DETERMINATION OF STRUCTURAL DYNAMIC PROPERTIES BY STATISTICAL ANALYSIS OF RANDOM VIBRATIONS

by

S. Cherry* and A. G. Brady**

Abstract

The concept of autocorrelation and its application in the study of experimental data is first reviewed. The suitability of this technique as a method for deducing basic structural dynamic properties is then tested. For this purpose, the results obtained from an analysis of the vibrations recorded in randomly excited, real and analog-modelled structures are compared with the known properties of these structures. The influence of data sample size on the outcome of autocorrelation is demonstrated; the studies show that although reasonable estimates of structural period can be secured from an analysis involving small amounts of data, relatively large sample sizes are required if meaningful estimates of structural damping are expected.

1. GENERAL

1.1 Introduction: — The response of a structure to dynamic loading is strongly dependent on such basic properties as the fundamental period of vibration and the damping of the structure. Modern seismic codes recognize the importance of these parameters and many earthquake provisions are now made to depend, in some manner, on them. This has stimulated an interest in the development of new methods for securing much needed information on the period and damping of existing structures. The work reported in this paper is intended as a contribution to this general problem.

Studies to determine the period and damping of structures normally involve full-scale, dynamic tests. An outline of the experimental methods frequently employed in such investigations, together with a discussion of their relative merits, is offered in recent papers by Hudson (1,2). Except for the newly proposed "man-excited" method (3), tests to obtain the dynamic properties of a building require elaborate equipment and must be carried out under controlled input conditions.

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[1] Hudson, D. E., "Dynamic Tests of Building and Special Structures," Experimental Techniques in Shock and Vibration, ASME Colloquium, Nov. 1962.

[2] Hudson, D. E., "A New Vibration Exciter for Dynamic Tests of Full-Scale Structures," Earthquake Eng. Research Lab., Calif. Inst. of Tech., Sept. 1961.

[3] Hudson, D. E., Keightley, W. O. and Nielsen, N. N., "A New Method for the Measurement of the Natural Periods of Buildings," Bull. Seism. Soc. Amer., Vol. 54, No. 1, Feb. 1964.

By comparison, the vibrations induced in a structure by wind and other natural phenomena can be obtained with relative ease and, as a result, period measurements have mainly been determined from wind excited tests. However, the irregular nature of the response records secured from such disturbances sometimes limits the accuracy with which building periods can be estimated and completely obscures any measure of the damping. A procedure which would enable the dynamic properties of a structure to be recovered from vibration records exhibiting stochastic features could provide much useful information and is therefore very desirable.

Takahashi and Husimi (4) have proposed the application of autocorrelation techniques as a means of extending the worth of experimental data possessing irregular characteristics, such as might be obtained from wind excited tests, traffic vibrations, microtremor studies and natural earthquakes, by yielding information on both the damping and period of the test structure. These authors set down the basic theory of the underlying principles involved in this approach, and used the results to examine the behavior of certain simple systems exposed to irregular disturbances. Although this method has been employed in the study of many types of scientific problems, its application in the analysis of vibration records of civil engineering structures has received scant attention in the literature. Hatano and Takahashi (5) were perhaps the first to apply this concept to full-scale structures. They used the computational procedure described in the earlier paper by Takahashi and Husimi (4) to secure estimates of the fundamental period and damping of an arch dam from an earthquake record obtained at its crest. Similar studies were later conducted with records taken at a second arch dam by Takahashi, Tsutsumi and Mashuko (6). Autocorrelation data of microtremor measurements recorded on tower, dam and building structures were summarily reported by Shima, Tanaka and Den (7) in a paper describing some instruments used in earthquake engineering research.

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[4] Takahashi, K., and Husimi, K., "Method to Determine Frequency and Attenuation Constant from the Irregular Motion of an Oscillating Body," Jour. Inst. Physical and Chem. Research, (Japan), Vol. 14, No. 4.

[5] Hatano, T., and Takahashi, T., "The Stability of an Arch Dam Against Earthquakes," Tech. Rept. C-5607, Central Res. Inst. of Elec. Power, Feb. 1957.

[6] Takahashi, T., Tsutsumi, H. and Mashuko, Y., "Behaviors of Vibration of Arch Dam," Tech. Rept. C-5905, Central Res. Inst. of Elec. Power, Dec., 1959.

[7] Shima, E., Tanaka, T., and Den, N., "Some New Instruments Used in Earthquake Engineering in Japan," Proc. 2nd World Conf. on Earthquake Engineering, Vol. II, 1960.

1.2 Object and Scope:- The work reported herein was undertaken to examine the feasibility of utilizing the statistical concept of autocorrelation in the analysis of vibration records of civil engineering structures. The first part of the paper is devoted to a background review of essential theory. The suitability of the autocorrelation technique as a method for predicting structural dynamic properties is then tested by comparing the period and damping values derived from analyses of the random vibrations recorded in different structures, with the known dynamic properties of these structures. This study was performed by simulating structural models on an electronic analog computer in which random inputs were generated by a "white noise" source; the recorded outputs were then processed for autocorrelation with the aid of a digital computer. Comparable studies were also carried out with records of wind vibration measured in real structures.

2. REVIEW OF THEORY

2.1 Autocorrelation Functions and Relationships:— The ability of the autocorrelation process to predict the physical properties of a mechanical system is related to the fact that the form of an autocorrelation response function depends on the nature of the system through which an input signal has passed. This relationship can be exhibited in the following manner.

A linear time invariant system may be specified by its impulse response or weighting function, $h(t')$, which is defined as the system response at t' time units after the occurrence of a unit impulse of the idealized or Dirac delta type. In a physically realizable system, the output at time t, $y(t)$, due to an arbitrary input, $x(t-t')$, is therefore given by the superposition integral

$$y(t) = \int_0^{\infty} h(t')\, x(t-t')\,dt' \qquad (1)$$

The response function for a single degree of freedom system having an undamped natural frequency, ω, and an underdamped fraction of critical damping, ξ, is known to be (8):

$$h(t') = \frac{e^{-\xi\omega t'}}{\omega\sqrt{1-\xi^2}} \sin\sqrt{1-\xi^2}\,\omega t \qquad \text{for } t' \geq 0$$

$$= 0 \qquad \text{for } t' < 0 \qquad (2)$$

Equation (2) is essentially an oscillation, at frequency $\omega\sqrt{1-\xi^2}$, which exhibits an amplitude envelope whose decay is dependent on ω and ξ.

