EARTHQUAKE RESPONSE OF APPENDAGE ON A MULTI-STORY BUILDING

by Joseph Penzien[1] and Anil K. Chopra[2]

ABSTRACT

Presented is an approximate method of analysis of the seismic effects on appendages located on top of multi-story buildings which are subjected to a base acceleration corresponding to the N-S component of the El Centro 1940 earthquake. This method is based on the forced response of separate two degree of freedom systems, one for each normal mode of the building. Response spectra for a two degree of freedom system are included to facilitate the use of this method. A comparison of the results are made with an accurate solution and a more approximate solution using single degree response spectra.

INTRODUCTION

One of the most significant contributions to the analysis of forces developed in structures during an earthquake has been the introduction of the concept of response spectra(3). These spectra established for a specific earthquake make it possible to immediately determine the maximum response of a single degree of freedom system to that earthquake. Thus, earthquake response spectra provide a direct means of determining the maximum response of a multi-story building in any one of its natural or normal modes of vibration. Maximum response due to all modes of vibration is often obtained approximately by taking the square root of the sums of the squares of the individual mode contributions(4).

Standard response spectra can also be used by the structural designer to predict seismic forces in an appendage to a multi-story building when the stiffness of that appendage is such that it moves essentially as a rigid body during the period of the earthquake. If the mass of the

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(1) Professor of Civil Engineering, University of California, Berkeley, California.

(2) Graduate Student in Civil Engineering, University of California, Berkeley, California.

(3) Alford, J. L., Housner, G. W., and Martel, R. R., Spectrum Analyses of Strong-Motion Earthquakes, Earthquake Research Laboratory, California Institute of Technology, August, 1951.

(4) Clough, R. W., Earthquake Analysis by Response Spectrum Superposition, Bulletin of the Seismological Society of America, Vol. 52, No. 3, July, 1962.

appendage is small compared to the mass of the building, the frequencies and shapes of the lower modes of vibration will be little affected by the presence of the appendage. Under these conditions, a standard modal analysis making use of response spectra will yield the maximum absolute acceleration at the location of the appendage; thus, giving a direct measure of the maximum seismic forces involved.

However, when the stiffness of the appendage is such that its own natural frequency (appendage is herein considered as a single degree of freedom system) is of the same order of magnitude as one of the frequencies of the lower modes of the building, the relative displacements within the appendage may be large compared with the relative displacements within the building itself which causes difficulty when using a single degree response spectra in predicting maximum appendage seismic force.

It is the purpose of this paper to discuss several methods for predicting maximum seismic forces in appendages when their supporting structures are subjected to known earthquake inputs.

METHODS OF ANALYSIS

A. Multi-Degree of Freedom Analysis - Assuming an N story building sufficiently symmetrical in its geometry to permit a planar dynamic analysis, its mass usually can be lumped at each floor level resulting in an N degree of freedom system. If an appendage which can be considered as a single degree of freedom system is attached to the top of this building, an N+1 degree of freedom system results. By standard methods, N+1 frequencies and N+1 mode shapes can be determined and the forced response to a horizontal ground acceleration $\ddot{u}_g(t)$ can be obtained using the generalized equations of motion. For example, letting $u_i(t)$ represent the horizontal displacement of mass m_i with respect to the moving base of the building, one can state

$$u_i(t) = \sum_{n=1}^{N+1} Y_n(t)\, \varphi_{in} \qquad i = 1,2,---, N+1 \tag{1}$$

where $Y_n(t)$ is the generalized coordinate of the nth normal mode and φ_{in} is the dimensionless nth mode shape quantity for location i. The time history of the generalized coordinate $Y_n(t)$ is obtained by solving the following generalized equation of motion; namely,

$$\ddot{Y}_n(t) + 2\omega_n\, \xi_n\, \dot{Y}(t) + \omega_n^2\, Y_n(t) = P_n(t)/M_n \qquad n = 1,2,---,N+1 \tag{2}$$

where

ω_n is the circular frequency of the nth mode

ξ_n is the damping ratio for the nth mode

and where

$$P_n(t) = -\ddot{u}_g(t) \sum_{i=1}^{N+1} m_i \varphi_{in} \tag{3}$$

$$M_n = \sum_{i=1}^{N+1} m_i \varphi_{in}^2 \tag{4}$$

Solving for the entire time history of each generalized coordinate as given by Eqs. (2), the desired time history of response as defined by Eq. (1) can now easily be obtained and its maximum value noted. Other response quantities such as $\dot{u}_i(t)$, $\ddot{u}_i(t)$, etc. can be obtained in a similar manner. Thus, the maximum seismic force in an appendage can likewise be obtained.

