.98$ $\beta_2 = -1.51$ $\beta_4 = -0.38$

which means that the foundation's rotation in our case has increased these factor s by approximately:

1,07 times in the first mode
1,39 times in the second mode
2,62 times in the third mode
8,06 times in the fourth mode

Based on this, and taking into account what was seen in (3.2.9), we can say that the foundation's rotation must mean, when compared to the perfect embedding, a considerable increase in the shear forces and bending moments introduced by the higher modes in the chimney; specially the third and fourth mode. On the other hand, the action of the first mode is practically uninfluenced by the foundation's rotation. These effects of the foundation's rotation will increase as the foundation's mass increases in relation to the chimney's mass.

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(1) For example in the first mode, the foundation's rotation produces an increase of these values of approximately 6% in $\int_0^L \mu \phi_1 dx$ and of 4% in $\int_0^L \mu \phi_1^2 dx$

4.- Shear forces and bending moments in the chimney.

4.1.- Calculations.

Determination of the response Z_n max.- The "Average velocity spe ctrum" given by Housner in "Shock and Vibration Handbook", applied to the "El Centro" earthquake in California, 1940, was used. The damping used was 2% of the critical damping for each of the system's mode shapes.

The following values were thus obtained:

Z_1 max = 16.0920 cm.
Z_2 max = 3.5764 cm.
Z_3 max = 0,6062 cm.
Z_4 max.= 0,0380 cm.

Calculation of maximun displacements at the end of the chimney.- See table 1.

Calculation of maximun shear forces and bending moments. See tables 2, 3 and 4.

Figure 7 shows the four mode shapes calculated ϕ_1, ϕ_2, ϕ_3 AND ϕ_4, and the curve of the relative shears and bending moments $V_n(x)$ and $M_n(x)$, associated to said mode shapes.

Table 1.- Maximun displacements at the end of the chimney.
Table 2.- Common factors.
Table 3.- Maximun shear forces
Table 4.- Maximun bending moments.

4.2.- Analysis of results. Comparison with results obtained without considering foundation rotation.-

The above results check with what was said in (3.4), as respects the increase in the internal stresses produced by the higher modes in the chimney, as a consequence of the foundation's movement.

We can point out the following specially significant facts which can be observed from tables 3 and 4, and which confirm what was said before:

1) The stresses produced by the second mode shape are now greater than those produced by the first mode.

2) The stresses produced by the fourth mode are on an average approximately equal to 15% of the total maximun stresses (20% in the shears and 10% in the bending moments). This is surprising, as in structures such as ours, but embedded at their base, the influence of the fourth mode is absolutely unimportant.

3) For design purposes, the stresses introduced in the top part of the chimney by the third and fourth mode shapes are decisive.

Figure 8 shows our resulting curve of maximun moments (full line) and the one that results from considering the chimney's base embedded (dotted line), for the purpose of comparing the resulting internal stresses.

We can observe:

First, that along the length of the chimney, the resulting internal stresses in our case are very much greater tran those that result from considering an embedded base. This is due to the considerable increase in stresses introduced in the chimney by the higher modes due to the foundation's rotation, as has already been established.

Furthermore this difference is specially great in the top half of the chimney, due to the action of the third and fourth mode shapes which in our case have great importance in this part of the chimney, and which in the case of an embedded base have practically no influence. Observe for example that at elevation 65, the maximun moment due to our calculation is three times higher than that which results from considering the chimney's base embedded. This clearly shows the importance of the foundation's rotation in very flexible slim structures as the one we have just analyzed. This effect will increase when there is an increase in the ratio between foundation mass divided by the upper structure mass, or in other words, as the upper structure becomes slimmer, and furthermore, as this structure becomes more flexible.

6.- Conclusion.

From the above comparison we come to the conclusion that in the dynamic calculation of very slim and flexible continuous structures shaken by seismic movement, the foundation's rotation must necessarily be considered, when it is permitted by the soil.

This is because, in the type of structure mentioned, the foundation's mass is a great part of the total mass, so that the inertial force and moment introduced in it due to small rotation, can become appreciable with respect to the rest of the inertial forces produced. The effect of said inertial forces of the foundation on the system's vibration is felt in two aspects:

1) It alters the system's mode shapes, increaing the corresponding natural periods.

2) It increases the system's generalized force and generalized mass in each mode shape, in increaing proportion to the modes' degree.