Let the input $x(t-t')$ be a random disturbance possessing stationary and Gaussian properties and having a zero mean. If the symbol $\langle\ \rangle$

[8] See, for example, Crandall, S. H. and Mark, W. D., "Random Vibrations in Mechanical Systems," Academic Press, 1963.

denotes a time average of the expression within the angular brackets, the average output of the product of a pair of functions given by equation (1) and separated by a time interval, τ, is

$$\langle y(t)y(t+\tau) \rangle = \langle \int_0^\infty \int_0^\infty h(t_1')h(t_2')x(t-t_1')x(t+\tau-t_2')dt_1' dt_2' \rangle \qquad (3)$$

where t_1' and t_2' are dummy variables of integration.

For any general stationary time function, z(t), the mathematical expression

$$R_z(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} z(t)z(t+\tau)dt = \langle z(t)z(t+\tau) \rangle \qquad (4)$$

defines the autocorrelation function of the variable z. T is the averaging time and τ the correlation time interval, which is held constant during averaging. Using this notation, equation (3) may be rewritten as

$$R_y(\tau) = \int_0^\infty \int_0^\infty h(t_1')h(t_2') R_x(\tau+t_1'-t_2')\, dt_1'dt_2' \qquad (5)$$

where $R_x(\tau+t_1'-t_2')$, is the autocorrelation function of the excitation with time interval $(\tau+t_1'-t_2')$ and $R_y(\tau)$ is the autocorrelation function of the system response, which is seen to depend on the input and on the dynamic properties of the system as specified by ω and ξ through equation (2). If R_x and R_y are known, or can be measured, these dynamic properties may be established.

For the case of an ideal, "white-noise" excitation, which is characterized by a constant spectral density, S, the input autocorrelation function is (9),

$$R_x(\tau + t_1' - t_2') = 2\pi S\, \delta(\tau + t_1' - t_2') \qquad (6)$$

where δ is the Dirac delta function. Upon substitution of equations (6) and (2) into equation (5), the corresponding output autocorrelation, which is an even function of τ, takes the form

[9]Bendat, J. S. "Principles and Applications of Random Noise Theory," John Wiley, 1958.

$$R_y(\tau) = \frac{\pi S}{2\xi\omega^3}\left[e^{-\xi\omega\tau}\left(\cos\sqrt{1-\xi^2}\,\omega\tau + \frac{\xi}{\sqrt{1-\xi^2}}\sin\sqrt{1-\xi^2}\,\omega\tau\right)\right] \quad (7)$$

for $\tau > 0$.

The above analysis reveals that this output autocorrelation function combines exponential decay with periodicity; indeed, we recognize that the expression within the square bracket of equation (7) defines the undistorted, damped, free vibrations of the system. Therefore, application of the method of autocorrelation analysis, as represented by equation (4), to an output record of random response, provides a $R(\tau)$ versus τ decay curve from which the dynamic structural properties, ω and ξ, can be estimated.

2.2 Autocorrelation Approximations:— A practical and approximate solution to equation (4) can be obtained by treating the experimental data as being composed of N discrete values, separated by equally spaced time intervals λ, and forming the mean lagged products

$$R_z(m) = \frac{1}{N-m}\sum_{k=1}^{N-m} z(k)z(k+m) \quad (8)$$

for $m \geq 0$. The lag numbers, m, are selected to correspond to whole multiples of λ and are related to the desired correlation times, τ, through $\tau = m\lambda$. This equation loses its reliability for large m/N ratios.

Equation (8) is the normal or unbiased definition used to approximate the autocorrelation function. However, Parzen (10) has shown that a biased estimate, in which the divisor is simply taken as N, has a smaller mean square error than the unbiased estimate. A more reliable form for equation (8) is therefore

$$R_z(m) = \frac{1}{N}\sum_{k=1}^{N-m} z(k)z(k+m) \quad (9)$$

The differences between equations (8) and (9) will be small for $m/N \ll 1$.

At this point it is advantageous to introduce the notion of the autocorrelation coefficient. This is a normalized autocorrelation function which, for a zero mean stationary process, is defined as

[10] Parzen, E., "Mathematical Considerations in the Estimation of Spectra," Stanford Univ. Tech. Rept. No. 3(Contract DA-04-200-ORD-996)Aug. 1960.

$$\rho_z(m) = \frac{\langle z(k)z(k+m)\rangle}{\langle z^2(k)\rangle} = \frac{R_z(m)}{\sigma_z^{\,2}} \qquad (10)$$

where $\sigma_z^{\,2}$ is the variance of the function. When dealing with equi-spaced discrete data, equation (10) may be approximated by

$$\rho_z(m) = \frac{\sum_{k=1}^{N-m} z(k)z(k+m)}{\left[\sum_{k=1}^{N-m} z^2(k) \sum_{k=1}^{N-m} z^2(k+m)\right]^{\frac{1}{2}}} \qquad (11)$$

in which the denominator is taken as a running mean variance in preference to the assumption of a constant variance.

Correlation measurements derived from an application of either equations (8), (9) or (11) are influenced by the size of the data sample, N, used in the analysis. However, the variance of the coefficient ρ_z with time, and hence with sample size, is smaller than the corresponding variance of the function R_z. This is qualitatively demonstrated in Fig. 1, which reproduces, to the same scale, records of comparative correlations performed with an analog computer. Because of this superior statistical behavior, the coefficient form of the auto-correlation was favored in the following prediction studies and equation (11) is used in subsequent data analysis.

Equations (9) and (11) define autocorrelation in forms which are convenient for investigation by digital computers. When hand computation is planned, if one is willing to accept an analysis based on equation (9), the work involved can be relieved considerably by replacing z(k) by its sign alone (4), that is by sgn z(k). This reduces the calculations of sums of products, which represent the major computation effort, to that of sums. As shown in the appendix, the validity of this simplification relies on the input excitation being Gaussian.

In multi-degree of freedom systems, the presence of high frequency vibrations may mask the individual character of the fundamental response. When these higher modes are unimportant, it is likely that an analysis of the preceding type will be sufficient. In other cases, the component mode shapes may have to be separated. A Fourier transform of the autocorrelation into a power spectrum (9) may be employed to provide the basic information on the physical properties of the contributing periodicities.