If the single degree relative displacement response spectrum, i.e. $S_D(\omega)$ vs. ω, is known for the specified earthquake acceleration input, one can find the absolute maximum value of $Y_n(t)$ reached during the period of the earthquake directly using the relation

$$Y_n(t)_{max.} = S_D(\omega_n) \left[\sum_{i=1}^{N+1} m_i \varphi_{in} \Big/ \sum_{i=1}^{N+1} m_i \varphi_{in}^2 \right] \tag{5}$$

It might seem appropriate to now find the maximum value of each term in the series of Eq. (1) and to approximate the absolute maximum value of $u_i(t)$ by simply taking the square root of the sums of the squares of the individual maxima. This approach gives good results when the natural frequency of the appendage, by itself, differs considerably from the natural frequencies of the building. However, when the appendage frequency is close to one of the lower mode frequencies of the building, this approach gives poor results.

From a designer's point of view, the above well known general multi-degree of freedom analysis is usually not practical as the work involved is far too great. It is, therefore, desirable to establish a simplified method which will yield good accuracy even for those cases where the appendage frequency is close to a lower natural frequency of the building.

B. Two-Degree of Freedom Analysis - The mass of an appendage is usually very small compared with the mass of the building which supports it. In such cases, the dynamic response of the appendage during an earthquake has little effect on the dynamic response of the building; however, the dynamic response of the building completely controls the dynamic response of the appendage. This condition suggests that one could get an approximate solution to the problem of determining seismic forces in an appendage by using a separate two degree of freedom system for each of the N normal modes of the building (without appendage) as shown in Fig. 1.

In Fig. 1, the terms M_n, K_n, and C_n represent the generalized mass, generalized spring constant, and generalized damping factor, respectively, for the nth building mode (without appendage). The terms m_a, k_a, and c_a represent corresponding quantities for the appendage. The following relations apply to these quantities:

$$M_n = \sum_{i=1}^{N} m_i \, \theta_{in}^2 \quad ; \quad K_n = \omega_n^2 M_n \quad ; \quad C_n = 2M_n \, \omega_n \, \xi_n$$
$$k_a = \omega_a^2 m_a \quad ; \quad c_a = 2m_a \, \omega_a \, \xi_a \tag{6}$$

where θ_{in} is the dimensionless mode shape quantity for point i due to the nth mode of the building itself without appendage attached, ω_n is the corresponding nth mode frequency, ξ_n is the damping ratio for the nth building mode, ω_a is the frequency of the appendage by itself; i.e., $\omega_a^2 = k_a/m_a$, and ξ_a is the damping ratio for the appendage by itself.

The two degree system of Fig. 1 is excited through its support by the motion $u_s(t)$ which is related to the ground motion $u_g(t)$ by the relation

$$u_s(t) = \left[\sum_{i=1}^{N} m_i \, \theta_{in} \Big/ \sum_{i=1}^{N} m_i \, \theta_{in}^2 \right] u_g(t) \equiv -\gamma_n \, u_g(t) \tag{7}$$

The constant quantity in the brackets of Eq. (7) is a participation factor γ_n defined so that the resulting support displacement time history $u_s(t)$, if applied to the generalized single degree system representing the nth mode of the building (with the appendage removed) will produce a displacement time history of mass M_n identical to the displacement time history of the top of the building in its nth mode.

Using the relations of Eqs. (6) and (7), and introducing a mass ratio $\beta_n = m_a/M_n$, the coupled equations of motion for the two degree of freedom system shown in Fig. 1 may be expressed as

$$\ddot{X}_n + 2\omega_n \, \xi_n \, \dot{X}_n + \omega_n^2 X_n - 2\beta_n \, \omega_a \, \xi_a \, (\dot{X}_a - \dot{X}_n) - \beta_n \, \omega_a^2 \, (X_a - X_n) = -\gamma_n \, \ddot{u}_g(t)$$
$$\ddot{X}_a + 2\omega_a \, \xi_a \, (\dot{X}_a - \dot{X}_n) + \omega_a^2 \, (X_a - X_n) = -\gamma_n \, \ddot{u}_g(t) \tag{8}$$

The above coupled equations of motion can be solved numerically(5) for any

(5) Wilson, E. L., and Clough, R. W., Dynamic Response by Step by Step Matrix Analysis, Symposium on the Use of Computers in Civil Engineering, Lisbon, Portugal, October, 1962.

prescribed ground acceleration $\ddot{u}_g(t)$ and for any set of parameters ω_n, ω_a, ξ_n, ξ_a, β_n, and γ_n to yield the desired appendage seismic coefficient C_{an} which is defined herein as

$$C_{an}\,(\omega_n,\omega_a,\xi_n,\xi_a,\beta_n,\gamma_n) \equiv |(X_a - X_n)\,\omega_a^2/g\,|_{max}. \tag{9}$$

The seismic coefficient as defined is the ratio of the maximum dynamic appendage spring force to the static appendage spring force produced by a 1g load.

