THE DETERMINATION OF THE NORMAL MODE PROPERTIES OF MULTISTORY RECTANGULAR RIGID FRAMED STRUCTURES USING AN ELECTRICAL ANALOGY.

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SYNOPSIS

The use of a simple electrical structural analogy to evaluate the normal mode properties of rigid frameworks is described.

It was found that the analogy may be used satisfactorily in a manner similar to the classical Stodola numerical method to derive the fundamental mode properties of flexible girder buildings and that it may also be used to evaluate the flexibility matrix of a framed structure and so reduce the amount of numerical computation necessary in the determination of higher mode properties.

NOMENCLATURE

E = Young's modulus of elasticity.

F_r = force at floor r.

I = moment of inertia.

K = flexural stiffness = $\frac{EI}{h}$

K_{AB} = flexural stiffness for member AB.

L = length of girder or width of bay.

M_{AB}, M_{BA} = end moments at member ends A & B respectively.

$\overline{M}_{AB}$, $\overline{M}_{BA}$ = fixed-end moments at member ends A & B respectively.

R = analogue main network resistors.

S = shear force.

S_r = shear force in story r.

V_A, V_B = voltages at nodes A & B respectively.

W_r = weight of building story r concentrated at floor r.

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f_m = natural frequency of vibration in mode m.

g = acceleration of gravity.

h = height of section column or story.

i = general designation for analogue currents.

i_{AB}, i_{BA} = current flowing towards nodes A & B respectively.

i'_{AB}, i'_{BA} = feed currents at nodes A & B respectively.

m_r = mass assumed concentrated at station or floor r.

p = analogue scale factor for current.

q = analogue scale factor for voltage.

r_1, r_2, r_3 = analogue shear circuit resistor values.

v = analogue shear circuit terminal voltage.

x_r = horizontal displacement at station r or floor r.

x_N = horizontal displacement of top floor of N story frame.

$x_{i,j}$ = horizontal displacement of floor i at end of step j in an iteration process.

z_r = displacement ratio of floor r = $\frac{x_r}{x_N}$.

δ = interstory displacement or relative displacement of ends of members or sections.

δi_r = story difference current for story r.

δ_r = building interstory displacement for story r.

θ_A, θ_B = rotations of member ends A & B respectively.

ω_m = angular frequency of vibration in mode m.

$[B]$ = lateral flexibility matrix.

$[M]$ = building mass matrix.

$[x]$ = column modal matrix.

1. INTRODUCTION

Numerous electrical analogies have been used to facilitate the analysis of structures. One such analogue which proved of significant value was developed at the New Zealand Dominion Physical Laboratory for the determination of the earthquake response of bending structures.(1)

In the investigation described below the simple electrical analogy described by J.W. Bray(2) was chosen for the purpose of determining the modal properties of rectangular rigid frame structures.

Previous to the work reported here being undertaken, an iteration procedure suitable for determining the normal mode properties of shear buildings involving the use of a static system equivalent to the vibrating structure had been developed. Using this method typical structures had been analysed both by numerical iteration(3) and by Bray's electrical analogy.(4)

The modal analysis of flexible girder frames by the application of Bray's analogy has since been investigated.(5) The utility and limitations of the techniques developed are described in this paper.

2.1 THE ANALOGY

Bray's analogy is based upon the simulation of the slope deflection equations by use of simple passive electrical networks. In the application of this analogue to the solution of rigid rectangular frameworks each structural member is represented by a unit circuit of three of four resistors. The relationships between the currents, voltages and resistances in each unit circuit are identical in form to the two slope deflection equations describing the structural parameters of each member.

To simulate the action of a rectangular rigid frame under load the unit circuits representing each member are connected in a similar configuration to the structure, and currents are fed to the circuits representing the fixed-end restraints. The electrical network is solved by measurement of the current distribution and the potential differences at the junction or node points of the unit circuits.

Solution of the analogous framework is accomplished by scaling of the measured currents and voltages to give values of moments and deflections in the structure.

2.2 Isolated Uniform Member

Figure 1 (a) shows an isolated structural member removed from a rectangular rigid frame of uniform members. The rotations of the ends A and B are denoted by θ_A and θ_B respectively and the corresponding end moments necessary to maintain equilibrium by M_{AB} and M_{BA}. Initially ignoring relative lateral displacement of the ends, the slope deflection equations for the isolated member AB are seen to be

$$M_{AB} = \overline{M}_{AB} - 2K_{AB} \; (2\,\theta_A + \theta_B) \qquad \text{(i)}$$

$$M_{BA} = \overline{M}_{BA} - 2K_{AB} \; (2\,\theta_B + \theta_A) \qquad \text{(ii)}$$

where $\overline{M}_{AB}$ and $\overline{M}_{BA}$ are the fixed end moments for ends A and B respectively and K_{AB} equals the flexural stiffness of the member.

The analogous unit circuit for the isolated frame member is shown in figure 1 (b). Currents i'_{AB} and i'_{BA} are fed into the circuit as shown and resulting currents i_{AB} and i_{BA} flow towards the node points A and B.