3. APPLICATIONS

3.1 Application to Analog-Modelled Structures: — The suitability of the autocorrelation method as a prediction mechanism was first examined by analyzing the records of simple systems in response to random disturbances. For this purpose, six different single-degree of freedom structures were simulated on an electronic analog computer and excited by a random input supplied by an Elgenco Type 321A Low Frequency Noise Generator. This noise source exhibited a Gaussian (normal) amplitude distribution and provided a constant power spectral density of approximately 0.6 volts^2/cps over the frequency range 0-100 cps.

The system constants, ω and ξ, were approximated by selective settings of the decade switches of interconnected Philbrick K5-U Universal Linear Operators; these electronic units simulate the mechanical systems in the analog. Accurate estimates of the damping were obtained from logarithmic decrement measurements of free vibration-decay records; these were secured for each system by introducing initial displacements in the analog circuit and automatically monitoring the response on a direct writing recorder. The natural frequency of each model was carefully established by first observing the resonant response to a variable frequency sinusoidal forcing function and then measuring the corresponding input frequency with an electronic counter. The results of these calibrations are summarized in Fig. 2. It may be seen that three of the models had periods of 0.063 secs. and damping values which varied between 2 and 12% of critical; the corresponding values for the three remaining systems were 0.12 secs and 5 to 20% of critical.

The response of each system to the random input signal was automatically recorded on a Sanborn direct writing oscillograph. An example of a typical output record is reproduced in Fig. 1b. Response values at equally spaced time intervals were carefully read from such records with the aid of a hand glass. A digital computer was then employed for the solution of the autocorrelation coefficient defined by equation (11), using N discrete observations and m equally spaced lags. (N and m variable).

The results of the analyses of the tests are presented in Figs. 3 to 8, which show the correlograms obtained by plotting the autocorrelation coefficients as ordinates against the correlation times as abscissa. These correlograms appear as damped exponentials with uniform distances between successive peak amplitudes. They form the basis for establishing the dynamic characteristics of the test structures. Damping values were estimated by measuring the logarithmic decrements of the curves; the periods were evaluated by measuring the intervals between zero crossings of the time axis. The experimental results secured in this manner are included in Fig. 2 for convenient comparison with the actual properties of the model structures. The tabulations in column (a) show the range of period and damping values found when considering a number of different successive cycles for each curve; those in column (b) correspond to the results found using only the first cycle of the appropriate correlogram. These different approaches yield period estimates which are essentially alike, but result in significant differences in the damping estimates. For reasons outlined below, the damping derived from initial cycle decrements provides the more accurate indication of the true damping. With this in mind, it may be seen that the autocorrelation process can provide a reliable estimate of the period and a measure of the order of damping present in single-degree of freedom structures excited by "white noise" disturbances. It should be noted, however, that the correlograms do not damp in the expected exponential manner as the time lag increases and, as a result, the damping estimated from other than the first cycle decrement may be entirely misleading.

In a record of finite length, the autocorrelations cannot be estimated for arbitrarily long lags. Blackman and Tukey (11) have recommended that the truncation point on τ should normally be taken at from 5 to 10% of the record length. Jenkins (12) is of the opinion that correlation may sometimes behave very well for lags up to 30% of the total observation sample. Some of the preceding correlograms exhibit a slightly unstable behavior in the 10% truncation zone; the instability becomes more pronounced if the time interval is increased beyond this region or, alternatively, if the sample size is reduced. As the initial or small time lag region of the correlogram is based on the largest number of observations and is the most stable, the amplitude decrement measured during the first cycle of the curve would be expected to yield the most reliable estimate of the damping value.

An indication of the influence of sample size, N, on the appearance of the correlogram is provided by the curves illustrated in Figs. 9-11, which are typical for the series. For large lag-to-sample size ratios, the coefficients do not damp in the expected manner and may oscillate irregularly. However, it should be noted that meaningful estimates of the period can still be obtained from curves derived from relatively small sample sizes. On the other hand, the very nature of the fluctuations preclude the securing of reliable damping estimates from samples containing a small number of observations.

3.2 Application to Real Structures:— The results of the analog tests seemed sufficiently promising to warrant proceeding with a comparable study involving real structures. A partially constructed steel frame building and a completed nine-story steel frame building were selected to provide the data for this phase of the investigation. The dynamic properties of the latter structure were accurately known from existing steady-state forced vibration tests (13) and hence a good standard was available for judging the reliability of any results which might be secured.

In the case of the partially constructed building, the autocorrelation analysis was performed on data taken from an existing record of wind-induced vibrations. This material was on file in the laboratory, having been measured at some earlier date with a portable Sprengnether seismograph located at the eighth floor of the nineteen-story structure. Similar information was not available for the completed steel frame building,

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[11] Blackman, R. B., and Tukey, J. W., "The Measurement of Power Spectra," Dover Publications Inc., 1959.

[12] Jenkins, J. M., "General Considerations in the Analysis of Spectra," Technometrics, Vol. 3, No. 2, May, 1961.

[13] Nielsen, N. Norby, "Dynamic Response of Multi-Story Buildings," Ph. D. Thesis, Calif. Inst. of Tech., Pasadena, Calif., 1964

and a response record of the vibrations due to strong wind excitation was secured at the ninth floor with an instrumentation system which employed a Statham 1/4g A-4 accelerometer for detecting the motion, a Brush RD 5612-11 carrier amplifier for its subsequent amplification, and a CEC model 5-124 direct writing oscillograph for automatic and continuous monitoring of the response. Sample tracings of the measured building vibrations are reproduced in Fig. 12.

The correlograms derived from an analysis of the data are shown in Fig. 13. Any application of the theory of section 2.1 tacitly implies that the input is characterized by a constant power spectral density and that the response possesses stationary properties. When considering wind induced vibrations, the questionable nature of the assumptions necessary to satisfy these requirements should be appreciated.

The correlogram for the partially constructed building is shown in Fig. 13a. It is characterized by an oscillation whose amplitudes alternatively grow and diminish, although the correlogram period itself remains nearly constant. This behavior is frequently noticed in analyses involving inadequate sample sizes (14). Satisfactory damping estimates cannot be secured from a curve exhibiting this trend. However, the regular oscillations of the correlogram permit a more accurate period estimate of the vibrating system to be made than would normally be possible by direct observation of the original vibration data. Autocorrelation analysis reveals a building period of 1.81 secs. An estimated value of 1.78 secs. was assigned to this parameter on the basis of direct observation of the wind record.