Obtaining C_{an} for n = 1,2,---,N, it is shown subsequently that the true seismic coefficient C_a for the appendage including all modes of the building can be obtained approximately by the relation

$$C_a = (C_{a1}^2 + C_{a2}^2 + --- + C_{aN}^2)^{1/2} \tag{10}$$

Usually only a few of the lower building modes need be considered in evaluating C_a by Eq. (10).

C. Single-Degree of Freedom Analysis - For easy application by the designer of the previously described two degree of freedom analysis, it would be necessary to have available response spectra for the two degree of freedom system. Since response spectra are readily available only for the single degree system, one might consider reducing the two degree of freedom system of Fig. 1 to two generalized single degree of freedom systems representing its first and second modes of vibration, see Fig. 2. The maximum response in each of these two modes can, of course, be obtained directly using the single degree response spectrum corresponding to the ground motion being considered. Hopefully, one might then take the square root of the sums of the squares of the two maximum appendage spring forces to obtain the true maximum spring force produced by the nth building mode.

Letting ω_{n1}, Φ_{n1}, Φ_{a1} and ω_{n2}, Φ_{n2}, Φ_{a2} represent the frequency and dimensionless mode shape values for the first and second modes, respectively, of the system shown in Fig. 1, the generalized quantities for the two single degree systems of Fig. 2 are given by the relations

$$\begin{aligned}
M_{n1} &= M_n\,\Phi_{n1}^2 + m_a\,\Phi_{a1}^2 & M_{n2} &= M_n\,\Phi_{n2}^2 + m_a\,\Phi_{a2}^2 \\
C_{n1} &= C_n\,\Phi_{n1}^2 + c_a\,(\Phi_{a1} - \Phi_{n1})^2 \;; & C_{n2} &= C_n\,\Phi_{n2}^2 + c_a\,(\Phi_{a2} - \Phi_{n2})^2 \\
K_{n1} &= \omega_{n1}^2\,M_{n1} \;; & K_{n2} &= \omega_{n2}^2\,M_{n2} \\
P_{n1} &= -(M_n\,\Phi_{n1} + m_a\,\Phi_{a1})\,\ddot{u}_s(t) \;; & P_{n2} &= -(M_n\,\Phi_{n2} + m_a\,\Phi_{a2})\,\ddot{u}_s(t)
\end{aligned} \tag{11}$$

Making use of the available single degree relative displacement response spectrum $S_D(\omega)$ for the specified $\ddot{u}_g(t)$(6), the absolute maximum values for the generalized corrdinates $Y_{n1}(t)$ and $Y_{n2}(t)$ are

$$|Y_{n1}(t)|_{max.} = \gamma_n \, S_D(\omega_{n1}) \left[(M_n \, \Phi_{n1} + m_a \, \Phi_{a1})/(M_n \, \Phi_{n1}^2 + m_a \, \Phi_{a1}^2) \right]$$

$$|Y_{n2}(t)|_{max.} = \gamma_n \, S_D(\omega_{n2}) \left[(M_n \, \Phi_{n2} + m_a \, \Phi_{a2})/(M_n \, \Phi_{n2}^2 + m_a \, \Phi_{a2}^2) \right] \qquad (12)$$

Accepting at this point the square root of the sums of the squares approach, the seismic coefficient C_{an} as previously defined becomes

$$C_{an} = (\omega_a^2/g) \left[|Y_{n1}(t)|^2_{max.} \, (\Phi_{a1}-\Phi_{n1})^2 + |Y_{n2}(t)|^2_{max.} (\Phi_{a2}-\Phi_{n2})^2 \right]^{1/2} \qquad (13)$$

The total appendage seismic coefficient would now be obtained using Eq. (10).

COMPARISON OF METHODS OF ANALYSIS

To compare the three methods of analysis previously described, an appendage was assumed to be placed on top of the six-story shear building shown in Fig. 3 and its base subjected to a horizontal ground acceleration $\ddot{u}_g(t)$ corresponding to the N-S component of the 1940 El Centro earthquake. The results of this study are shown in Figs. 4 and 5 where the appendage seismic coefficient C_a is plotted against period of the appendage T_a for each of the three methods. The ratio of the appendage mass to the total building mass is 0.001 in Fig. 4 and 0.010 in Fig. 5. The appendage damping ratio ξ_a is equal to 0.02 in each case. The period of vibration for each of the six building modes are indicated along the abscissa.

A comparison of the results in Fig. 4 shows reasonable agreement between the three methods of analysis except in those regions where the appendage frequency approaches the frequency of one of the lower building modes. In these regions, the multi-degree and two degree of freedom methods agree quite well; however, the single degree method is obviously considerably in error. In Fig. 5, which represents a larger appendage, by a factor of ten, the agreement of all three methods is reasonably good over the entire period range.

