DYNAMICAL ANALYSIS OF STEEL STRUCTURE BY ELECTRONIC ANALOGY

BY

TAKEO NAKA*, TUTOMU KATO**, AND MAKOTO YUASA***

ABSTRUCT

This paper is on the electrical passive analog-circuit, to study structural response subjected to random wave. First a brief descriptions of the device are informed, but the present paper will give mainly a summary of the principles about the electrical circuit. Complete details of the circuit arrangements are available in a special report. Then responses of one mass system, including non-linear zone, are reported, and responses of five mass representing twenty-one storied steel structure are also described.

INTRODUCTION

To use electrical circuit for the purpose of analysing structure problem is now common. But we had few examples that electrical circuit was directly used to study dynamical problem of structures. This report is the first step of the above mentioned purpose. As T.K. Caughey and other people indicated (1) in their paper the special features of the analog circuit design which make the device particularly useful for dynamical analysis applications are : (1) The circuits are arranged so that natural period can be adjusted throughout its range by the variable reactance coil, the variable resister and condensers. (2) Various prescribed values of damping can be obtained without changing other circuit characteristics. A simple test is available to check the actual damping present at any time. (3) The time scale of the electrial circuit is speeded up compared with the structural prototype so that large number of spectrum points can be calculated in a relatively short time.

SIMULATION

When structure (shown in figure 1) is subjected earthquake, we have the following equation

$$m y'' + C_s y' + k(y - D_0) = 0 \qquad '' : \frac{d^2}{dt^2} \qquad ' : \frac{d}{dt} \tag{1}$$

while the voltage of the electrical circuit (shown in figure 2) is expressed as

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = E_0 \tag{2}$$

---

* a professor of Tokyo University
** an associate professor of Tokyo University
*** a student in doctor course of Tokyo University

(1) T. K. Caughey, D.E. Hudson, and R.V. Pawell "The C.I.T. Mark II Response Spectrum Analyzer for Earthquake Engineering Studies" IIWCEE P 130 July 14th, 1960.

considering that $\int i\,dt = Q$, equation (2) is

$$L\ddot{Q} + R\dot{Q} + \frac{1}{C}(Q - CE_o) = 0 \qquad (3)$$

we find quation (3) is analogous to equation (1) seeing that m corresponds to L, C to R, K to $\frac{I}{C}$ and y to Q. The equation (1) and the equation (3) can be also rewritten in the following equations.

$$\ddot{y} + \frac{4\pi}{T}\left(\frac{C}{C_s}\right)\dot{y} + \frac{4\pi^2}{T^2}y = \frac{4\pi^2}{T^2}D_o f(t) \qquad (4)$$

$$\ddot{Q} + \frac{4\pi}{T}\left(\frac{C}{C_s}\right)\dot{Q} + \frac{4\pi^2}{T^2}Q = \frac{E_o f(t)}{LN^2} \qquad (5)$$

as these equations have been expressed in the same real time t, corresponding terms can be directly identified. The displacement response can be obtained as:

$$\frac{y}{Q} = \frac{\frac{4\pi^2}{T^2}D_o f(t)}{\frac{E_o}{LN^2}f(t)} \qquad (6)$$

while the vo tage Ec across the measuring condenser is $Ec = \frac{Q}{C}$ and $T^2 = 4\pi^2 N^2 LC$ equation (6) becomes

$$y = \frac{D_o}{E_o}E \qquad (7)$$

Likewise, verocity and acceleration response can be obtained as:

$$\dot{y} = \frac{D_o}{E_o}\,\frac{E_R}{R}\,\frac{2\pi}{T}\sqrt{\frac{L}{C}} \qquad (8)$$

$$\ddot{y} = \frac{4\pi^2}{T^2}\,\frac{D_o}{E_o}E_L \qquad (9)$$

In the same procedure, general multimass structure (figure 3) can be analogous to electrical system (figure 4). Equation (10) is representing structure response and equation (11), (12) *electrical relation.*

$$\begin{aligned}
&m_1\ddot{y}_1 + C_{s1}\dot{y}_1 + k_1(y_1 - D_o) + k_2(y_1 - y_2) = 0\\
&m_2\ddot{y}_2 + C_{s2}(\dot{y}_2 - \dot{y}_1) + C_{s3}(\dot{y}_2 - \dot{y}_3) + k_2(y_2 - y_1) + k_3(y_2 - y_3) = 0\\
&\vdots\\
&m_i\ddot{y}_i + C_{si}(\dot{y}_i - \dot{y}_{i-1}) + C_{si+1}(\dot{y}_i - \dot{y}_{i+1}) + k_i(y_i - y_{i-1}) + k_{i+1}(y_i - y_{i+1}) = 0\\
&m_n\ddot{y}_n + C_{sn}(\dot{y}_n - \dot{y}_{n-1}) + k_n(y_n - y_{n-1}) = 0
\end{aligned} \qquad (10)$$