Figure 13b, which pertains to the completed structure, provides an example of the appearance of a correlogram of a vibration record whose wave form contains both periodic and random components. For such cases, the combined autocorrelation coefficient is equal to the sum of the autocorrelation coefficients of the individual components. If the delay time is sufficiently long, the contribution of the random part to the total would appear as a damped exponential; periodic contributions would persist as periodic functions. In the example under consideration it is possible to detect the presence of two high frequency oscillations superimposed on a base vibration of lower frequency. The latter corresponds to the autocorrelation coefficient resulting from the random wind excitations induced in the building, and its period is a measure of the fundamental period of the building. By scaling the distance between peak spikes this is established as 0.95 secs., which is in good agreement with the steady-state test result of 0.88 secs. obtained for this structure. The measured frequencies of the high frequency oscillations are 300 cps and 90 cps. These two periodic components result from the apparent contamination

[14] Kendall, M. G., "Contributions to the Study of Oscillatory Time-Series," Cambridge Univ. Press, 1946.

of the basic wind data by extraneous "noise", which can be attributed to motors and air conditioning fans operating in the building at nominal rated speeds of 290 cps and 80 cps respectively. Although an estimate of building damping is not directly recoverable from a multi-component correlogram of this type, it is possible that a Fourier transformation of the autocorrelation into a power spectrum would provide basic information on this parameter, as well as clearer evidence of the frequencies of the contributing components.

4. Summary and Concluding Remarks: — The possibility of employing autocorrelation procedures for the prediction of structural dynamic properties has been demonstrated. When direct and simpler methods for securing these properties are ineffective, such as in the case of large dams or other massive structures, the application of this statistical technique offers some promise and is worthy of consideration. However, caution should be exercized in the interpretation of an autocorrelation analysis since the results obtained are strongly affected by practical limitations.

The influence of sample size on the appearance of the correlogram has been illustrated and is particularly emphasized. Small sample sizes may contain trends which are reflected in the correlogram and which, in turn, may obscure or give a false impression of the actual characteristics of the underlying phenomena.

The damping exhibited by a correlogram is especially susceptible to the sample size used in the analysis. For small data fields it is commonly observed that correlograms do not damp in the expected manner; this discrepancy diminishes as the sample size increases. The initial cycle decrement of a correlogram appears to provide a more reliable index of damping than that offered by later cycle decrements. On the basis of the studies reported in this paper, it must be concluded that damping estimates derived from an autocorrelation analysis based on small data fields, such as have been reported in the literature for certain civil engineering structures, should be treated with caution. In such cases, the apparent tendency for correlograms to damp in the expected manner is likely more the exception than the rule.

The correlogram period is not as sensitive to sample size as the damping, and reasonable estimates of the period can usually be derived from a relatively small sample despite the accompanying amplitude fluctuations of the correlogram.

Unless automatic equipment is available for digitizing and correlating large samples of vibration data, practical working limitations impose severe restrictions on the use of the autocorrelation technique for other than period estimates.

Acknowledgments

The authors are grateful to Professors D.E.Hudson and T.K. Caughey, California Institute of Technology, for many helpful discussions and suggestions relating to this study.

APPENDIX

If hand computation of the autocorrelation function is contemplated, the use of a simplification which permits the z(k) term in equation (9) to be replaced by its sign alone, sgn z(k), is recommended. This alteration reduces the tedious nature of the arithmetic by replacing a computational scheme involving sums of products with one involving sums only. The validity of this substitution, which depends on the input excitation being Gaussian, is shown by the following theoretical analysis of a zero mean stationary random process which continues for an infinite length of time.

For ease of notation let the abbreviations z_1, z_2, ρ and σ signify $z(t)$, $z(t+\tau)$, $\rho_z(\tau)$ and σ_z respectively. Now express the mean or expectation of the product sgn $z_1 \cdot z_2$ by the double integral

$$\langle \operatorname{sgn} z_1 z_2 \rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \operatorname{sgn} z_1 \cdot z_2 \, p(z_1, z_2) dz_1 dz_2 \qquad (12)$$

When the distribution of z_1 and z_2 is Gaussian, $p(z_1, z_2)$, which is the joint probability density function of the paired values z_1 and z_2, is defined (8) as

$$p(z_1, z_2) = \frac{1}{2\pi\,\sigma^2(1-\rho^2)^{\frac{1}{2}}}\, e^{-\frac{1}{2(1-\rho^2)\sigma^2}(z_1^2 - 2\rho z_1 z_2 + z_2^2)} \qquad (13)$$

For a linear system, this condition is satisfied so long as the input is Gaussian.

To perform the calculations described by equation (12), first complete the square on z_2, then make the substitution

$$\sigma_1^2 = \sigma^2(1-\rho^2) \qquad (14)$$

and rewrite equation (13) as

$$p(z_1, z_2) = \frac{1}{2\pi\sigma\,\sigma_1}\, e^{-\frac{z_1^2}{2\sigma^2}} \cdot e^{-\frac{(z_2-\rho z_1)^2}{2\sigma_1^2}} \qquad (15)$$

Next set equation (15) into equation (12) to obtain

$$\langle \operatorname{sgn} z_1 z_2 \rangle = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \frac{z_2}{\sqrt{2\pi}\,\sigma_1}\, e^{-\frac{(z_2-\rho z_1)^2}{2\sigma_1^2}} dz_2 \right] \frac{\operatorname{sgn} z_1}{\sqrt{2\pi}\sigma}\, e^{-\frac{z_1^2}{2\sigma^2}} dz_1 \qquad (16)$$

After making the substitution,

$$z_2 - \rho z_1 = x \tag{17}$$

the integral in the above parenthesis reduces to

$$\int_{-\infty}^{\infty} (x + \rho z_1) \frac{e^{-\frac{x^2}{2\sigma_1^2}}}{\sqrt{2\pi}\,\sigma_1} dx = \rho z_1 \tag{18}$$

and equation (16) then takes the form

$$\langle \text{sgn } z_1 \, z_2 \rangle = \int_{-\infty}^{\infty} \frac{\rho z_1 \text{ sgn } z_1}{\sqrt{2\pi}\,\sigma} e^{-\frac{z_1^2}{2\sigma^2}} dz_1 \tag{19}$$

with solution

$$\langle \text{sgn } z_1 \, z_2 \rangle = \frac{2\rho}{\sqrt{2\pi}\,\sigma} \int_{0}^{\infty} z_1 \, e^{-\frac{z_1^2}{2\sigma^2}} dz_1 = \sqrt{\frac{2}{\pi}}\,\sigma\rho \tag{20}$$

Finally, after introducing equation (10), we are led to the simple relationship

$$\langle \text{sgn } z_1 \, z_2 \rangle = \langle z_1 \, z_2 \rangle \sqrt{\frac{2}{\pi}} \cdot \frac{1}{\sigma} \tag{21}$$

in which we note the presence of the autocorrelation function in conjunction with a multiplication constant involving the standard deviation σ.