It may easily be shown that

$$i_{AB} = i'_{AB} - \frac{1}{R} \; (2\,V_A - V_B) \qquad \text{(iii)}$$

and

$$i_{BA} = i'_{BA} - \frac{1}{R} \; (2\,V_B - V_A) \qquad \text{(iv)}$$

where R equals the magnitude of the resistances used, and V_A and V_B are the voltages at nodes A and B respectively.

Apart from sign, equation (i) and (ii) are identical in mathematical form to equations (iii) and (iv). For an analogy to exist the following quantities must be proportional to each other:

Network current proportional to end moment
Feed current proportional to fixed-end moment
Voltage proportional to rotation
Resistance proportional to $L/_{2EI} = \frac{1}{2K}$

These relationships may be expressed as

$$\begin{aligned} i_{AB} &= +\,p\,M_{AB} & \qquad i_{BA} &= -\,p\,M_{BA} \\ i'_{AB} &= +\,p\,\overline{M}_{AB} & \qquad i'_{BA} &= -\,p\,\overline{M}_{BA} \\ V_A &= +\,q\theta_A & \qquad V_B &= -\,q\theta_B \end{aligned}$$

$$R = \frac{q}{p} \cdot \frac{L}{2EI}$$

where p = the analogue scale factor for current
q = " " " " " voltage

2.3 Sign Convention

The following quantities were chosen as having positive signs; clockwise moments, anticlockwise rotations, currents directed towards the nodes and voltages above earth potential. Adopting this convention and considering the equations presented above, it becomes evident that in order to obtain the correct analogy between the two systems it is necessary to attach alternately positive and negative signs to the nodes of the networks, and to use the following rules for establishing signs when relating corresponding quantities:

Sign of feed current = (Sign of fixed-end moment). x (Sign of node)
Sign of end moment = (Sign of network current). x (Sign of node)
Sign of rotation = (Sign of voltage). x (Sign of node)

2.4 Equilibrium and Continuity

It may readily be demonstrated that, in addition to the behaviour of a single member being correctly simulated by the analogous circuit, the conditions of equilibrium and continuity are satisfied when simple unit circuits are connected together to form the electrical analogy of a complex framework.

2.5 Boundary Conditions

At the fixed end of a structural member it is assumed that no rotation occurs and this is simulated on the analogue by earthing the corresponding node point. At a pinned end the moment is zero and so the corresponding node point on the circuit is left open.

2.6 Sway

Lateral sway of the columns in a rectangular rigid asymmetrically loaded frame can be simulated by the following circuit modifications:

(i) A fourth resistor, having a resistance value equal to each of the three resistors, is added to each column group.

(ii) An additional circuit, termed a shear circuit, is provided for each story in the frame.

The shear circuit and column modifications are illustrated in figure 2, which shows the analogous circuit for a uniform laterally loaded fixed base rectangular portal.

The shear circuit terminals a and c are connected alternately to the upper and lower resistors of the adjacent column groups in each story to establish the correct sway analogy. The shear circuit itself consists of three resistors r_1, r_2, and r_3 which must be in the ratio 1 : 1 : $\frac{4}{3}$, and are supplied with current from a battery B through a variable resistor P. The junction between r_1 and r_2 is earthed and X and Y denote the take-off points for inputs to a meter which records the difference between the currents flowing at the two points.

Provided r_1, r_2 and r_3 are small in comparison with the resistors in the main network, and that the voltage drops across the difference meter inputs X and Y are negligible, then the terminals a and b have potential equal in magnitude and opposite in sign to each other.

When the shear circuit is used in the analogue solution of laterally loaded frames, the additional circuit enables the setting of a known value of shear force into each story and the determination of the magnitude of the resulting story sways.

If the slope deflection equations for the column subjected to a sway δ shown in figure 3 (a) are compared with the equations describing the electrical behaviour of the analogous network (figure 3 (b)):

$$M_{AB} = -2\,K_{AB}\left(2\theta_A + \theta_B + \frac{3\,\delta}{h}\right) \qquad \text{(v)}$$

$$M_{BA} = -2\,K_{AB}\left(2\theta_B + \theta_A + \frac{3\,\delta}{h}\right) \qquad \text{(vi)}$$

and

$$i_{AB} = -\frac{1}{R}\left(2\,V_A - V_B + v\right) \qquad \text{(vii)}$$

$$i_{BA} = -\frac{1}{R}\left(2\,V_B - V_A + v\right) \qquad \text{(viii)}$$

it can be shown that the story difference current $(i_2 - i_5) = \frac{1}{3}(i_{AB} - i_{BA}) = \frac{P}{3}.Sh.$

When the story difference current $(i_2 - i_5)$ is set proportional to the story moment for each story in a multi-story laterally loaded frame, the shear circuit terminal voltages v must be proportional to the quantity $\frac{3\delta}{h}$ for each story. The scale ratio between the quantities v and $\frac{3\delta}{h}$ must be in the same ratio as for the rest of the circuit hence

$$v = q \cdot \frac{3\delta}{h} .$$

In general the sway of all the columns in a particular story will be equal. This effect may be simulated on the analogue by supplying each story from a single independent shear circuit which induces the same voltages $\pm$ v to all the column resistor groups of a given story.