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(6) Jenschke, V. Clough, R. W., and Penzien, J., Analysis of Earth Motion Accelerograms, Institute of Engineering Research, Report No. SESM 64-1, January, 1964.

SINGLE DEGREE OF FREEDOM ANALYSIS NEAR RESONANCE

For small appendages, the single degree of freedom method of analysis as previously presented is considerably in error when the appendage frequency ω_a is near the frequency of one of the lower modes of the building. To provide additional information on the behavior in such cases, the two degree of freedom system shown in Fig. 1 was subjected to a support acceleration $\ddot{u}_s(t)$ corresponding to the N-S component of the El Centro 1940 earthquake; i.e., γ_n was set equal to unity, and its forced response was determined by solving the two generalized equations of motion.

The coupling of the above mentioned generalized equations of motion due to damping was neglected for this particular study; thus, yielding equations of motion for two separate single degree of freedom systems which are characterized by the generalized quantities of Eqs. (11). The error introduced by neglecting the coupling of the generalized equations of motion due to damping was investigated and found to be small for the damping ratios considered in this particular study.

The results of the above general investigation to determine the so-called "resonance" effects are presented in Figs. 6 and 7. The appendage seismic coefficients are plotted as ordinates in each of these figures with period T_n as the abscissa in Fig. 6 and mass ratio β_n as the abscissa in Fig. 7.

In Fig. 6, the mass ratio β_n, the appendage period T_a, damping ratio ξ_n, and damping ratio ξ_a are held constant and equal to 0.001, 0.40 seconds, 0.05, and 0.02, respectively. Curve No. 1 shows the contribution of the first mode of vibration to the appendage seismic coefficient, Curve No. 2 shows the contribution of the second mode, Curve No. 3 shows the total coefficient based on the square root of the sums of the squares of the 1st and 2nd mode maxima, and Curve No. 4 is the true total coefficient. As the period T_n approaches T_a it is clear that the square root of the sums of the squares approach is appreciably in error. This error results because the frequencies of the two degree system are very close to each other and are greatly out of phase during the critical period of the earthquake; thus, giving first and second mode contributions which are of opposite sign.

It is evident that when the earthquake ground motion retains its high intensity level over a relatively long duration, the two above mentioned mode contributions will come more into phase with each other; thus, producing very large appendage seismic coefficients. This behavior can be clearly demonstrated by calculating the response of the two degree system of Fig. 1 to a stationary Gaussian, random support acceleration which has a constant power spectral density; i.e., is a "white" input.

Crandall(7) and Mark have thoroughly studied this problem and show, for example, that the mean square acceleration of the secondary mass m_a becomes extremely large as the frequency ratio ω_2/ω_1 approaches unity. This increase in response at "resonance" is even more pronounced for the stationary random input than for the transient El Centro 1940 input previously discussed.

In Fig. 7, "resonance" is maintained; i.e., $T_a = T_n = 0.40$ seconds; therefore, this graph shows the effect of mass ratio β_n on the appendage seismic coefficient for this resonant condition. Damping ratios are similar and Curves 1-4 represent the same quantities as in Fig. 6. Large errors in the single degree method of analysis are observed for $\beta_n < 0.005$.

TWO DEGREE OF FREEDOM RESPONSE SPECTRA

The two degree of freedom method of analysis has been shown to yield good results even in the vicinity of the so-called "resonance" range; therefore, this method can be used reliably by the structural designer.

Since many structural designers base their designs on the N-S component of El Centro 1940 earthquake, the appendage seismic coefficient as given by Eq. (9) has been determined for a wide range of the parameters ω_n, ω_a, ξ_n, ξ_a and β_n and for $\gamma_n = 1$ using the N-S component of El Centro earthquake as the specified ground acceleration $\ddot{u}_g(t)$. These results are shown in Figs. 8-16 (see Table I for summary of basic parameters). To obtain C_{an} for $\gamma_n \neq 1$ simply multiply the value of C_{an} taken from Figs. 8-16 by γ_n.

All values of C_{an} in Figs. 8-16 have been obtained by solving numerically the coupled equations of motion; i.e., Eqs. (8).

CONCLUDING REMARKS

Based on the results of the investigation reported herein, the following conclusions have been deduced:

(1) the two degree of freedom method of analysis quite accurately predicts the maximum dynamic response of an appendage attached to the top of a multi-story building even when its period of vibration coincides with the fundamental period of the building; thus,

(7) Crandall, S. H., and Mark, W. D., Random Vibration in Mechanical Systems, Academic Press, New York and London, pgs. 80-102.

the availability of two degree of freedom response spectra makes this method of analysis practical;

(2) the single degree method of analysis becomes considerably in error when the period of the appendage is near the period of one of the lower building modes; therefore, this method of analysis should not be used in such cases;

(3) to greatly reduce the seismic forces in an appendage, it should be designed so that its period of vibration differs considerably from the first mode of vibration of the building and also does not coincide with other lower building modes; and,

(4) the seismic forces developed in an appendage, even when designed in accordance with the recommendations of (3) above, are larger than code values; therefore, the desirable effects of inelastic deformations must be considered as is standard practice in the design of buildings.