$$\begin{aligned}
&L_1\frac{di_1}{dt} + R_1 i_1 + R_2(i_1 - i_2) + \frac{1}{C_1}\int i_1\,dt + \frac{1}{C_2}\int (i_1 - i_2)\,dt - E_o = 0\\
&L_2\frac{di_2}{dt} + R_2(i_2 - i_1) + R_3(i_2 - i_3) + \frac{1}{C_2}\int (i_2 - i_1)\,dt + \frac{1}{C_3}\int (i_2 - i_3)\,dt = 0\\
&\vdots\\
&L_i\frac{di_i}{dt} + R_i(i_i - i_{i-1}) + R_{i+1}(i_i - i_{i+1}) + \frac{1}{C_i}\int (i_i - i_{i-1})\,dt + \frac{1}{C_{i+1}}\int (i_i - i_{i+1})\,dt = 0\\
&L_n\frac{di_n}{dt} + R_n(i_n - i_{n-1}) + \frac{1}{C_n}\int (i_n - i_{n-1})\,dt = 0
\end{aligned} \qquad (11)$$

$$L_1 \ddot{Q}_1 + R_1 \dot{Q}_1 + R_2(\dot{Q}_1 - \dot{Q}_2) + \frac{1}{C_1}(Q_1 - Q_0) + \frac{1}{C_2}(Q_1 - Q_2) = 0$$
$$L_2 \ddot{Q}_2 + R_2(\dot{Q}_2 - \dot{Q}_1) + R_3(\dot{Q}_2 - \dot{Q}_3) + \frac{1}{C_2}(Q_2 - Q_1) + \frac{1}{C_3}(Q_2 - Q_3) = 0 \qquad (12)$$
$$L_i \ddot{Q}_i + R_i(\dot{Q}_i - \dot{Q}_{i-1}) + R_{i+1}(\dot{Q}_i - \dot{Q}_{i+1}) + \frac{1}{C_i}(Q_i - Q_{i-1}) + \frac{1}{C_{i+1}}(Q_i - Q_{i+1}) = 0$$
$$L_n \ddot{Q}_n + R_n(\dot{Q}_n - \dot{Q}_{n-1}) + \frac{1}{C_n}(Q_n - Q_{n-1}) = 0$$

On the condition that structure members are all elastic zone, equations (1)-- (12) are compatible. When we think of the responses of structures of which members were to be yielded, and spring constant varies K to Kp by bilinear, we have non-linear differential equation as following

$$m\ddot{y} + C_s\dot{y} + \{k - F(y)\}(y_1 - y_0 - \varepsilon) = 0 \qquad (13)$$

$$y_{p-} \leqq y_1 - y_0 \leqq y_{p+} \quad F(y) = 0$$
$$y_1 - y_0 > y_{p+} \text{ or } y_1 - y_0 < y_{p-} \quad F(y) = k - k_p \qquad (14)$$

Simulating equation can be found in the electrical system shown figure 5, namely

$$L\ddot{Q} + R\dot{Q} + \frac{1}{C - G(Q)}(Q - Q_p) = E_0 \qquad (15)$$

$$E_{p-} \leqq E_c = \frac{C}{Q} \leqq E_{p+} \quad G(Q) = 0$$
$$E_c > E_{p+} \text{ or } E_c < E_{p+} \quad G(Q) = C - C_p \qquad (16)$$

In the equation (15) we must note that external relative displacement changes its intensity at the point Ec=Ep and that coefficient to calculate intensity of acceleration from the voltage across L changes its value at the point Ec=Ep (refer to equation (8) and (9)) and same with velocity. So being dealt with plastic zone, the electrical wave should be traced elaborately to detect the point Ec=Ep.

As for earthquake simulation the following serial hypotheses are available.

1) In the seismic centre, all kinds of vibration are generated so that frequency spectrum is almost even not having any exceeding period, and in the electrical system "white noise" generator corresponds to the seismic centre.

2) An intensity of the earthquake (1) damps as time goes on following an exponential function $I = I_0 e^{-\lambda t}$, and in the electrical system, white-noise source is modulated by periodical pulse and charged in the condenser to be discharged following an exponential function $E_a = E_{a0} e^{-C_a R_a t}$ Varying Ra corresponds varying damping ratio of the intensity of the earthquake.

3) Spectrum of intensity of earthquake affecting the structure depends on the foundation condition and the foundation is classified in four kinds:

I) In mountainous district, an exceeding period is between about 0.05 second and 0.2 second.

II) In plateau district, an exceeding period is between about 0.2 second and 0.4 second.

III) In plain district, there is not so accurate exceeding period as I and II but lenient exceeding period between about 0.4 second and 0.8 second.

IV) In soft foundation, there is more lenient exceeding period as III between about 0.6 second and 0.8 second.

Corresponding circuits can be obtained by using electrical filter circui and resister shown in figure 6.

Thus the whole diagram of the electrical system comes to be figure 7.

ACTUALIZATION

I) White noise generator: In order to get white noise we use thyratoron tube and amplify its output.