3. DESCRIPTION OF APPARATUS USED

3.1 Analogous Circuit Panel

The circuit panel used is shown in figure 4. Turret lugs enabled resistors to be soldered to the panel as desired in order to represent each structural member of rectangular frames of up to ten stories and five bays in size. Open and closed circuit jacks at each node point on the panel enabled currents and voltages in the network to be measured conveniently. Shear circuits for each story were permanently connected on the panel; each shear circuit was supplied with six $1\frac{1}{2}$ volt dry cells in series producing a total supply of 9 volts.

3.2 Resistors

The resistors used in the analogous circuits were $\pm$ $\frac{1}{4}$% tolerance wire wound precision resistors.

3.3 Difference Current Meter

A chopper amplifier difference current meter, was designed and constructed by the Electronic Developments and Applications Company of Wellington for use with the analogue equipment. The chopper amplifier circuits were modifications of a circuit originally presented in a Mullard Technical Communications Journal(6).

The instrument was provided with two difference current ranges, 0 to 1.0 m.a. and 0 to 10.0 m.a. It was found that linearity and matching of the choppers on the 1 to 1.0 m.a. range were within 2% of exact. Although a 2% variation in the linearity of the individual chopper performances may appear insignificant, it is in fact possible for such a variation to produce errors of the order of 10% or more in the measurement of a small difference current obtained from two relatively large input currents. In order that accurate analogue solutions can be achieved for a wide variety of structure it appears necessary to have linearity and matchability of the chopper amplifiers to an accuracy of 0.1%.

Tests revealed that it was not practical to greatly improve the linearity and matchability of the particular instrument used in this investigation.

3.4 Metering Bridge Circuits

To enable analogue network currents and voltages to be measured without introducing spurious voltages into the circuits two simple bridge circuits were used, one for voltage and one for current measurement.

The bridge circuit diagrams are shown in figure 5. Each circuit contains a centre zero microammeter for balancing and an Avometer on which the current&voltage values may be read. A modified milliammeter was used in the bridge circuit for measuring voltages from 0 to 1.0 volts and was found to be considerably more accurate over this range than the Avometer. In operating a bridge circuit, the potentiometer was adjusted until no current flowed through the microammeter. When this condition was achieved no potential drop occurred in the network current being measured, and no current flowed from the circuit at the point at which the voltage was being measured.

The manufacturers claim the following accuracies for the Avometers; D.C. current, 1% of the full-scale value over the effective range; D.C. voltage, 2% of indication between full-scale and half-scale deflection and below half-scale deflection, 1% of the full-scale value.

The milliammeter used for voltage measurement was a British Physical Laboratories instrument having an accuracy of $\pm\ \frac{1}{2}\%$ of the full-scale value over the effective range.

3.5 Circuit Errors in the Analogy of Laterally Loaded Frames

In the preliminary examination of the analogue it was found that circuit errors were present and that these were significantly larger than the metering errors. Subsequently a theoretical investigation showed that the errors observed in analogue solutions could be accounted for by an approximation in Bray's theory.

The derivation of the relationship between the current difference $(i_2 - i_5)$ and the story moment $\frac{Sh}{3}$ (figure 3):

$$(i_2 - i_5) = p.\frac{Sh}{3}$$

relies on the shear circuit resistors r_1, r_2 and r_3 being small in comparison with the main network resistors, thus giving the potentials of the shear circuit terminals a, b and c as v, - v and $\frac{7}{3}$v respectively. Analysis showed that deviation in the relationship is considerable even when r_1, r_2 and r_3 are of the order of 1% or less of the main network resistors. This is because although the currents which are compared in determining the story difference current may have no appreciable individual errors due to the approximation it is possible for the resulting difference current value to be a relatively small difference of two comparatively large currents and a significant proportionate error can then result in the difference current.

Nevertheless it was found possible to keep the errors arising from the shear circuit approximation to insignificant proportions by selecting the resistor values so that the ratio of the shear circuit resistances to the main analogue resistances was sufficiently low. However there are practical limits to which the ratio can be reduced as lowing the ratio also lowers the range of difference currents available. Furthermore, if a limit exists on the shear circuit current available a low ratio of shear circuit resistances to main analogue resistances may reduce the main network currents to values lower than can be accurately measured.

4. FUNDAMENTAL MODE DETERMINATION BY ANALOGUE ITERATION

4.1 Experimental Procedure

The analogue was used in an analogous manner to the classical Stodola numerical method to give the fundamental mode properties of multi-story framed structures. The process of successively applying inertia loads to the frame and solving for deflections is a tedious numerical procedure for typical framed structures but can be simulated on the analogue to give a rapid convergence.