The quantitative results presented in this paper are based on the N-S component of ground motion recorded during the 1940 El Centro, California earthquake. Considering the fact that such results differ appreciably when using other earthquake ground motions of similar intensity, one must use the data presented herein with caution and with good engineering judgement.

REFERENCES

1. Alford, J. L., Housner, G. W., and Martel, R. R., Spectrum Analyses of Strong-Motion Earthquakes, Earthquake Research Laboratory, California Institute of Technology, August, 1951.

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5. Crandall, S. H., and Mark, W. D., Random Vibration in Mechanical Systems, Academic Press, New York and London.

6 Biot, A. M., Analytical and Experimental Methods in Engineering Seismology, Transactions, ASCE, 1943, pp. 377-379.

CURVE NO.	$T_a = 2\pi/\omega$	β_n
1 2 3	0.20	0.002 0.010 0.025
4 5 6	0.40	0.002 0.010 0.025
7 8 9	0.60	0.002 0.010 0.025
10 11 12	0.80	0.002 0.010 0.025
13 14 15	1.00	0.002 0.010 0.025

TABLE I - VALUES OF PARAMETERS FOR TWO DEGREE RESPONSE SPECTRA FIGS. 8 – 16

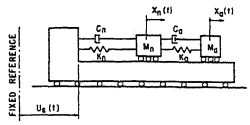


FIG. 1 - TWO DEGREE OF FREEDOM SYSTEM

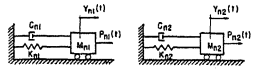
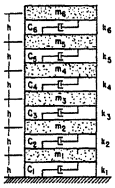


FIG. 2 - GENERALIZED SINGLE DEGREE OF FREEDOM SYSTEMS FOR THE SYSTEM OF FIG. 1



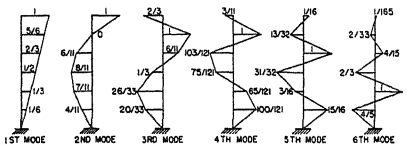
FLOOR	m_i/m_1	k_i/k_1	c_i/c_1	STORY
6	1	6/21	6/21	6
5	1	11/21	11/21	5
4	1	15/21	15/21	4
3	1	18/21	18/21	3
2	1	20/21	20/21	2
1	1	21/21	21/21	1

WHERE:

$M = \sum_{i=1}^{6} m_i$

$k_1 = \frac{16\pi^2 M}{T_1^2}$

$c_1 = \frac{16\pi M}{T_1}\,\xi_1$

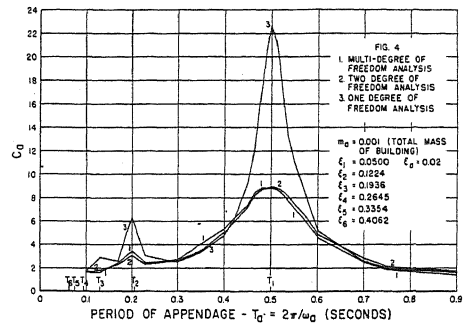


MODE NO.	$(T_1/T_i)^2$
1	1
2	1/6
3	1/15
4	1/28
5	1/45
6	1/66

$T_1^2 = \frac{16\pi^2 M}{k_1}$

LET $T_1 = 0.5$ SEC.

FIG. 3 - PROPERTIES OF 6-STORY BUILDING



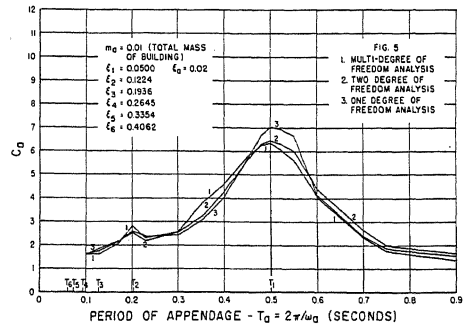
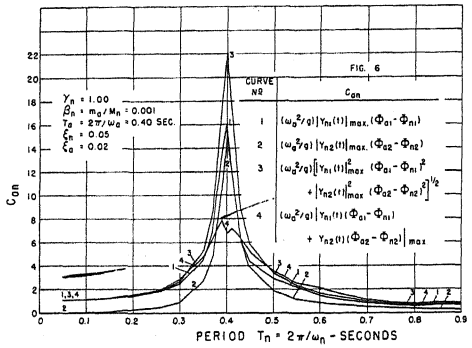