This last aspect is specially interesting, due to its greater influence on the system's internal stresses. In this manner, the increases in the generalized forces bring about proportional increases in the internal stresses; on the other hand, the increases in the generalized masses produce a decrease in these stresses. However, the fundamental fact is that the increase in the system's generalized force, in the first four modes, is much greater than the increase in the generalized mass. The consequence of this is that in said modes the system's internal stresses increase. Although the increase of these stresses in the first mode is small, it is considerable in the second, third and fourth modes, to a point where it is decisive for the structure's stability. This increase will be greater or smaller, depending on whether the structure is more or less slim and flexible.

In summary we can say that the rotation of the foundation, due to its mass, which is proportionately great in the type of structure we have studied, permits the higher mode shapes to introduce stresses which are vitally important in the structure, which have to be considered in its design.

Table 1.- Maximun displacements at the end of the chimney.

n	Z_n max.	β_n	$Y_{n max} = Z_{n max} \cdot \beta_n$
1	16,0920	2,0460	32,9240 cm.
2	3,5764	−2,0954	−7,4940 cm.
3	0,6062	2,5657	1,5554 cm.
4	0,0380	−3,1771	−0,1207 cm.

Table 2.- Common factors.

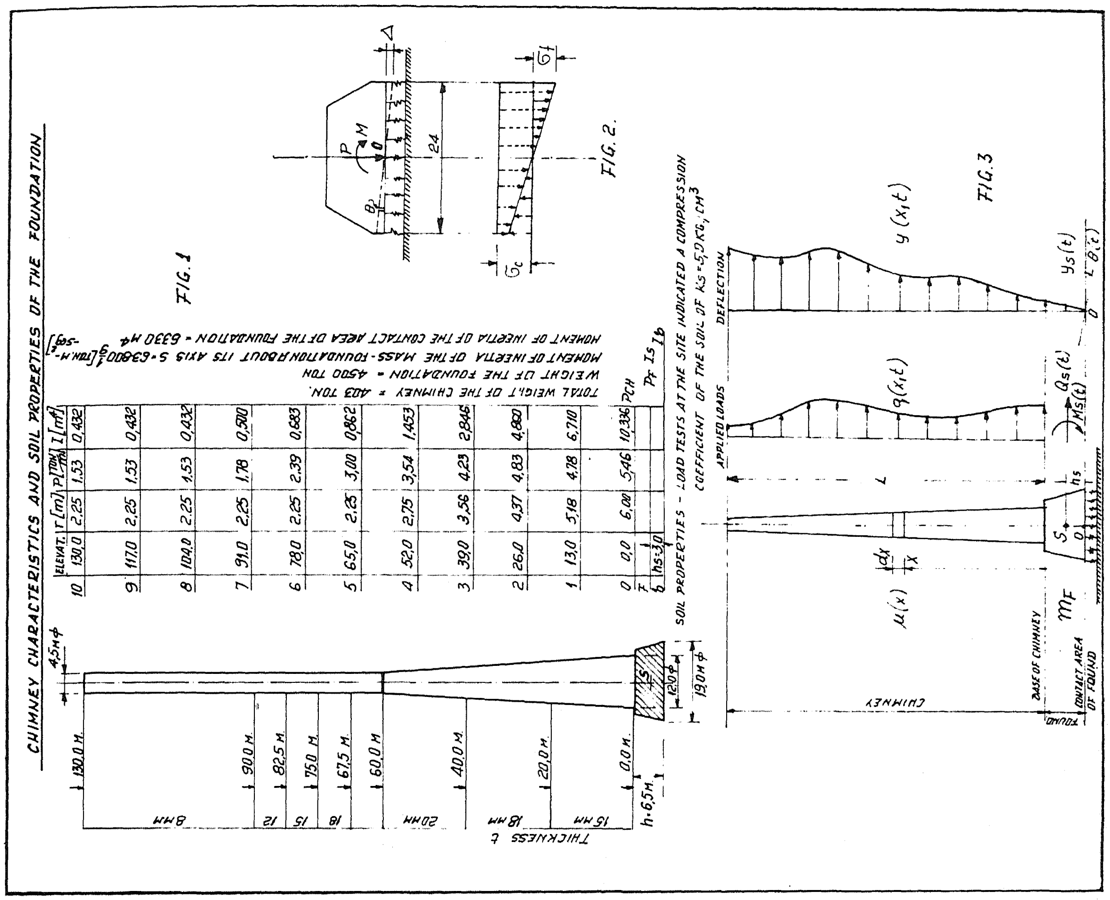
n	Y_n max [m]	$\frac{\lambda \omega_n^2}{12g} Y_n$ max.	$\frac{\lambda^2 \omega_n^2}{12g} Y_n$ max.
1	0,329240	0,5590	7,2673
2	−0,074940	−1,5763	−20,4925
3	0,015554	2,1446	27,8803
4	−0,001207	−2,2112	−28,7456

Table 3.- Maximun shear forces.