It was found convenient to start the analogue iteration process by loading the frame with the story weights, to measure the deflections x_r, and to take these deflections as the initial deflections for the convential Stodola process. The method adopted and expressed in terms of the analogue parameters was as follows:

(i) The analogous circuit representing the frame was set up on the analogue.

(ii) From the frame story weights initial story moments were calculated, and by suitable choice of the scale factor p initial story difference currents were established.

(iii) The story difference currents were then applied to the analogous circuit; shear circuit voltages were measured and scaled to give the frame deflections.

(iv) Deflections obtained from the above solution were divided by the top-story deflection to give deflection ratios for the structure.

(v) The following new load system was then applied to the structure

$$F_r = z_r \cdot m_r \cdot g \qquad (x)$$

where F_r is the lateral force at floor r

z_r is the displacement ratio at floor r

m_r is the mass of floor r

and g is the acceleration due to gravity.

Hence a second set of trial values of difference currents was established. The loads were taken proportional to deflection ratios rather than deflections in order to keep difference currents within the same range for each trial load system.

(vi) Steps (iii) to (v) were repeated until convergence was complete.

The fundamental mode deflection ratios were obtained directly from the final deflections, and the fundamental frequency was established from the following considerations.

At any cycle after the initial one, the load applied to the i^{th} story in the analogue iteration process is

$$F_i = z_i \cdot m_i \cdot g = \frac{x_i}{x_N} \cdot m_i \cdot g \qquad \text{(xi)}$$

Now assuming vibration at the fundamental mode frequency, and assuming a deflection curve x_r, the inertia load at story i is given by

$$F_i = x_i \cdot m_i \; \omega_1^2 \qquad \text{(xii)}$$

Comparing expressions (xi) and (xii) it can be seen that the analogue applied loads must be multiplied by a factor of

$$\frac{x_N \; \omega_1^2}{g}$$

to establish the true inertia loading in any assumed deflection curve. Furthermore, the measured analogue deflections produced by a set of the applied loads must be multiplied by the same factor to establish the true deflections. Let the subscript j denote the final set of displacements obtained at convergence of the analogue iteration - then,

$$x_{i,\ j-1} = x_{i,\ j} \cdot \frac{x_{N,\ j-1}}{g} \cdot \omega_1^2$$

If the convergence is exact within the limits of accuracy of the analogue

$$x_{i,\ j-1} = x_{i,\ j}$$

Thus

$$\omega_1^2 = \frac{g}{x_{N,\ j}} \qquad \text{(xiii)}$$

This expression holds approximately if convergence is incomplete.

4.2 Experimental Results

In order to assess the accuracy and usefulness of the analogue technique described above the method was used to determine the basic fundamental mode properties of several typical frames. The investigation of one of these frames, a four-story three-bay frame original analysed by G.V. Berg (7), is described below. The frame is shown in figure 6 and the analogous circuit resistor values listed in figure 7.

The experimental results and their theoretical counterparts are tabulated in tables 1 and 2. Theoretical solutions were accomplished using a matrix iteration process on a digital computer.

4.3 Comments on Results

Excluding analogue setting up time the fundamental mode displacement ratios and frequencies were established in under half an hour for Berg's frame. The analogue iteration gave the fundamental mode displacement ratios and frequency within a few percent of the theoretical values. The results were typical of those obtained for all the frames analysed using the analogue.

Since a knowledge of only the fundamental mode properties is frequently sufficient to allow the application of the many current building codes which use a dynamic approach, the analogue iteration process described above may well prove of value.

5.1 ANALOGUE DETERMINATION OF THE LATERAL FLEXIBILITY MATRIX

The application of matrix algebra to the Stodola method(8) of normal mode property evaluation reduces the analysis to the problem of solving the expression:

$$[x] = \omega^2 . [B] . [M] . [x] \qquad \text{(xiv)}$$

where $[x]$ is the column modal matrix

ω^2 is the square of the mode frequency

$[B]$ is the square symmetric matrix of lateral flexibility coefficients

and $[M]$ is the diagonal square mass or inertia matrix

Evaluation of the $[B]$ matrix forms an indispensable preliminary to this matrix iteration process.

The analogue described above was found suitable for the evaluation of [B] . The matrix can be formed by this method reasonably accurately and in considerably less time than is possible by hand computation. It appears that a valid method of solving equations (xiv) is to evaluate [B] using the analogue and then to iterate the equations by hand to give the normal mode properties.

5.2 Experimental Procedure

The analogue method of forming the lateral flexibility matrix [B] is exactly analogous to the static analysis method of applying a unit load at each floor level or mass point in turn and determining the deflections at every floor level for each load.

The set of deflections obtained for each loading case provides one column of the lateral flexibility matrix.

By assuming a suitable scale factor p story difference currents were established and applied to the analogous circuits. Shear circuit voltages were measured to give inter-story deflections which were then used to give the deflection curve for the load position considered. These deflections gave one column of the flexibility matrix and by repeating the process with a unit load applied at each story in turn, the complete experimental matrix was obtained.

Theoretically the matrix [B] is symmetrical and as the experimental technique adopted gave the complete matrix, averages of the symmetrically placed terms were used in the subsequent iteration process.