FIG. 4,5 - COMPARISON OF METHODS OF ANALYSIS



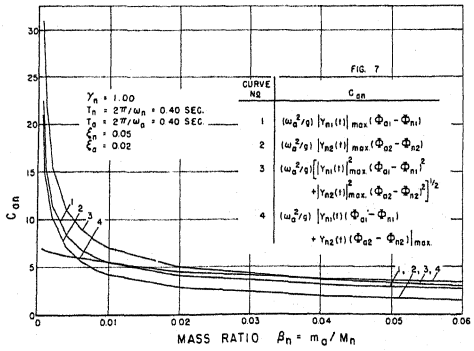
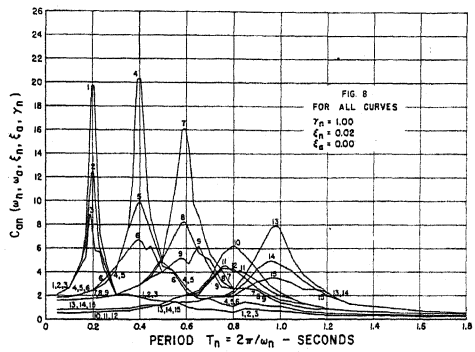


FIG. 6,7 RESPONSE STUDIES- TWO MASS SYSTEM OF FIGURE I.



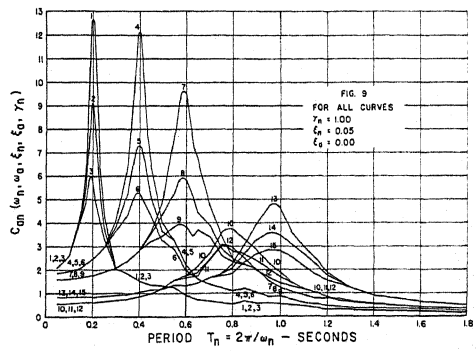
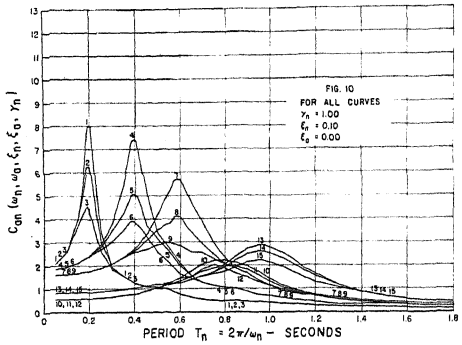


FIG. 8,9 - TWO DEGREE RESPONSE SPECTRA N-S COMPONENT, EL CENTRO EARTHQUAKE, 1940



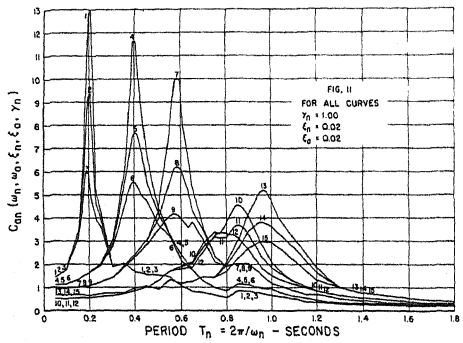
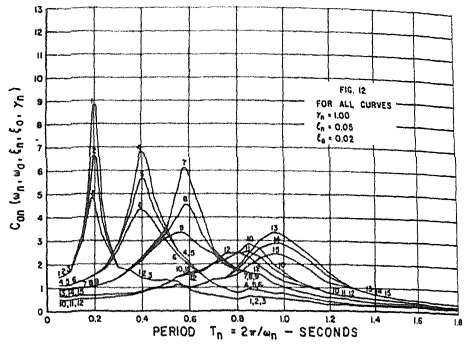


FIG. 10,11 - TWO DEGREE RESPONSE SPECTRA
N-S COMPONENT, EL CENTRO EARTHQUAKE, 1940



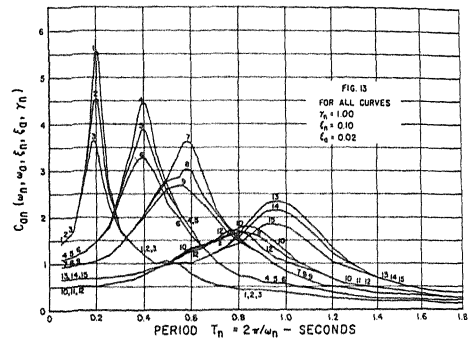
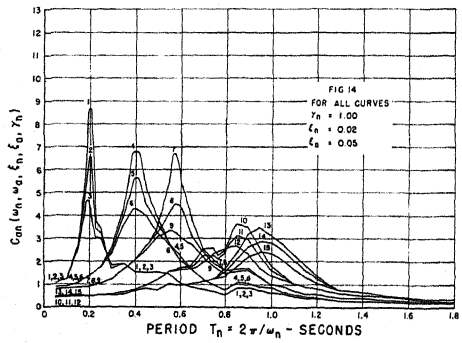


