BEHAVIOUR OF AN ELASTO-PLASTIC OSCILLATOR

ACTED BY RANDOM VIBRATION

by

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ABSTRACT

Results concerning the elastic and elasto-plastic behaviour of structures under the action of earthquakes are obtained in the framework of the theory of stochastic processes.

By integrating the Fokker-Planck equation the statistics of the magnitudes involved are determined and the mean maximum values of the response are obtained for oscillators with different natural frequencies and different yielding factors.

1 - INTRODUCTION

The need of a reliable theoretical basis for the description of the behaviour of oscillators under the action of random loadings is for long recognised. This would allow a better understanding of the seismical behaviour of structures, and makes easier the interpretation of the experimental tests.

The analytical study performed being rather lengthy (1) it is only possible to present a short summary of the methods adopted and the results obtained.

2 - GENERAL

2.1 - Seismic action

From an engineering point of view, an earthquake can be regarded as an acceleration applied on the foundation of the structure. Its random nature excluding a priori a deterministic description in the time domain, naturally suggests the application of the theory of shochastic processes.

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From the numerical analysis, performed at our Laboratório, on several earthquake records two main reasonable conclusions can be drawn.

a) - The acceleration time function corresponds to a sampling, with a time duration T, obtained from a family of functions with the same power spectral density S(f). (2,3).

b) - The probability distribution of the instantaneous values is of Gaussean type (2).

2.2 - Structural response

To judge the dynamic behaviour of the structure, a statistical parameter must be judiciously sought. This parameter can be chosen as follows:

c) - The interesting statistical value of the random variable describing the dynamic behaviour of the structure is the mean maximum value in the time interval T. (4).

2.3 - Typical values

Curve I of fig. 1 shows the power spectral density of acceleration computed at our Laboratório for the N-S component of El-Centro, 1940, earthquake (2). Its mean value in the bandwidth of 0 to 5 Hz is $S_o = 675 \ cm^2 sec^{-4} Hz^{-1}$, this value being adopted as reference.

The time duration T is taken as 30 sec.which seems a reasonable estimate.

3 - ELASTIC BEHAVIOUR

3.1 - Synopsis of the analytic study

The theory of stochastic processes can be easily applied in this case of linear behaviour. From the differential equation of movement the transfer functions of relative displacement, velocity and acceleration are derived and the root mean square values of those variables calculated. It can be mathematically proved that the distribution of maxima practically coincides with the distribution of the envelope,so that the mean number of oscillations in the interval T can be obtained from this distribution (of the Rayleigh type).Those events being assumed as independent, the ratio between the mean maximum and the root mean square value is obtained in function of the natural frequency of the oscillator.

If the structure has several degrees of freedom, the technique of modal analysis applies and the oscillator is resolved into N oscillators with a single degree of freedom. It can be easily shown that a statistical weighting of the sum of the N solutions defines a local mean number of oscillations per second and so the problem is reduced to the previous one (1).

3.2 - Results obtained

It seems important to ascertain the influence of the particular shape adopted for the power spectrum density of the standard earthquake. So the computations of the maximum values of displacement and velocity were carried out for the three different spectral curves shown in fig.1. First were calculated the mean square values of displacement and velocity whose values can be represented with great accuracy by the formulas

$$\overline{z^2} = \frac{S_o}{8nw_o^3} \qquad \overline{\dot{z}^2} = \frac{S_o}{8nw_o}$$

where n is the percentual damping, w_o the natural angular frequency of the structure and S_o the power spectral density of the earthquake. Those formulas, which are correct for case III, are also valid for cases I and II provided $w_o < w_L$.

The ratio between the maximum mean value and the root mean square value versus frequency, in the time interval T, is presented in fig. 2. These values were computed for two values of T, 25 and 30 sec respectively, and, as can be seen, the results obtained are pratically coincident.

In fig. 3 and 4 the maximum mean values of displacement and velocity are presented.

3.3 - Discussion of the results

The particular shape of the power spectrum curve does not seem to be critical as the values obtained for three different hypotheses are practically the same. What is relevant however is the knowledge of the limit frequency value f_L itself as the response of the system appreciably decreases in this neighbourhood.

The values of the seismic coefficient can be computed from the mean maximum relative displacement and are presented in fig.5. The high values obtained exclude the possibility from a pratical point of view of designing the structures to behave elastically

4 - NON-LINEAR BEHAVIOUR

The limit case of an elasto-plastic behaviour is considered. The non-linear characteristic between restitution force and deformation is maintained but losses by hysteresis are replaced by viscous losses, the suitable choice of a viscous parameter being discussed.