5.3 Experimental Results

Experimental flexibility matrices were evaluated for a series of structures but only the results of the analysis of the four-story, three-bay frame shown in figure 6 are recorded below for the purpose of illustration.

The theoretical flexibility matrix obtained from a digital computer analysis is also listed for comparison.

(a) Theoretical Lateral Flexibility Matrix

$$\begin{bmatrix} 2.203 & 2.695 & 2.743 & 2.755 \\ & 5.004 & 5.515 & 5.543 \\ & & 9.309 & 9.887 \\ \text{(Symmetrical)} & & & 13.998 \end{bmatrix} \times 10^{-5} \text{ in./lb.}$$

(b) Averaged Experimental Lateral Flexibility Matrix

$$\begin{bmatrix} 2.184 & 2.676 & 2.760 & 2.784 \\ & 4.908 & 5.436 & 5.520 \\ & & 9.324 & 9.852 \\ \text{(Symmetrical)} & & & 13.980 \end{bmatrix} \times 10^{-5} \text{ in./lb.}$$

5.4 Normal Mode Properties

The theoretical and experimental flexibility matrices were iterated using a digital computer to give normal mode properties and the normal mode responses to a static acceleration of one g were evaluated. The results obtained from the two matrices listed above are presented in table 3.

5.5 Comment on Results

The normal mode responses tabulated were typical of those obtained for the range of frames analysed using the analogue method. It can be seen that a flexibility matrix evaluated using this technique provides results sufficiently accurate for practical purposes. In fact it is apparent that the shear forces in the lower more important modes can be obtained within a few per cent of exact values. The higher modes are obtained rather less accurately, but this is of little practical significance.

6. CONCLUSIONS

The analogue described is of practical value for the evaluation of both static moments and fundamental normal mode properties.

Direct evaluation of modes higher than the fundamental mode appear impracticable. A trial and error method might be adaptable, but such a method would be tedious and probably inaccurate because of analogue errors and the predominance of the fundamental mode. Direct iteration of higher modes by a similar process to that used for the fundamental mode is impossible because of the predominance of the fundamental mode and the lack of any physical method of removing its effect.

A combination of an analogue iteration process with a numerical "sweeping" process might be successful although it seems doubtful if such an approach would be superior to the analogue evaluation of the lateral flexibility matrix with subsequent hand iteration.

Although evaluation of the lateral flexibility matrix using the analogue appeared to be successful, a comparison of this approach with the method of evaluating the normal mode properties of frames by reducing them to an equivalent shear building[5] was made. The equivalent shear building approximation method consists of reducing the structure to an equivalent single-bay frame with pins inserted at each floor level in such a manner that the frame is reduced to a series of pinned bays. Using this analogue equipment, the analogue evaluation of the lateral flexibility matrix followed by hand iteration would be both slower and less reliable than the equivalent shear building approach. For low frames the analogue lateral flexibility matrix may give rather more accurate higher mode properties than the equivalent shear building approach.

Because of the suitability of digital computers for static frame analysis and normal mode property determination the use of an electronic digital computer is likely to be preferred to the use of analogue equipment in the establishment of these properties for a chosen structure.

An efficient analogue might be of greater use in a design problem as the effect of modifications to a chosen structure can more readily be determined as the design progresses and an optimum condition may be achieved more rapidly than would be possible using a digital computer.

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8 REFERENCES

1. K.M. Adams, R.A. Morris and R.I. Skinner, "An Analogue Computer for the Determination of the Earthquake Response of Buildings in Bending and Shear Modes", Proceedings, Second World Conference on Earthquake Engineering, 1960.

2. J.W. Bray, "An Electrical Analyser for Rigid Frameworks", The Structural Engineer, Vol. 35, No. 8, 1957.

3. R.J. O'Driscoll and R. Shepherd, "Dynamic Response of Multistory Buildings", New Zealand Engineering, Vol. 18, No. 9, Sept., 1963.

4. R.J. O'Driscoll, "An Investigation of the Dynamic Response of Structures Using an Electrical Analogy", Unpublished M.E. Thesis, University of Canterbury, New Zealand, 1962.

5. J.H. Wood, "The Response of Structures Under the Action of Lateral Loading", Unpublished M.E. Thesis, University of Canterbury, New Zealand, 1963.

6. S. Guennon and W.L. Kemhadjian, "D.C. Amplifier with Balanced Chopper", Mullard Technical Communications, Vol. 5, No. 43, April 1960.

7. G.V. Berg, "Response of Multistory Structures to Earthquake", Journal, Engineering Mechanics Division, A.S.C.E., Vol. 87, No. EM2, April 1961.

8. R.H. Scanlan and R. Rosenbaun, "An Introduction to the Study of Aircraft Vibration and Flutter", MacMillan, New York, 1951.