FIG. 12,13 - TWO DEGREE RESPONSE SPECTRA
N-S COMPONENT, EL CENTRO EARTHQUAKE, 1940



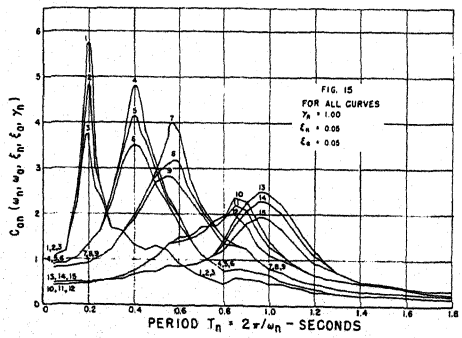


FIG. 14, 15 – TWO DEGREE RESPONSE SPECTRA
N-S COMPONENT, EL CENTRO EARTHQUAKE, 1940

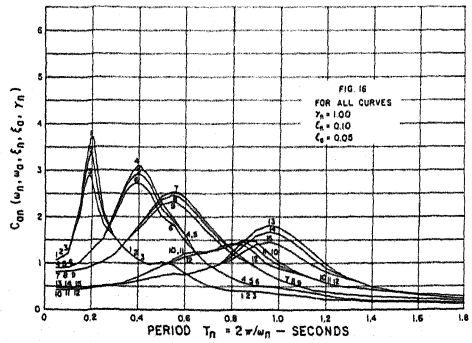


FIG. 16 - TWO DEGREE RESPONSE SPECTRA
N-S COMPONENT, EL CENTRO EARTHQUAKE, 1940

EARTHQUAKE RESPONSE OF APPENDAGE ON A MULTI-STORY BUILDING

BY J. PENZIEN AND A.K. CHOPRA

QUESTION BY: T. KATAYAMA - JAPAN.

Referring to Fig. 4 and 5 and equation 2. In your equation 2, we can see from Fig 4 and 5 the damping constant increases when the mode increases and with this type of damping that the higher mode of vibration is really negligible.

AUTHOR'S REPLY: In this investigation, the damping matrix for the six story shear building was assumed to be proportional to the stiffness matrix; thus, yielding damping ratios which are proportional to mode frequency. Therefore, the effects of higher modes are small. If, however, the damping matrix had been assumed proportional to the mass matrix, the damping ratios would be inversely proportional to mode frequency in which case the effects of higher modes would be much greater. By choosing damping proportional to stiffness, the authors do not wish to convey the thought that they consider this type of damping as most appropriate for structural systems. Considerable further experimental data are needed to clarify this point.

QUESTION BY: R.W. CLOUGH - U.S.A.

For what ratio of masses of appendage to structure are these results applicable, i.e. under what conditions can the appendage be expected to respond violently. This question refers specifically to the question of whether a "set back" tower on a tall building can be expected to act like an appendage.

AUTHOR'S REPLY: For the so-called resonance condition, i.e. where $T_a = T_n$ as shown in Fig. 7, the separate contributions of the first and second modes of a two degree system begin to increase very rapidly as the mass ratio β_n drops below approximately 0.008. One could, therefore, expect this same phenomenon to occur for a building having a large set back if (1) the ratio of the first mode generalized mass of the upper or set back portion of the structure to the first mode generalized mass of the entire structure falls much below this same value, and (2) the frequency of the set back portion of the structure (by itself) nearly coincides with the fundamental frequency of the entire structure. As to whether or not the large contributions of the two lowest modes of the building will actually combine so as to produce high seismic forces in the set back portion depends on the damping present and the duration of the oscillatory excitation.

EARTHQUAKE RESPONSE OF APPENDAGE ON A MULTI-STORY BUILDING

DISCUSSION

by H. Joseph Sexton[1] and Edward J. Keith[2]

INTRODUCTION

Messrs. Penzien and Chopra are to be congratulated on their very valuable paper dealing with earthquake response of two-mass systems. The writers feel that the paper was too abridged and that its contribution to the field of engineering may be overlooked. This discussion, therefore, is an attempt to expand on the possible applications to engineering systems.

DISCUSSION

A two-mass system can be considered as a mathematic idealization of any system, except of course, a one-mass system. One mass may represent a parapet, penthouse, or piece of equipment; the other mass may reflect the characteristics of the building or supporting structure in any one of its degrees of freedom.