In the prediction problem under consideration, only zero crossings and ordinate ratios are involved so that there is no need to evaluate this constant; it is the relative shape of the autocorrelation function which is of interest. Thus, the simplification resulting from a substitution of sgn z_1 for z_1 relieves the computational labor without theoretically affecting the final outcome of the analysis. However, in actual practice, the record is of finite length and statistical differences exist between $\langle \text{sgn } z_1 \, z_2 \rangle$ and $\langle z_1 \, z_2 \rangle$ due to fluctuations in σ. These differences may not be serious when the number of data points is reasonably large, but for small sample sizes an analysis based on $\langle z_1 \, z_2 \rangle$ is to be preferred.

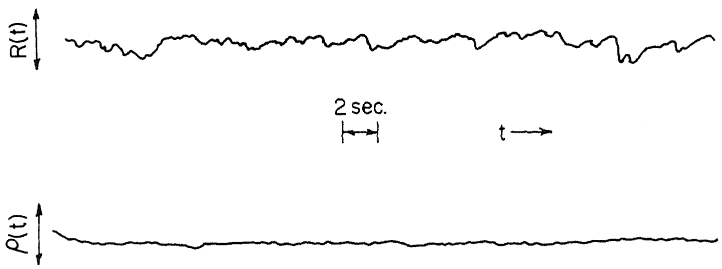


Fig. I(a) SAMPLES OF VARIATION OF CORRELATIONS WITH TIME (for system with ω=100 rad./sec, ξ=10% and $\tau = \pi/\omega$)

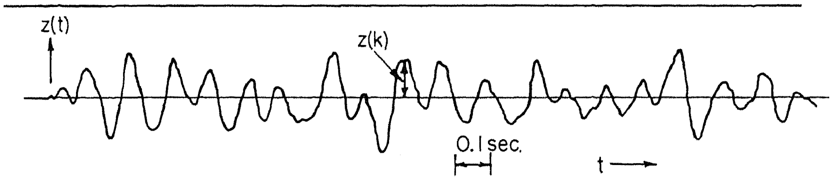


Fig. I(b) SAMPLE OF RESPONSE OF MODEL STRUCTURE TO RANDOM "NOISE" EXCITATION (TEST NO.6)

TEST NO.	TRUE PERIOD (SECS.)	CORRELOGRAM PERIOD (SECS.) (a)	(b)	TRUE DAMPING (% CRITICAL)	CORRELOGRAM DAMPING (% CRITICAL) (a)	(b)
1	0.0633	0.062 - .063	0.063	2.6	3.0 - 4.2	4.2
2	.0633	.061 - .064	.064	4.4	2.5 - 4.1	3.4
3	.0633	.060 - .064	.063	12.0	3.1 - 12.9	12.9
4	.1282	.122 - .126	.124	5.1	5.9 - 8.8	5.9
5	.1282	.127 - .134	.130	10.9	3.8 - 12.9	12.9
6	.1282	.112 - .120	.112	20.3	3.0 - 23.1	23.1

(a) Range of values as determined from decrements of a number of cycles along correlogram.

(b) Values as determined from first cycle decrement only.