TABLE 1: 1ST MODE BERG'S 4-STORY 3-BAY FRAME ANALOGUE ITERATION.

r	Story Weights W_r lbs.	Initial Shears S_r lbs.	δi_r m.a.	v_r volts	δ_r ins.	x_r ins.	$z_r m_r$ g lbs.
4	0.6125 x 10^5	0.6125 x 10^5	0.077	0.792	6.2	49.7	0.613 x 10^5
3	0.937 "	1.5495 "	0.19	1.65	12.9	43.5	0.820 "
2	0.937 "	2.4865 "	0.31	1.66	13.0	30.6	0.578 "
1	0.980 "	3.4665 "	0.52	1.87	17.6	17.6	0.347 "
		2nd Shears					
4		0.613 x 10^5	0.077	0.783	6.1	40.2	0.613 x 10^5
3		1.433 "	0.18	1.54	12.1	34.1	0.795 "
2		2.011 "	0.25	1.28	10.0	22.0	0.512 "
1		2.358 "	0.35	1.27	12.0	12.0	0.292 "
		3rd Shears					
4		0.613 x 10^5	0.077	0.783	6.1	39.6	0.613 x 10^5
3		1.408 "	0.18	1.55	12.1	33.5	0.792 "
2		1.920 "	0.24	1.27	9.9	21.4	0.507 "
1		2.212 "	0.33	1.22	11.5	11.5	0.285 "
		Final Shears					
4		0.613 x 10^5	0.077				
3		1.405 "	0.18				
2		1.912 "	0.24				
1		2.197 "	0.33				

$$p = 0.30 \times 10^{-6} \text{ m.a./lb.ft.}$$
$$\delta_{4-2} = 7.84 \times v \text{ ins.}$$
$$\delta_1 = 9.41 \times v \text{ ins.}$$
$$S_i = \sum_{r=i}^{4} z_r m_r g$$

TABLE 2 - FUNDAMENTAL MODE PROPERTIES

RESPONSE	EXPERIMENTAL	THEORETICAL
z_4	1.000	1.000
z_3	0.846	0.850
z_2	0.540	0.549
z_1	0.290	0.294
ω_1^2	9.75	9.63
f_1 cps	0.495	0.494

TABLE 3 - NORMAL MODE RESPONSE 4-STORY, 3-BAY FRAME.

r		MODE 1		MODE 2	
		THEO. MATRIX	EXPT. MATRIX	THEO. MATRIX	EXPT. MATRIX
	f_m cps.	0.493	0.494	1.361	1.375
4	deflns.	53.6	53.5	-2.48	-2.48
3		45.8	45.5	-0.12	-0.11
2	x_r ins.	29.8	29.4	2.12	2.09
1		15.8	15.8	1.78	1.76
4	shears	163×10^3	164×10^3	-57.5×10^3	-57.7×10^3
3		376 "	376 "	-61.7 "	-61.9 "
2	S_r lbs	514 "	514 "	13.5 "	13.9 "
1		591 "	591 "	79.4 "	80.6 "

r		MODE 3		MODE 4	
		THEO. MATRIX	EXPT. MATRIX	THEO. MATRIX	EXPT. MATRIX
	f_m cps	2.25	2.26	3.32	3.37
4	deflns.	0.322	0.306	-0.024	-0.020
3		-0.350	-0.336	-0.059	-0.053
2	x_r ins.	0.043	0.049	-0.143	-0.140
1		0.210	0.189	0.145	0.145
4	shears	20.4×10^3	19.5×10^3	-3.35×10^3	-2.81×10^3
3		-13.5 "	-13.2 "	9.13 "	8.70 "
2	S_r lbs	- 9.3 "	- 8.4 "	-21.1 "	-21.7 "
1		11.9 "	10.8 "	11.1 "	11.2 "