II) Varying intensity and damping ratio of the earthquake: multi-vibrator is used to have square wave and variable resisters are available to vary time of possitive side and negative side, possitive side corresponds to the energy of the earthquake and negative side to the whole time scale of earthquake. With this circuit white noise is modulated and comes to have intensity-time relation as shown in figure 8.

III) Foundation : An electrical C.L. filter and resister is used for the purpose of simulating foundation.

IV) Structure : Variable resister, variable inductance coil (inevitably having resister factor about 750 ohms) and a group of condenser consisting in considerable kinds of capacitance are used and the variable amplitude amplifier is also prepared. As for the non-linear element a couple of voltage sources and a couple of diodes are necessary.

V) Measuring apparatus : The oscillograph, the voltage metre, the camera, the movie camera and the data recorder are used to measure and to record the value, and stabilized electrical sources is also prepared for the white noise generator, multi-vibrator, amplifier, filter etc..

ONE MASS SYSTEM

A variety of period of structure, we are now going to analyse, is determined as T=0.25 sec., 0.5 sec., 1.0 sec., and 1.5 sec., and time factor N is decided to be 250. There is another freedom to determine the coefficient between mechanical and electrical system. As before mentioned, m corresponds to L and relation between m and L is expressed as $m\alpha = L$.

Where α is converting coefficient having demension Henry centimetre/ton second . Once α is determined, the other converting coefficient between k and C ($RC = \beta$) is fixed as

$$\alpha\beta = \frac{1}{N^2} \qquad \because \frac{T^2}{N^2} = 4\pi^2 LC \qquad T^2 = 4\pi^2 \frac{m}{k}$$

$$\frac{1}{N^2} = \frac{L}{m} C k = \alpha\beta$$

In this report α is determined 40 H cm/sec^2 and $\beta = 4.0 \times 10^{-7}$ t.F./cm

Representing foundation condition, we adopted, as exceeding period, period of 0.2 second for mountainous district, 0.3 second for plateau district, 0.5 second for plain district and 0.8 second plus 20% mixed white noise (0.8 WN 20%) for soft foundation.

For the sake of reappearance ability of simulated white noise earthquake, we recorded some typical 6 types of them in the data recorder and reproduced them at analysis. Programming details is indicated in table 1.

On the screen of the oscillograph, we can see the curve of the result if sweep time of the oscillograph is speeded up. and if that of the oscillograph is slow down, we can observe the envelope of wave only on the screen. We took them by the still camera at the latter and by the movie camera at the former. The results are shown in figure 9, 10, 11 and photograph 2. These all results are the averages of 6 different white noise data and the value on the assumption that the maximum amplitude of ground displacement (Do) is to be 1cm. Steep two peaks in figure 9 are representing displacement including non-linear zone, assuming that the maximum elastic displacement is as long as 3cm, and that spring constant, when displacement is over 3cm, is reduced to 10%. In figure 10, we can observe that period becomes larger, velocity becomes smaller and this relation resembles to that of pure sine wave. For on sine wave of constant amplitude, the value of velocity and period is linear relation. Figure 11 shows percentage of acceleration to the maximum acceleration of the same type of ground displacement. The same thing, as indicated in figure 10, can be said about figure 11 ; the relation between the value of acceleration and period is almost square linear. Relative errors of this whole device and process at linear zone is believed under 10% but at non-linear zone, we cannot define it. Considering that the errors at detecting Ep=Ec point is not quite accurate and these inaccuracies are multiplied, relative errors at non-linear zone seem to be amounted to high value. In spite of the above mentioned fact, accuracies of this order are believed to be adequate for the engineering applications contemplated.

FIVE MASS SYSTEM REPRESENTING TWENTY-ONE STORIED STEEL STRUCTURE

Schematic plan and elevation of the structural frame are indicated in figure 13. Scale of columns and beams were designed by the following procedure, namely to decide previously seismic coefficient in accordance with the frame height from the ground, determine external force by multiplying the the decided seismic coefficient and weight of mass and calculate bending moment and normal force of columns and beams.

Three methods were performed to study this structure : (1) To use electronic digital computor according to the Rumge-Kutta's third order procedure. (2) modal analysis (3) electrical analogy.

(1) On the assumption of shear building, we obtain an equation for each moving mass. The force equibilium for the i th girder will require.

$$m_i \ddot{Y}_i + c_{si}(\dot{Y}_i - \dot{Y}_{i-1}) - c_{s\,i+1}(\dot{Y}_{i+1} - \dot{Y}_i) + k_i(Y_i - Y_{i-1} - \varepsilon_i)$$

$$- k_{i+1}(Y_{i+1} - Y_i - \varepsilon_{i+1}) + F_i - F_{i+1} = -m_i \alpha(\tau_j) \qquad (17)$$

Fi : Fi = Qyi if ki = 0
or Fi = 0 if ki $\neq$ 0
in this case Qyi is the maximum shear force of i th story.

The time interval adopted at analysis is 0.01 sec. and ground acceleration is the record of El Centro Earthquake (1940, N.S., max. acc. 330 gal). Table 2 is the results.