The system treated by the authors is a very special two-mass system. It is a system having one mass many times smaller than the other and having fundamental periods of vibration relatively close to each other. The paper, in fact, is a study of resonance of a special two-mass system which produces unusually large accelerations and forces on the smaller mass when subjected to seismic disturbances.

The examples of two mass systems previously cited are typical cases meeting the above conditions. Parapets, penthouses and equipment rarely contribute more than a few per cent to the total weight of a building, yet their periods of vibration may be very close to one of the periods of their supporting structure. In nuclear reactor plants, piping, storage tanks, and even the reactor itself seldom contribute more than five or ten percent to the total mass of the structure, which is heavily weighted with shielding and massive concrete sections. Furthermore, the fundamental periods of the

(1) Vice President and Chief Engineer, John A. Blume & Associates, Engineers San Francisco.

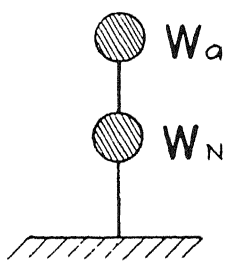
(2) Project Engineer, Dynamics, John A. Blume & Associates, Engineers, San Francisco.

reactor building and the associated equipment are usually very nearly the same, thus meeting the conditions treated in the authors' paper. In fact, applications of the authors' two mass response spectra can be found in almost any structure, large or small, simple or complex.

In general, building codes do not recognize the special considerations that should be given structural systems having the characteristics previously described. Parapet walls which are usually designed for a lateral force coefficient of 1.00 may be subjected to much greater forces during an earthquake.

The performance of structures during past earthquakes indicates that this problem cannot be considered lightly. Experiences with the Chilean earthquake of 1960, and the Anchorage earthquake of 1964 demonstrated dramatically the effect earthquakes can have on penthouses, parapets, and equipment on structures. In many instances total failure of these appendages was observed even though their supporting structures were only moderately damaged. This occurred not only because the accelerations were high at the top stories but because the building accelerations were greatly magnified in the appendages by a condition of resonance. It is apparent, therefore, that more refined design methods must be applied if failures are to be avoided in the future. This is particularly true in nuclear reactor projects, large processing facilities and any structure where damping is low, and the cost of damage in terms of loss of human life and property are high. The authors' paper presents just such a design approach, and one that is relatively easy to apply.

To illustrate how the authors' two-mass response spectra can be applied to the design of an appendage, consider the following problem:



W_a = 100 kips

ξ_a = 2% Damping

T_a = 0.2 sec.

W_n = 5000 kips

ξ_n = 5% Damping

T_n = 0.15 sec.

The first step is to calculate the mass ratio:

$$\beta_n = \frac{W_a}{W_n} = \frac{100}{5000} = 0.02$$

From FIG. 12 of the authors' paper, for Tn = 0.15 seconds and Ta = 0.2 seconds, interpolating between curves 2 and 3 (not really necessary in this case) obtain C_{an} = 3.0. Therefore design the appendage for a force at its center of gravity of three times the weight of the appendage, if the earthquake considered is applicable.

For design cases where the damping values or period ratios do not agree exactly with those given in the curves, interpolation will produce sufficiently accurate results for design purposes.

For multi-story buildings the same methods are used. The requirement here is a substitution of the "generalized mass" for the mass of the structure, M_n. The "generalized mass" should be calculated using the particular mode shape under consideration.

From FIG. 3 of the authors' paper the mode shape for the first mode of a six story building is presented as follows: 1/6, 1/3, 1/2, 2/3, 5/6, 1 for the first to sixth floors respectively. The "generalized mass", M*, can be calculated as follows:

$$M^* = m \left(1/6^2 + 1/3^2 + 2/3^2 + 5/6^2 + 1^2\right) = 2.53m$$

If the mass of each story, m, is assumed to weigh 3165 kips, then

$$M^* = 8000/32.2.$$

Assume the previously described appendage is located at the top of this building. Assume also that the period and damping for this building is the same as in the previous example. Then the mass ratio is identical to that previously determined, and the lateral force to be applied at the center of gravity of the appendage is three times the weight of the appendage.

In this way each mode of the building can be analyzed separately for the appendage problem. The total maximum design force can then be obtained by combining the maximum modal values on a root-mean-square basis. While this method of combining modes is only an approximation, it gives results sufficiently accurate for design purposes. For important structures, such as nuclear reactor plants, a more thorough dynamic analysis is required for critical pieces of equipment. For non-critical components the root-mean-square approximation is acceptable for design purposes.

In many cases, large pieces of important equipment are located at intermediate floors of buildings. For these cases, the method of determining the total maximum design force is the same as the multi-story case, however the result should be multiplied by the mode shape factor at the elevation of the equipment support.

It should be pointed out that the results of the authors' paper are based upon the El Centro Earthquake N S component 1940. It is hoped that the authors will extend this work to encompass other earthquakes, and relate the results.