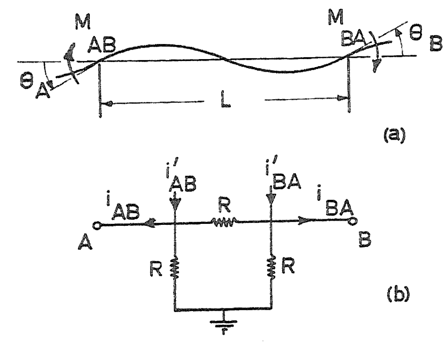


Fig. 1

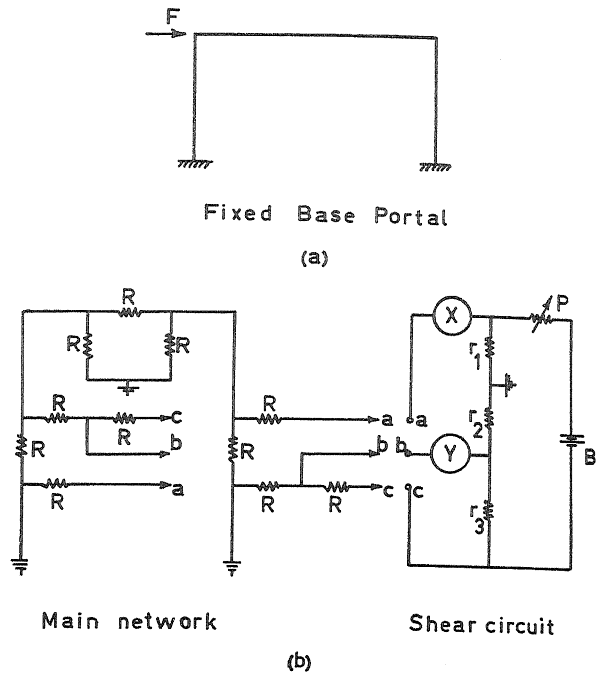


Fig. 2

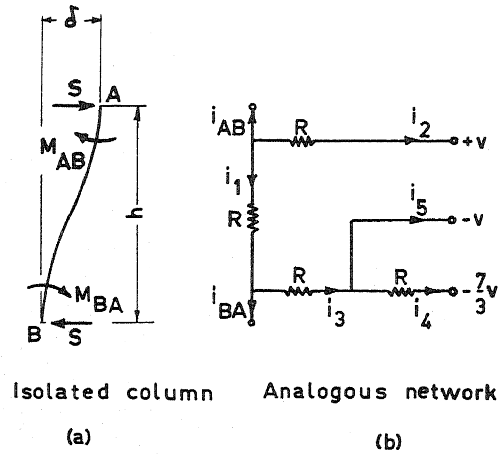


Fig. 3

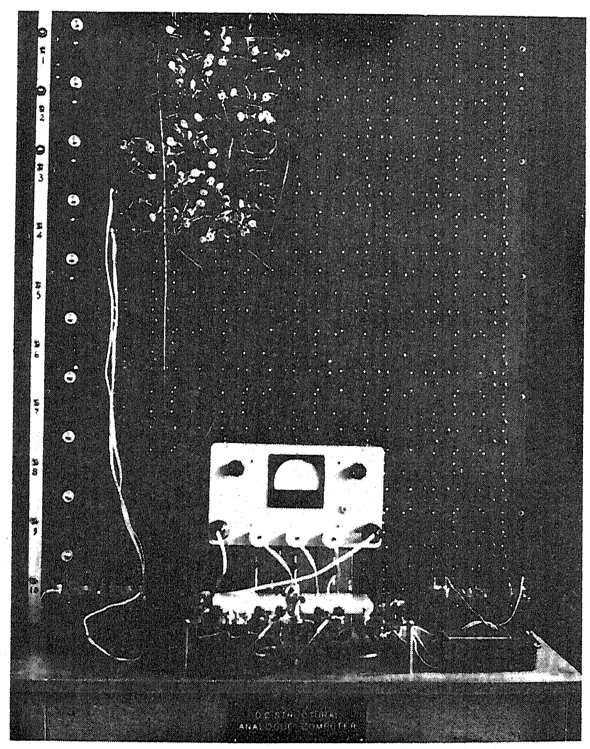
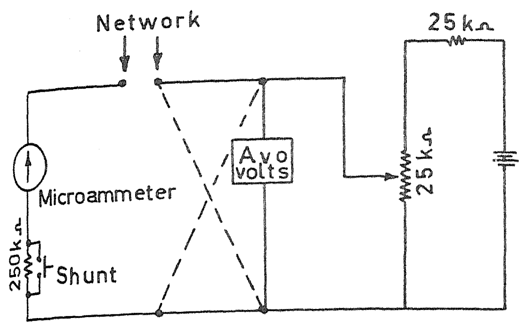
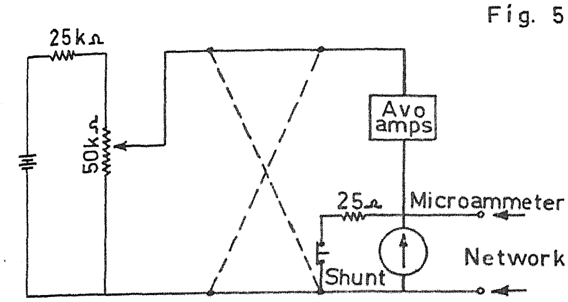