(2) Then the original building is reduced to the five mass system (table 3) and solved using the method of modal analysis. The results are in the table 4.

(3) Five mass system was adopted to simulate an electrical analogy. m and k were therefore used as shown table 3. From table 3, inductance, capacitance and resistance were decided as shown in table 5. As for ground displacement, five different types of displacement as voltage were tried to electrical analogy networks. (1) white noise (2) exceeding period of 0.8 second (3) that of 0.5 second (4) that of 0.313 second and (5) that of 0.2 second, and amplitude of each ground displacement is 5cm at maximum. Results of displacement of i th mass relative to the (i-1) th mass together with shear in the i th story at 0.8 second period is on figure 13. Figure 13 shows that maximum displacement occurs at 0.8 second and in this period ground acceleration comes to be 0.32g, assuming that displacement were sine wave having amplitude of 5cm.

NOMENCLATURE:

m	= concentrated mass
y	= absolute structure displacement
Cs	= damping factor
t	= time
k	= spring constant of the structure
D_o	= absolute ground displacement, a function of time
L	= reactance of coil
R	= resistance
i	= current

c = capacitance

Eo = external voltage, a function of time

Q = coulomb

T = period of structure

$\left(\frac{C}{C_s}\right)$ = fraction of critical damping of structure

N = time factor between structure and electrical system

Ec = the voltage across the condencer

Er = the voltage across the resister

E_L = the voltage across the reactor coil

m_i = concentrated mass of i th story

y_i = absolute displacement of i th mass

k_i = spring constant between i th and i-1 th mass

Csi = damping factor between i th and i-1 th mass

Li Ri Ci = as shown in figure 4

Qi = coulomb of i th circuit

kp = spring constant when frame is yielded

ε = remainded displacement

$yp_{+(-)}$ = relative displacement of structure at yield point of the frame to possitive side (to negative side)

Qp remainded coulomb

$Ep_{+(-)}$ = corresponding voltage to $yp_{+(-)}$ to possitive side (to negative side)

λ = constant of damping ratio

I = intensity of earthquake

Ea = the voltage to modulate white noise

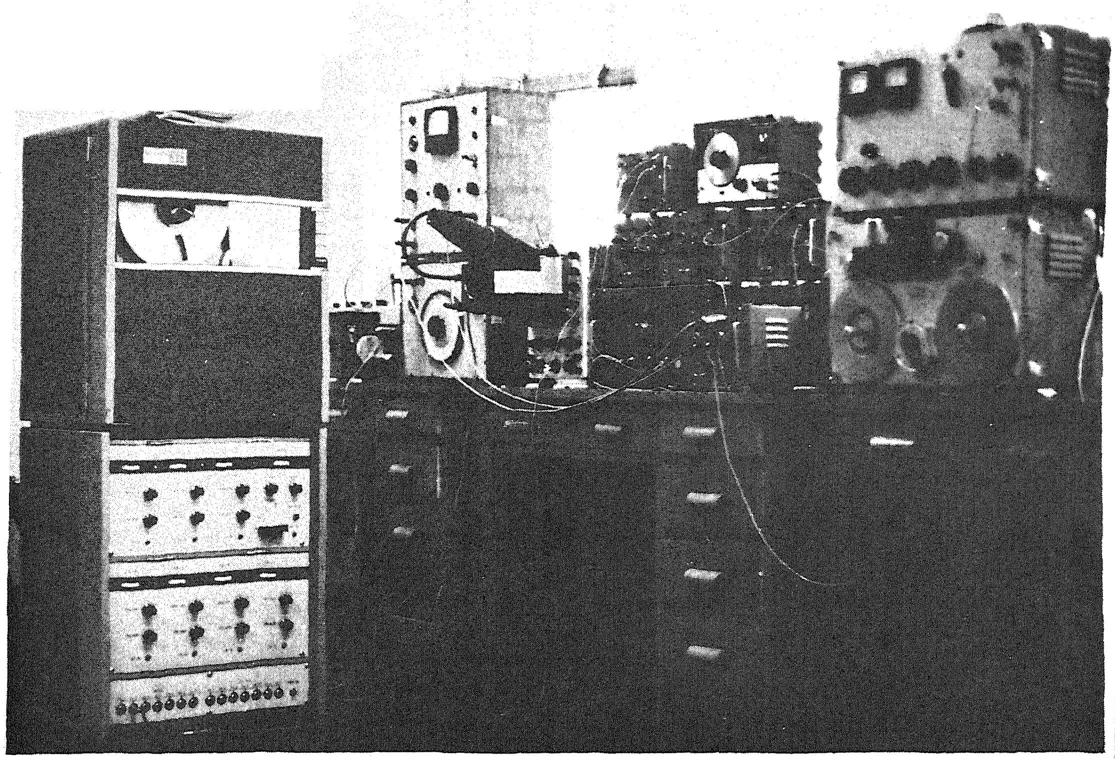
I_o = the maximum intensity of earthquake