FIG. 2. SUMMARY OF MODEL STRUCTURE TEST RESULTS

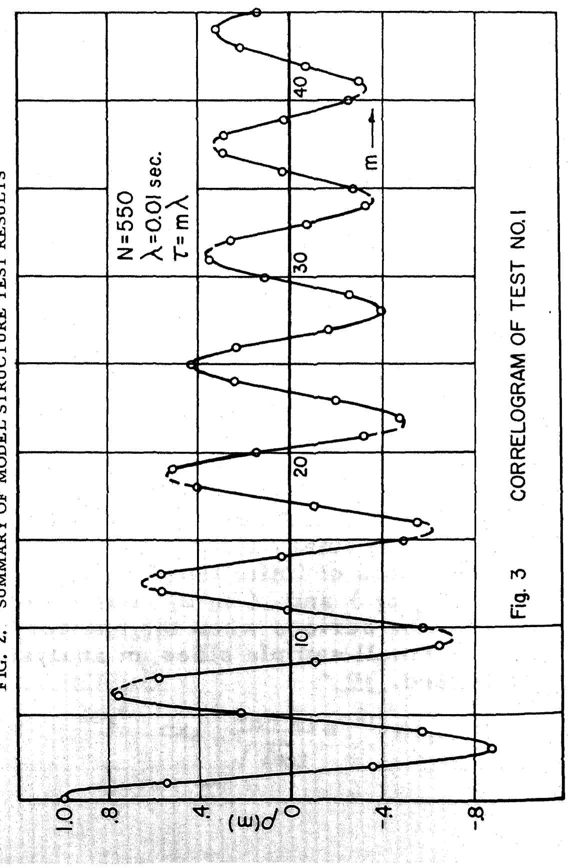


Fig. 3 CORRELOGRAM OF TEST NO.1

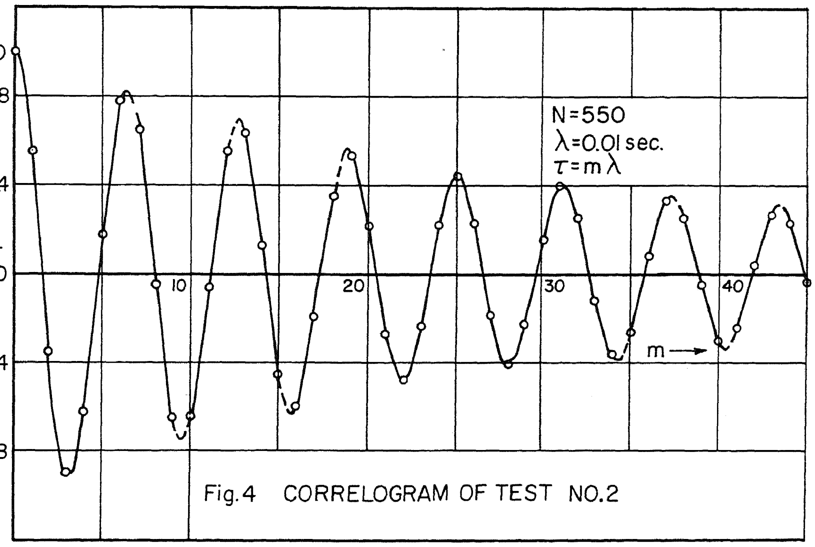


Fig. 4 CORRELOGRAM OF TEST NO.2

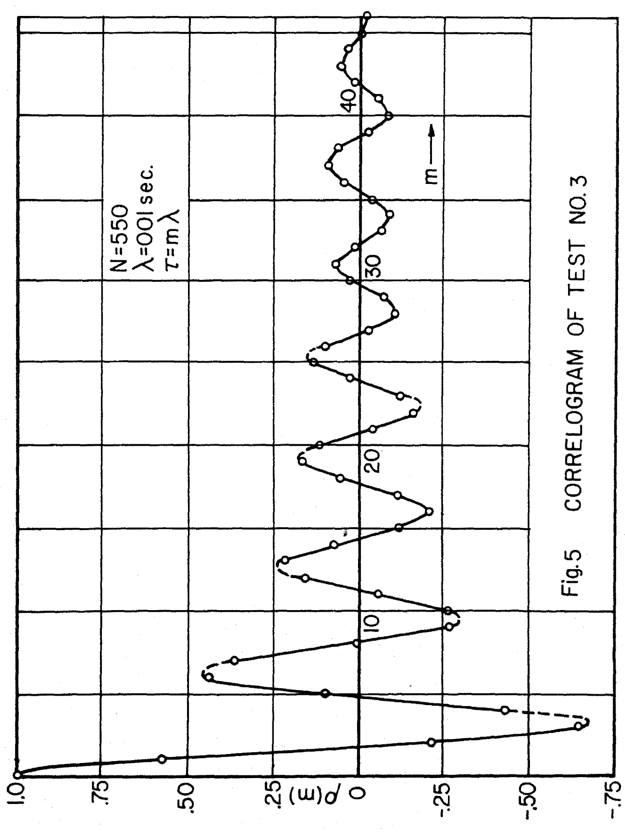


Fig. 5 CORRELOGRAM OF TEST NO.3

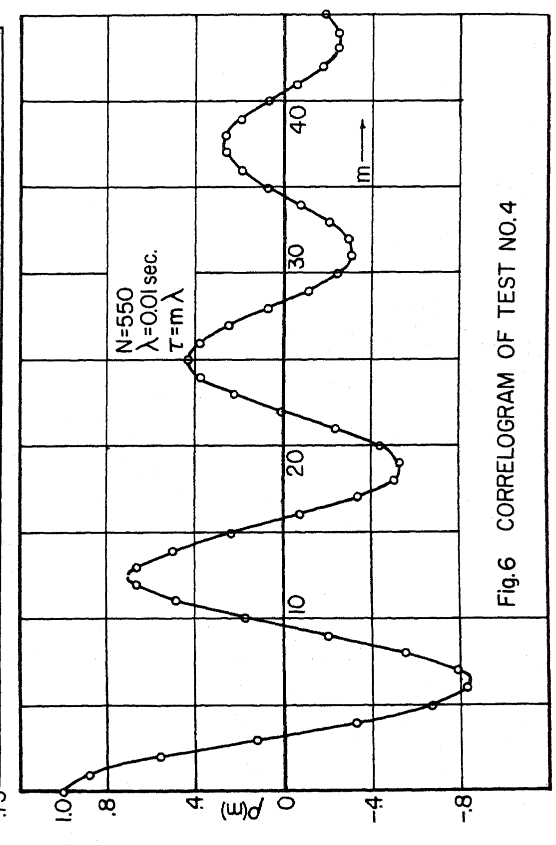


Fig. 6 CORRELOGRAM OF TEST NO.4

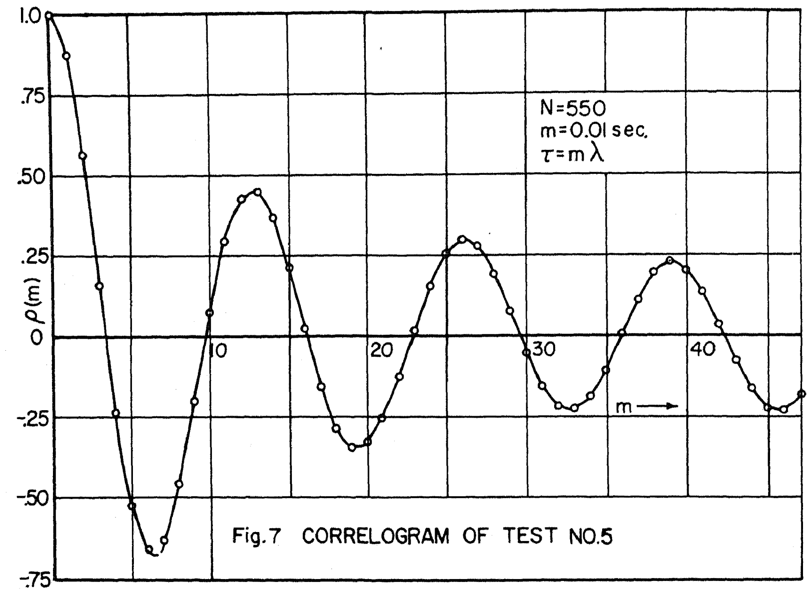