4.1 - Synopsis of the analytic study

The mathematical study is based on the theory of stochastic processes of the Markov type in which the present problem can be included once the hypothesis of univocity is accepted.

The solution depends on a second-order partial differential equation called Fokker-Planck equation, whose integration defines the conditional probability density of transition between states of the system. From the asymptotic behaviour of the solution (stationary state) it is possible to find an integral equation of Abel's type giving the probability distribution of maxima. As the equivalence between hysteresis and viscous losses depends on the maxima, it is possible to find an equation determining the value of the viscous parameter to be adopted from a statistical point of view. Finally the mean maximum displacements are calculated and compared with the elastic values.

4.2 - Results of the study

Defining the earthquake by a power spectral density curve of type III (fig. 1), and the structure by its natural frequency w_o, its percentual damping in the elastic domain and the factor G (quotient of the limit elastic acceleration by $g = 980$ cm/s^2) the idealized restitution force per unit mass is defined in fig. 6.

The mathematical study being performed with dimensionless variables the following parameters are defined

$$x = \frac{z}{z_o}$$

$$D_o = \frac{w_o \, S_o}{4 \, G^2 . g^2}$$

$$\psi = \frac{1}{2} \cdot \frac{D_o}{n}$$

$$z_o = \frac{G.g}{w_o^2}$$

The value of D_o can be computed from the assumed characteristics of the earthquake and structure. Its substitution in fig. 7 allows the determination of ψ and n_{eq} (sum of the linear viscous damping with the equivalent viscous one corresponding to hysteresis).The value obtained from this graph allows to determine the root mean square value of the dimensionless displacement x from fig. 8. Finally the ratio between the mean maximum and the root mean square value is obtained from fig. 9.

Multiplying this value by Z_o, the mean maximum value of the relative displacement in the time interval T = 30 sec is determined. It is represented in fig. 10 for an earthquake with S_o = 675 $cm^2 sec^{-4} Hz^{-1}$ and a structure with n = 5% (elastic damping).

4.3 - Discussion of the results

Although the behaviour of a structure presents quite different characteristics in the elastic and the inelastic ranges it is important to note that deformations are roughly the same in both cases.

This shows that the judgement of the seismic behaviour of a structure basically rests on its capacity to undergo deformations.

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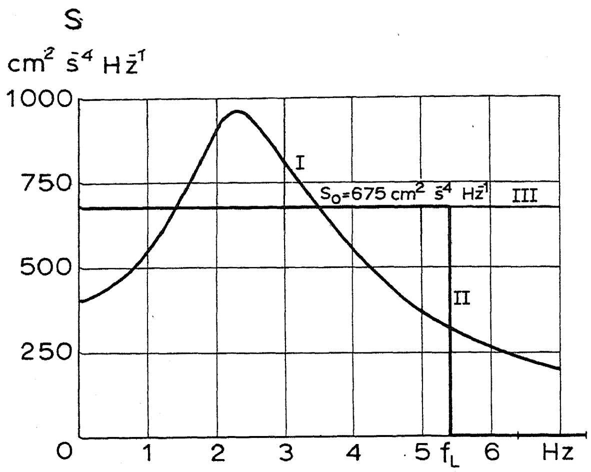