Eao = the initial voltage to modulate white noise

Ca, Ra = capacitance and resistance of modulation circuit

Yi = relative displacement of i th story to the ground

$\alpha(\tau_j)$ = ground acceleration at time



PHOTOGRAPH I ELECTRICAL ANALOGY SYSTEM

TABLE I

No	T SEC	F $\frac{I}{sec}$	m $\frac{tsec^2}{cm}$	k $\frac{t}{cm}$	α $\frac{Hcm}{tsec^2}$	β $\frac{Ft}{cm}$	L H	C $\times 10^{-7}$F	f $\frac{1}{sec}$	h	R KΩ	y(D) $\frac{cm}{Eo}$	y' (V) $\frac{cm}{Eosec}$	y'' (A) $\frac{g}{Eo}$
I	0.25	4	0.5	3.2	40	4.0×10^{-7}	20	1.25	1000	0.05	12.7	$\frac{Ec}{E}$	256	0.645
2				10						0.025	6.35	"	512	"
3	0.5	2	0.5	0.8	40	4.0×10^{-7}	20	5	500	0.05	6.35	"	126	0.162
4										0.025	3.18	"	256	"
5	1.0	1	0.5	0.2	40	4.0×10^{-7}	20	20	250	0.05	4.25	"	63	0.048
66				"						0.025	2.13	"	126	"
7	1.5	0.67	1	0.178	40	4.0×10^{-7}	40	22.4	167	0.05	3.18	"	42	0.018
8										0.025	1.59	"	84	"

TABLE 2

STORY	δ_i max	time (sec)	δ_Y	$\frac{\delta_{max}}{\delta_Y} = \mu$
21	0.96	6.45	5.25	0.183
20	1.53	6.51	3.32	0.461
19	2.05	"	"	0.617
18	2.52	"	"	0.758
17	2.92	"	"	0.878
16	2.73	6.56	2.98	0.915
15	2.33	"	"	0.782
14	2.50	6.61	2.91	0.858
13	2.36	"	2.53	0.933
12	2.19	6.66	2.84	0.770
11	2.31	"	2.74	0.842
10	2.15	5.61	2.48	0.866
9	1.83	"	2.53	0.719
8	1.93	5.56	2.41	0.809
7	1.97	"	2.19	0.899
6	1.88	"	2.52	0.745
5	1.95	"	2.30	0.847
4	1.78	"	2.09	0.[illegible]
3	2.02	"	2.84	0.710
2	2.05	5.51	2.60	0.788
1	1.95	"	2.42	0.[illegible]05

δ_{imax}: relative maximum deflection

time: the time when δ_{max} comes

δ_Y : the maximum elastic deflection

TABLE 3

STORY	i'	m_i	m_i'	k_i	k_i'
21		0.317		117	
20		0.563	0.88	196	
19		0.330		"	
18	5	"		"	41.9
17		0.330		196	
16		"	1.32	234	
15		"		300	
14	4	"		"	62.4
13		0.330		340	
12		"	1.32	376	
11		"		"	
10	3	"		421	85.8
9		0.330		553	
8		"	1.32	"	
7		"		572	
6	2	"		630	143.7
5		0.330		630	
4		"		718	
3		"	1.51	741	
2		0.382		"	
1	1	"		699	132.9

TABLE 4

i' (mass)	δ_i	shear force at i th mass (ton)
5	9.3	390
4	11.6	723
3	9.8	952
2	7.8	1158
1	9.5	1295

TABLE 5

i th mass	m $\frac{t\,sec^2}{cm}$	k $\frac{t}{cm}$	T sec	α $\frac{Hcm}{tsec^2}$	β $\frac{Ft}{cm}$	L H	C $\times 10^{-9}F$	f $\frac{I}{sec}$	R $K\Omega$
5	0.88	41.9	0.912	20		17.6	19	274	3.04
4	1.32	62.4	0.915	20	"	26.4	12.8	273	4.55
3	1.32	85.8	0.78	20	"	26.4	9.35	321	5.3
2	1.32	143.7	0.603	20	"	26.4	5.56	415	6.76
1	1.51	132.9	0.67	20	"	30.2	6.0	374	7.1