Fig.7 CORRELOGRAM OF TEST NO.5

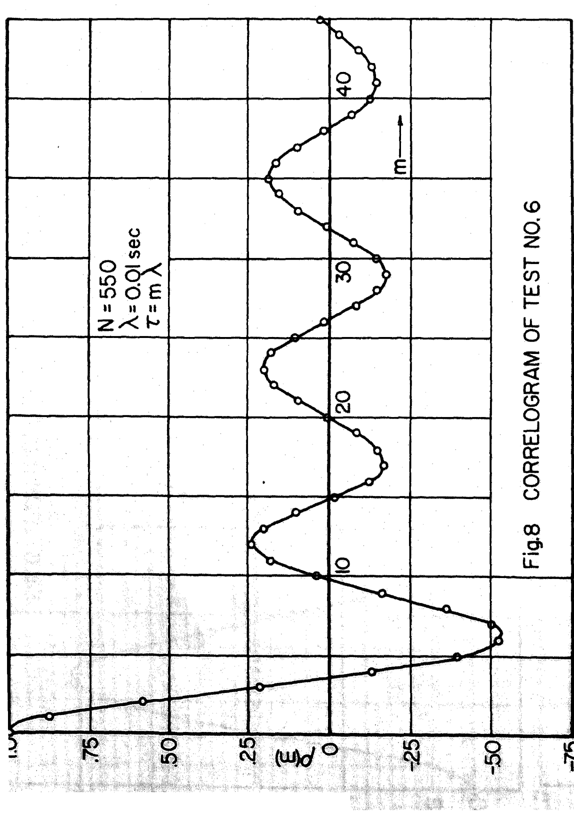


Fig.8 CORRELOGRAM OF TEST NO.6

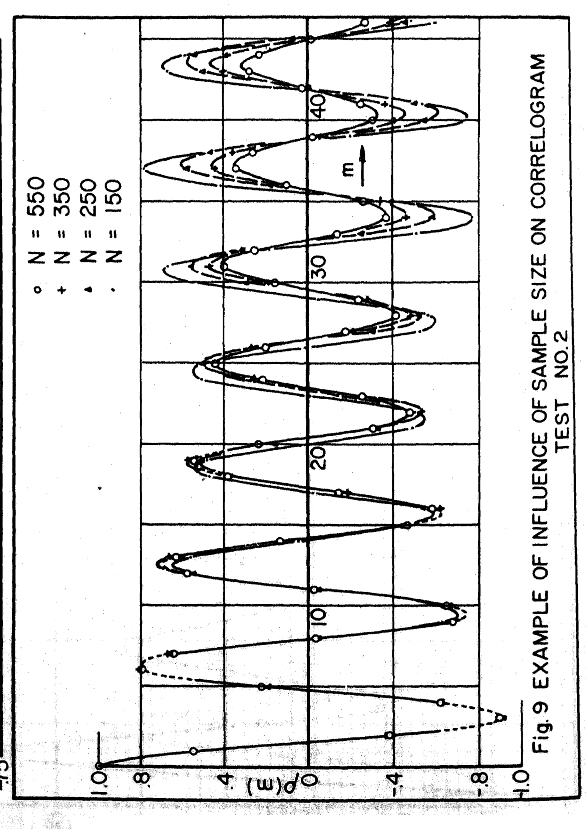


Fig. 9 EXAMPLE OF INFLUENCE OF SAMPLE SIZE ON CORRELOGRAM TEST NO.2

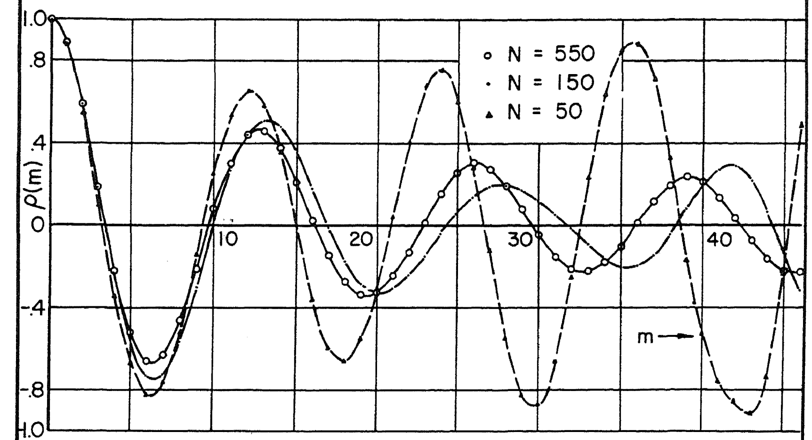


Fig. 10 EXAMPLE OF INFLUENCE OF SAMPLE SIZE ON CORRELOGRAM TEST NO. 5

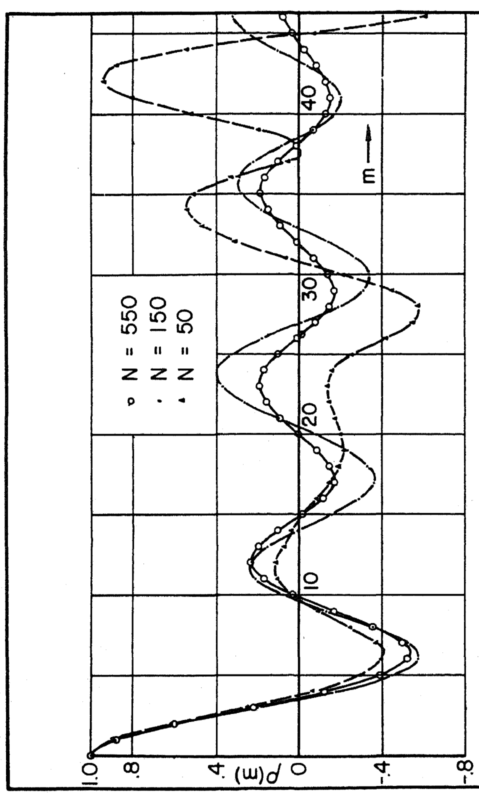
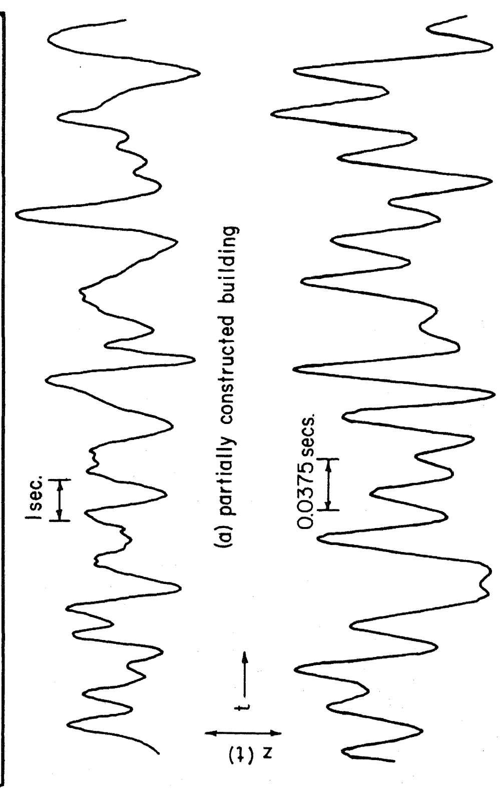