Fig. 1

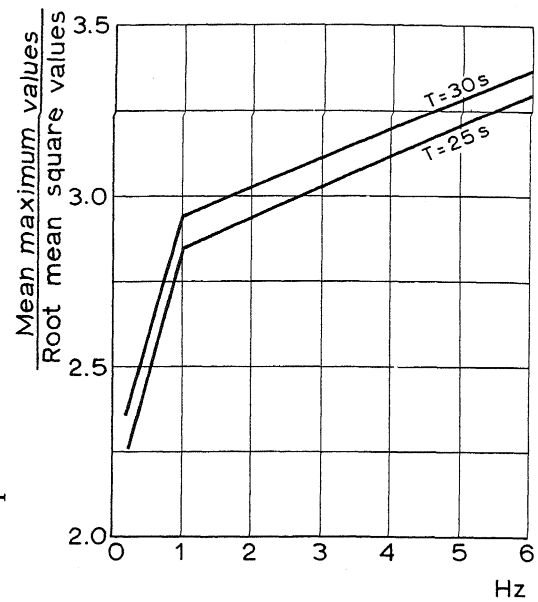

Fig. 2

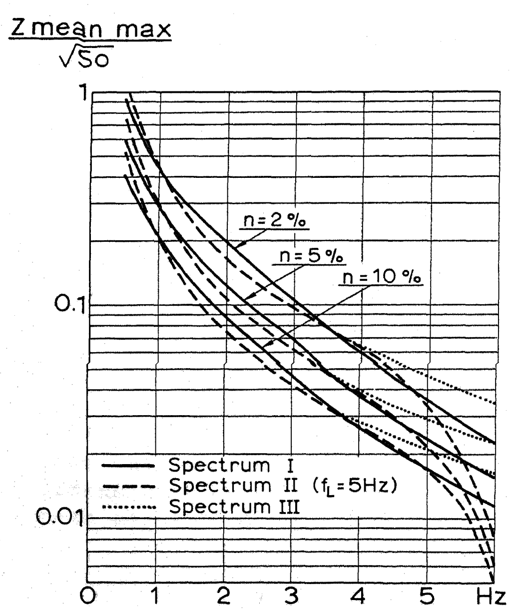

Fig. 3

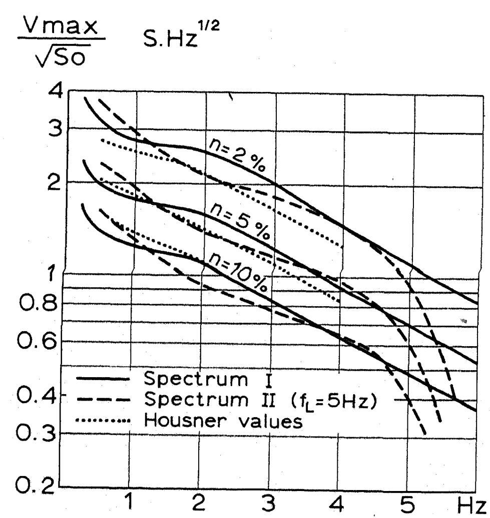

Fig. 4

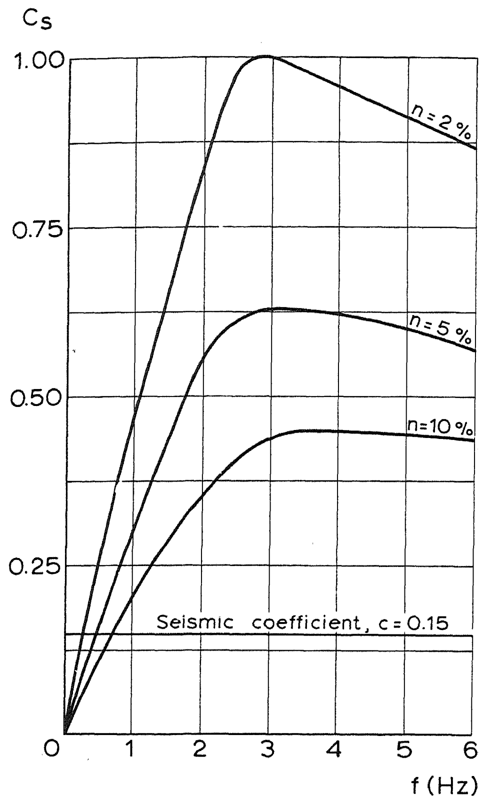

Fig. 5

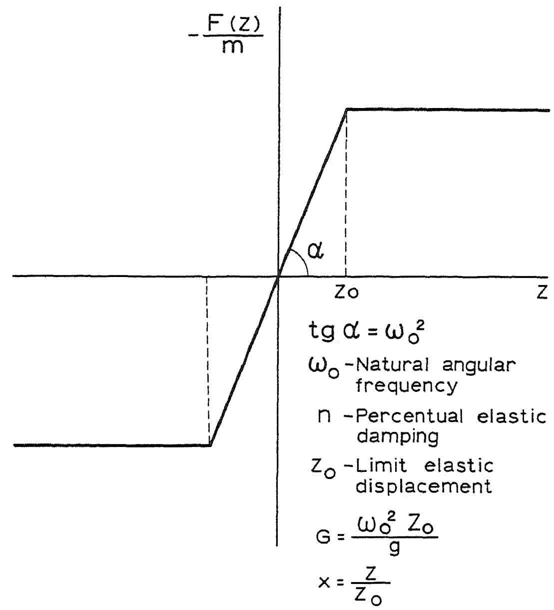

Fig. 6

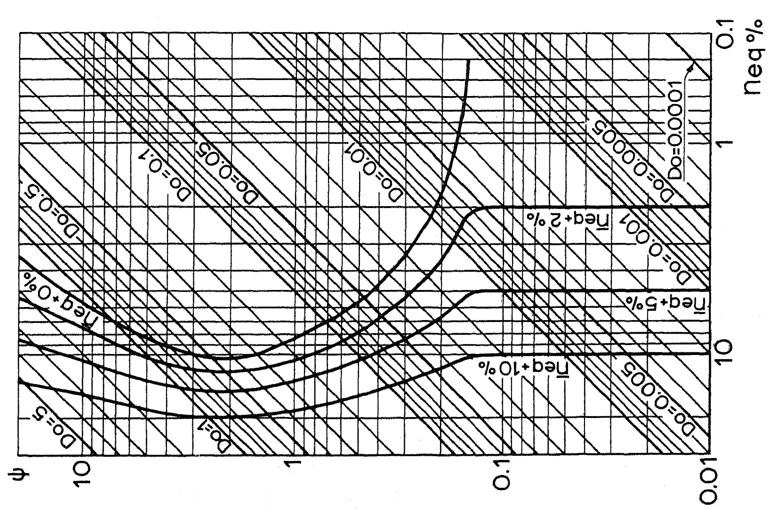

Fig. 7

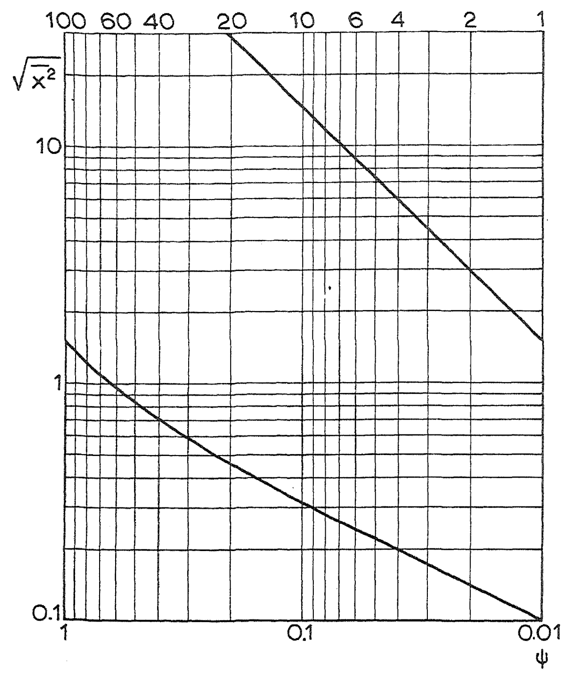

Fig. 8

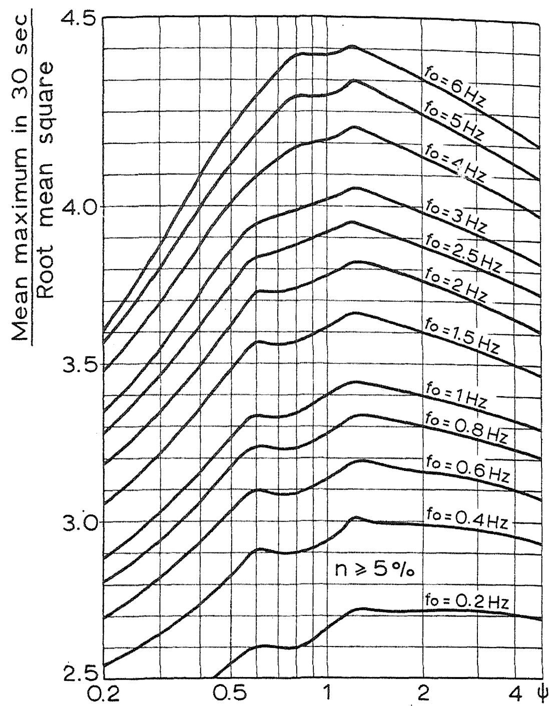

Fig. 9

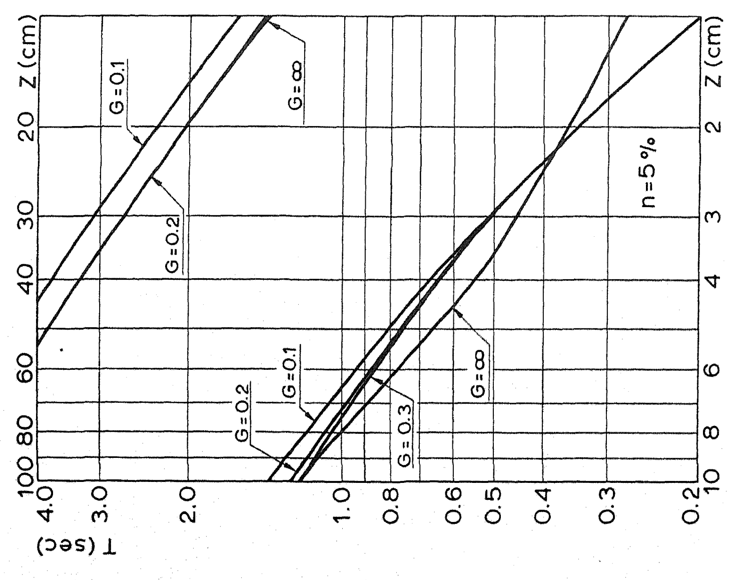

Fig. 10

BEHAVIOUR OF AN ELASTO-PLASTIC OSCILLATOR ACTED BY RANDOM VIBRATION

BY J.M.J. PEREIRA

QUESTION BY: G. GRANDORI - ITALY.

At the Politecnico of Milan we carried out a research project quite similar to that carried out by Mr. Pereira. It may be of some interest only to know the difference in the assumptions and, consequently, in the results.

The main difference in the assumptions consists in the statistical parameter chosen for the description of the dynamic behaviour of the structure. - We turned our attention to the probability distribution of the absolute maximum of the response during a given time interval in a long-type earthquake. Using the same mathematical theory used by Mr. Pereira (namely the Rice theory), we found that the probability we were looking for is well approximated for a linear system by the expression:

$$p(\alpha) \cong M e^{-\frac{\alpha^2}{2\bar{y}^2}}$$

where $p(\alpha)$ is the probability that the absolute maximum is $> \alpha$, M is the expected number of maxima in the time interval considered, $\bar{y}^2$ is the square mean value of the response.

The exact expression of $p(\alpha)$ is however not much more complicated.