h = 0.05

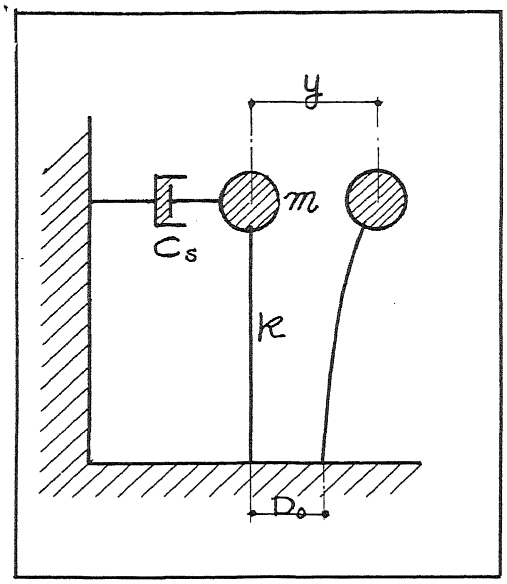


Fig.1

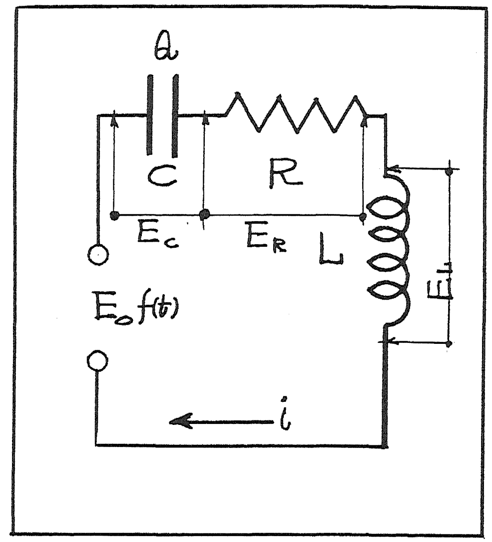


Fig.2

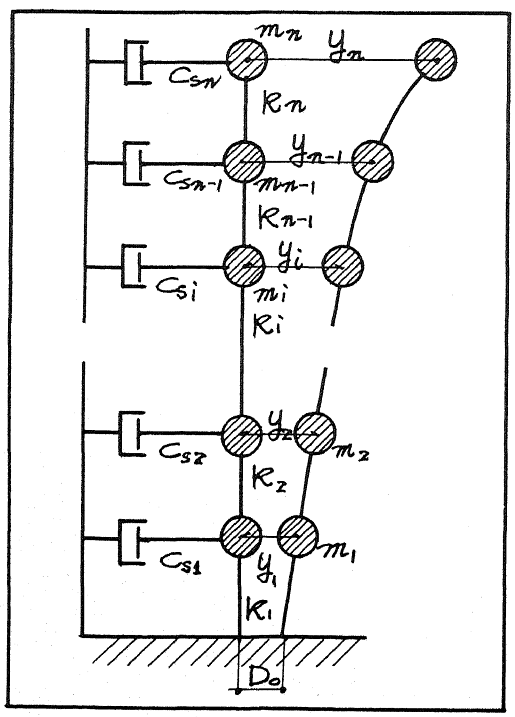


Fig.3

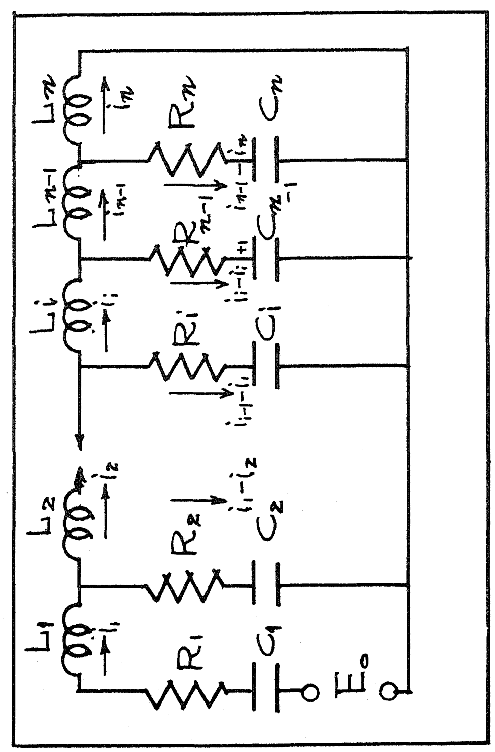