	1st. MODE		2nd. MODE		3rd. MODE		4th. MODE		TON.
Secc	V	x 0,559	V	x 1,576	V	x 2,145	V	x 2.211	Σ=V max.tot
9	8,61	4,81	7,58	11,95	6,49	13,93	5,56	12,30	42,98
8	23,57	13,18	16,58	26,14	9,95	21,35	5,11	11,30	71,96
7	35,33	19,75	16,98	26,77	2,10	4,50	−5,74	−12,69	63,70
6	45,34	25,34	9,83	15,50	−10,64	−22,81	−13,38	−29,59	93,24
5	54,43	30,43	−4,54	−7,16	−19,96	−42,82	−7,01	−15,50	95,90
4	61,78	34,54	−23,04	−36,31	−17,31	−37,13	10,14	22,42	130,40
3	67,05	37,48	−40,91	−64,49	−2,94	−6,31	20,76	45,90	154,19
2	70,58	39,45	−55,37	−87,28	16,43	35,23	15,63	34,56	196,51
1	72,56	40,57	−64,65	−101,92	33,08	70,94	−1,08	−2,40	215,82
0	73,33	40,99	−68,65	−108,21	42,03	90,14	−16,84	−37,24	276,59

Table 4.- Maximun bending moments.

Secc	1st. MODE		2nd. MODE		3rd. MODE		4th. MODE		TON.MT.
	M	x 7,267	M	x 20,493	M	x 27,88	M	x 28,746	Σ=M max.tot
9	8,61	62,57	7,58	155,28	6,49	181,05	5,56	159,89	558,79
8	32,18	233,86	24,16	495,10	16,45	458,53	10,67	306,72	1494,21
7	67,51	490,58	41,14	843,07	18,54	516,98	4,93	141,73	1992,36
6	112,84	820,05	50,97	1044,59	7,91	220,40	−8,45	−242,87	2327,91
5	167,27	1215,63	46,43	951,53	−12,06	−336,21	−15,46	−444,32	2947,69
4	229,06	1664,63	23,39	479,44	−29,37	−818,89	−5,32	−152,90	3115,86
3	296,11	2151,93	−17,51	−358,87	−32,32	−900,96	15,44	443,83	3855,58
2	366,69	2664,82	−72,88	−1493,45	−15,89	−443,02	31,07	893,06	5494,35
1	439,25	3192,17	−137,53	−2818,38	17,19	497,22	29,98	861,89	7351,65
0	512,58	3725,11	−206,18	−4225,15	59,22	1651,04	13,14	377,73	9979,03


CHIMNEY CHARACTERISTICS AND SOIL PROPERTIES OF THE FOUNDATION
4,5 M φ
130,0 M.
90,0 M.
82,5 M.
75,0 M.
67,5 M.
60,0 M
40,0 M.
20,0 M.
0,0 M.
h=6,5 M.
12,0 φ
19,0 M φ
THICKNESS t
8 MM
12
15
18
20 MM
18 MM
15 MM
ELEVAT. r [m] P [TON/M] I [m4]
10 130,0 2,25 1,53 0,432
9 117,0 2,25 1,53 0,432
8 104,0 2,25 1,53 0,432
7 91,0 2,25 1,78 0,500
6 78,0 2,25 2,39 0,683
5 65,0 2,25 3,00 0,862
4 52,0 2,75 3,54 1,453
3 39,0 3,56 4,23 2,846
2 26,0 4,37 4,83 4,860
1 13,0 5,18 4,78 6,710
0 0,0 6,00 5,46 10,336 PCH
hs=3,0
PF IS Ib
TOTAL WEIGHT OF THE CHIMNEY = 403 TON.
WEIGHT OF THE FOUNDATION = 4500 TON
MOMENT OF INERTIA OF THE MASS-FOUNDATION ABOUT ITS AXIS S = 63800 1/g [TON.M.SEC²]
MOMENT OF INERTIA OF THE CONTACT AREA OF THE FOUNDATION = 6330 M⁴
FIG. 1
P
M
O
θ
Δ
24
σc
σf
FIG. 2.
SOIL PROPERTIES - LOAD TESTS AT THE SITE INDICATED A COMPRESSION COEFFICIENT OF THE SOIL OF Ks = 5,0 KG/CM³
APPLIED LOADS
DEFLECTION
CHIMNEY
μ(x)
dx
x
L
q(x,t)
y(x,t)
BASE OF CHIMNEY
FOUND.
CONTACT AREA OF FOUND
mF
S
O
hs
Qs(t)
Ms(t)
ys(t)
θ(t)
FIG. 3