These results apply to one degree of freedom systems as well as to multi-degrees, of course when model analysis is possible. This may represent a good solution of the problem of superposition of normal modes.

Another comment to Mr. Pereira's paper is the following: in the paper it is assumed that the maxima of the response can be considered as independent. We think (but the mathematical proof has not yet been fully developed) that they must be considered as independent, this being a consequence of the general assumptions of the theory.

AUTHOR'S REPLY: The reasons that justify the choice of the mean maximum values to describe the dynamic behaviour of structures have been presented by J.F. Borges in recent papers *. The physical and statistical reasoning, underlying this choice, is that variability associated with the seism intensity is much more important than the one associated with the structure.

The assumption of independence between successive maxima has given fruitful results that well agree with experimental values. Even so, it seems difficult that a mathematical proof of this hypothesis may be found.

* Borges, J.F. "Statistical Estimate of Seismic Loading", Preliminary Publication, V Congress of the International Association for Bridge and Structural Engineering, Lisbon, 1965.

Borges, J.F. - "Structural behaviour and Safety Criteria", Preliminary Publication, VII Congress of the International Association for Bridge and Structural Engineering, Rio de Janeiro, 1964.

QUESTION BY: G.W. HOUSNER - U.S.A.

That this paper shows that the response is relatively insensitive to the shape of the power spectral density, can be understood for damped systems, but as damping approaches zero one must see a marked difference between it when you have different power spectral densities. Since some of our structures have damping less than 2% did you follow down to lower damping to see if and at which point the differences become obvious.

AUTHOR'S REPLY: As it is very difficult from a mathematical point of view to consider simultaneously the free and the forced vibrations, we assumed in the paper that the free vibrations could be neglected. This hypothesis, which is reasonable for dampings greater than, say, 2%, showed the relative insensitivity of the mean maximum values of the displacements to the shape of the acceleration of the power spectrum curve. This result was also obtained by seismic tests on models.

The limit case of zero damping was also analytically studied, and confirmation was obtained of the values presented by Professor Housner.

QUESTION BY: J. PENZIEN - U.S.A.

Was the linealised model used in the elasto-plastic studies a viscous dash pot that absorbed the same energy assuming you had a certain mean square response? The response will go into the inelastic range with only a few excursions into the plastic range. How do you account for this in getting an adequate model; also, did you treat the rigid plastic system?

AUTHOR'S REPLY: From the relation between the hysteretic and viscous damping in a closed cycle (which is a function of amplitude) and from the probability density of maxima in the non linear case, it was possible to define, by integration, an equivalent viscous damping.

The case of the rigid plastic system was not considered.