Fig.4

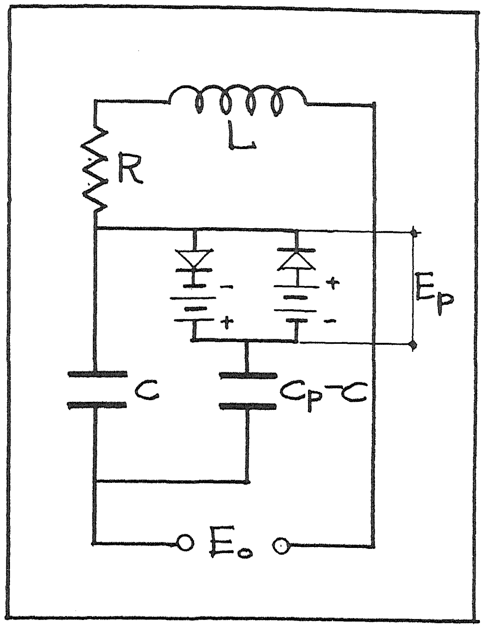


Fig.5

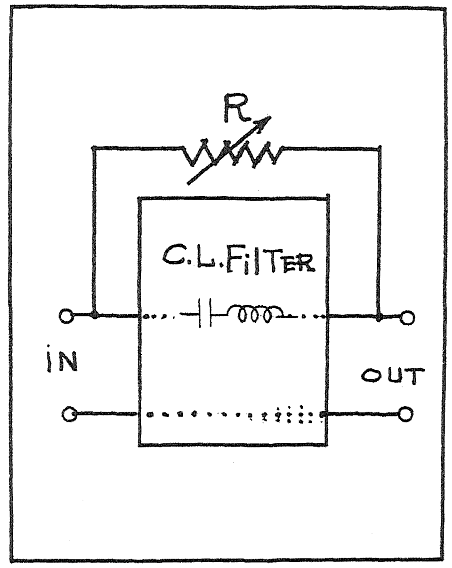


Fig.6

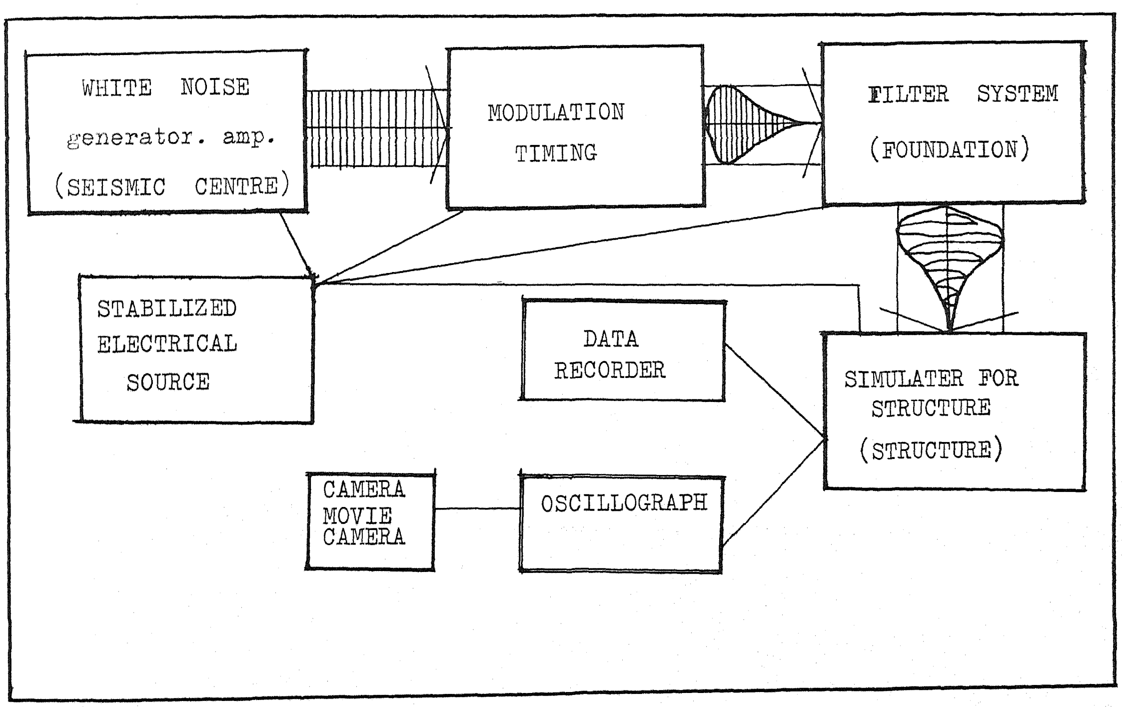
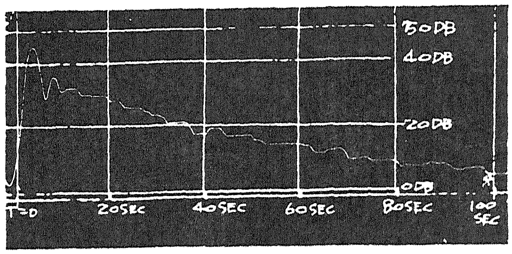