Fig. 4

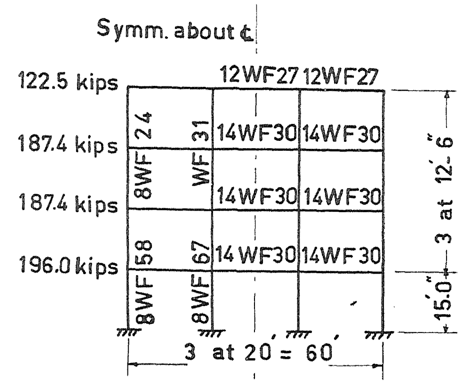


Voltage Measuring Circuit

Fig. 5

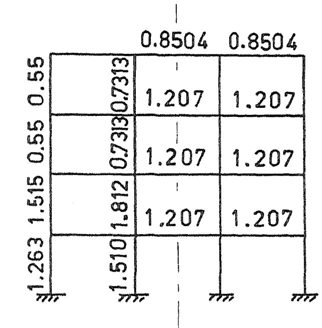


Current Measuring Circuit

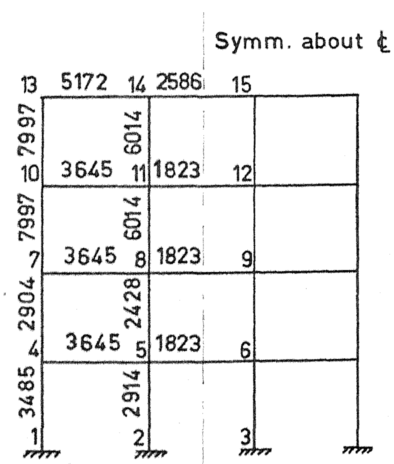


(a) Framing Diagram

Fig. 6



(b) $\frac{I}{L}$ Values ins.^{3}



Resistor values in ohms for analogue solution of 4 - Story 3 - Bay frame

Fig. 6(c)

THE DETERMINATION OF THE NORMAL MODE PROPERTIES OF MULTISTORY RECTANGULAR RIGID FRAMED STRUCTURES USING AN ELECTRICAL ANALOGY.

BY R.J. O'DRISCOLL, R. SHEPHERD AND J.H. WOOD.

QUESTION BY: D.G. ELMS - NEW ZEALAND

Have the Authors any comments on the relative advantages of using an electrical analogue such as they have described over a digital computer in normal mode determination?

AUTHORS' REPLY: The analogue described was constructed before digital computers were available in New Zealand and, had a digital machine been available the analogue may not have been used.

The analogue cost approximately £250 (N.Z.) to construct and so, if it was proposed to analyse a large number of simple rigid frameworks, the analogue described could well prove more economic than digital computation. However, it should be pointed out that the analogue described was rather limited in its application and the construction of a more satisfactory and versatile analogue, such as would be required for commercial use, may well cost considerably more and so the apparent economic advantage of the analogue is perhaps rather doubtful.

An efficient analogue would appear to be more useful as a design media in structural analysis rather than merely used as an analysis tool. The properties of the individual frame members can be rapidly varied on the analogue and the critical points in the design checked without the need for a completely new analysis as would be required on a digital computer. The operator of the analogue would certainly have a better understanding of what effect his modifications were having on the structure as his design progressed, and might produce optimum results more rapidly than would be the case with a digital computer.

Analogue computers are inherently suitable for the solution of differential equations or for problems in which integration plays an important part since integration can be carried out as a continuous distinct operation. Whilst the need to solve differential equations can often arise in both static and dynamic structural analyses, the evaluation of normal mode properties for rigid frames does not require such solutions, and so the analogue method has no particular merit. In the evaluation of normal mode properties for frames sets of linear simultaneous equations will generally have to be solved. Sets of equations of this form are readily solved by simple repeated arithmetic steps, and consequently, are well suited for adaption to digital computer methods.

QUESTION BY: S. TEZCAN - CANADA.

1. Are rotational stiffness of members considered?
2. What is the maximum degree of freedom that can be handled?

AUTHORS' REPLY:

1. The rotational stiffness of the members are considered as the analogue simulates exactly the familiar slope deflection equations for each member of the frame. In Section 2.2 in the paper the analogy is demonstrated for a member subjected to rotational deformations at either end.
2. The particular analogue described could be used for frames in sizes up to 10 stories and 3 bays. A structure with up to ten degrees of lateral freedom could thus be analysed. There would be no real difficulty in extending the analogue for larger structures; however, more elaborate equipment than that described would be required for supplying and setting the story difference currents in order to keep experimental errors within acceptable limits.

COMMENT BY: R. FLORES - CHILE

I want to point out that at the University of Chile, we have been using electrical analogies since 1947, to solve structural problems. In recent years we have worked out some dynamical problems along similar lines to those presented in this paper. Actually a paper offered to this Conference by Mr. Urioste deals with a similar approach to this problem.*

* Editor's Note: This paper appears in Abstract form.

AUTHORS' REPLY: The authors regret they were not previously aware of the work on electrical analogues carried out at the University of Chile. They look forward to receiving further information.

QUESTION BY: N.M. NEWMARK - U.S.A.

(1) The use of the Analogy is limited to the first mode, apparently.

(2) What about the symmetry of the flexibility matrix obtained from the Analogy? It should be symmetrical.

AUTHORS' REPLY: The analogue was used in an analogous manner to the classical Stodola numerical method to give the fundamental mode properties, but the problem of how to evaluate the higher mode properties directly using the analogue was not solved. As none of the mathematical processes normally applied in numerical analyses for this purpose can be physically simulated on the analogue, this problem is indeed a difficult one. By the indirect approach of evaluating the lateral flexibility matrix on the analogue the numerical computations normally required for evaluating higher mode properties could be considerably reduced.

The analogue was in fact used to derive the complete lateral flexibility matrix, which was not exactly symmetrical because of small experimental errors. Averages of the symmetrically placed terms in the experimental matrix were used in subsequent iteration processes to evaluate the normal mode properties. Use of the averaged matrix gave results in close agreement with theoretical values.