Fig. 11 EXAMPLE OF INFLUENCE OF SAMPLE SIZE ON CORRELOGRAM TEST NO. 6



(a) partially constructed building

(b) completed building

Fig. 12 SAMPLE OF RESPONSE OF REAL STRUCTURES TO WIND EXCITATION

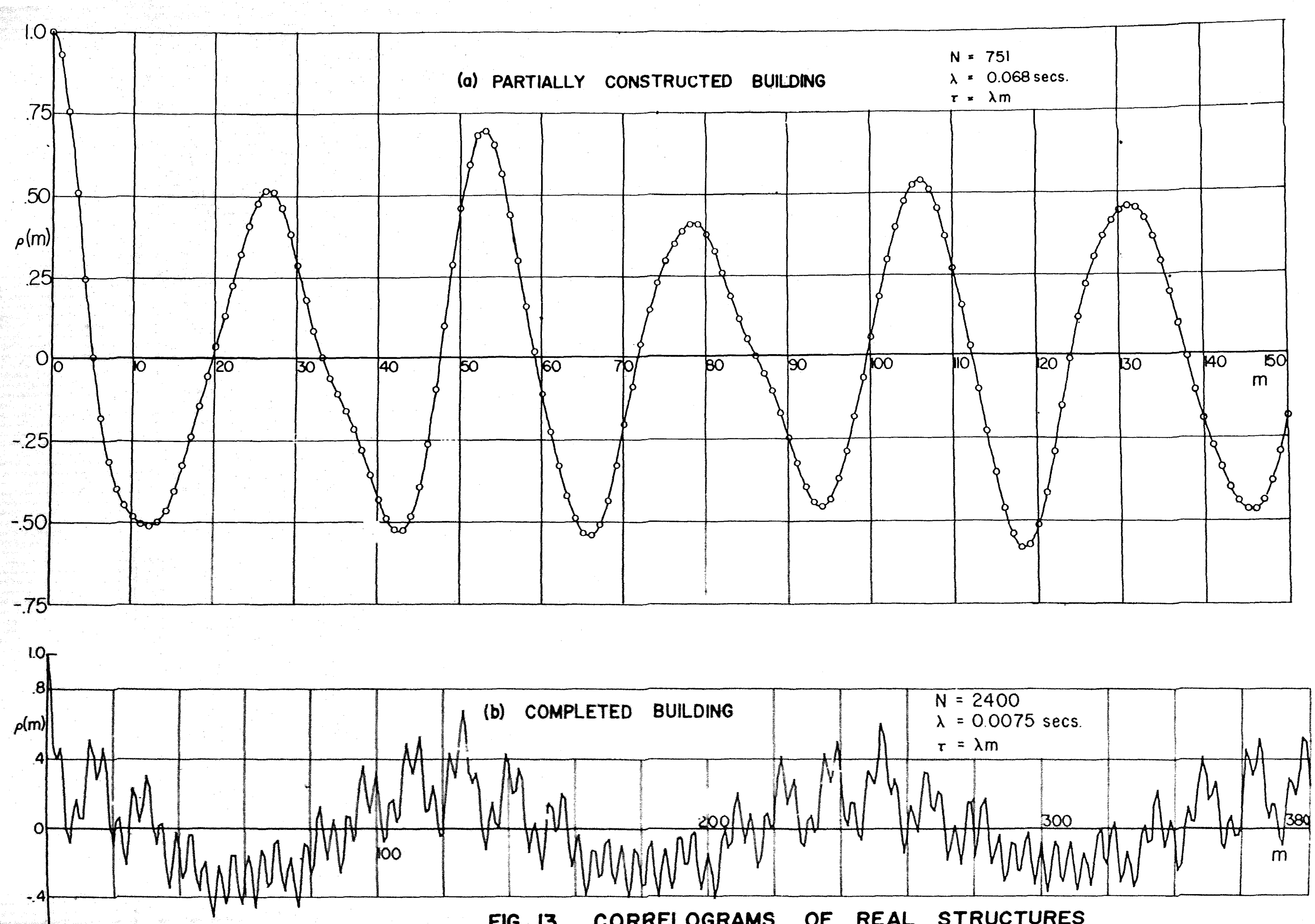


FIG. 13. CORRELOGRAMS OF REAL STRUCTURES

DETERMINATION OF STRUCTURAL DYNAMIC PROPERTIES BY STATISTICAL ANALYSIS OF RANDOM VIBRATIONS

BY S.CHERRY AND A.G. BRADY

QUESTION BY: G.W. HOUSNER - U.S.A.

Would it be feasible to obtain a high degree of accuracy by using a very long record?

AUTHORS' REPLY: Yes, but this can only be achieved practically by using automatic correlation equipment for processing the large amounts of data associated with long records. Otherwise, the man-hours involved in the manual reading of such vibration records would become excessive, and this would impose practical working limitations on the auto-correlation technique.

INELASTIC EARTHQUAKE RESPONSE OF TALL BUILDINGS

by

Ray W. Clough*, K.L. Benuska** and E.L. Wilson***

ABSTRACT

A digital computer procedure for evaluating the inelastic forces and deformations developed in each column and girder of any arbitrary building frame subjected to earthquake motions is described. A special bi-linear moment-rotation property may be prescribed independently for each member. The distribution of maximum deformations and forces produced in two different 20 story building frames by the El Centro 1940 earthquake, computed by this program, are discussed and compared with results obtained in a purely elastic analysis. Three different earthquake intensities, approximately 2/3, 3/3 and 4/3 of El Centro, are considered.

INTRODUCTION

Great advances have been made during recent years toward a more complete understanding of the behaviour of structures subjected to earthquake excitation. The introduction two decades ago of the elastic response spectrum concept(1), which provides a convenient means for representing the elastic behaviour of simple structures, was followed by recognition of the fact that the forces predicted by such spectra far exceed normal design requirements(2). Because structures having much less strength than is prescribed by the spectral values were observed to have performed satisfactorily in rather severe earthquakes, it became apparent that the elastic response spectrum is not a direct measure of the significant earthquake behaviour of many structures. Even moderate earthquakes may be expected to produce inelastic deformations in typical buildings, and it is now understood that the plastic energy absorbed by the structure has a controlling influence on the deformation amplitudes which it may develop.

Recognition of the important role played by ductility in the earthquake performance of structures led to initiation of research programs directed toward the quantitative study of simple elasto-plastic systems subjected to earthquake motions(3,4,5). These investigations demonstrated that the maximum structural displacement amplitudes produced by a given earthquake tend to be reasonably independent of the yield strength of the structures(3). In other words, the maximum displacement in a simple structure was found to be about the same whether it remained elastic or yielded.

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