Fig.7



RELATION BETWEEN TIME AND INTENSITY
OF SIMULATED EARTHQUAKE

Fig.8

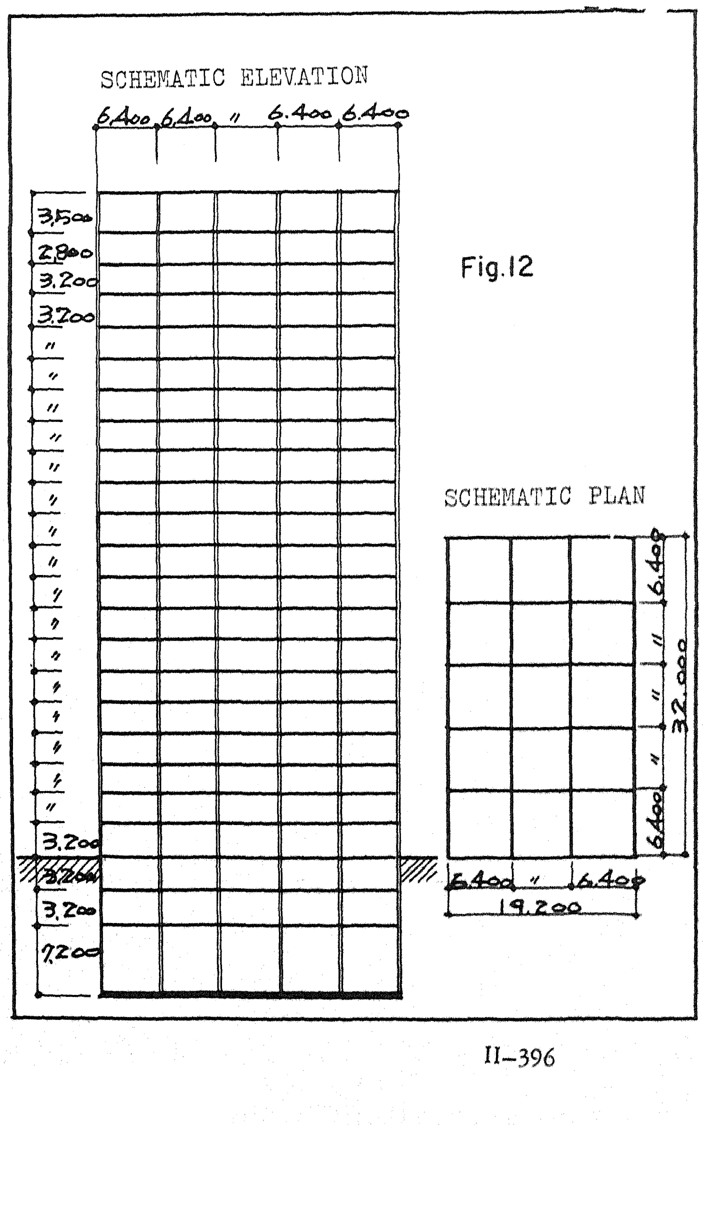


Fig.12

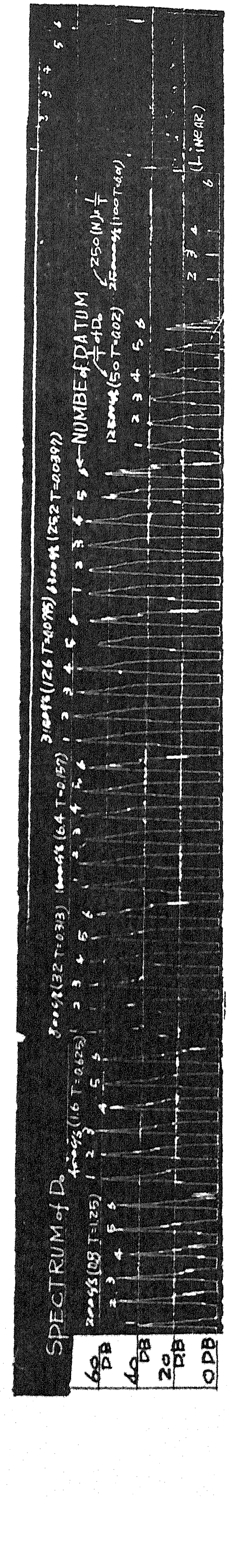
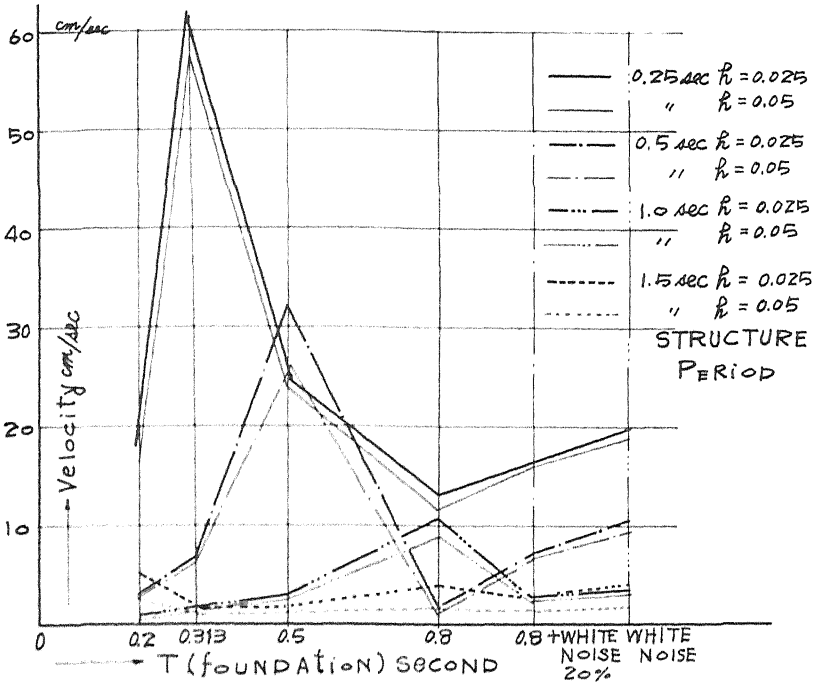
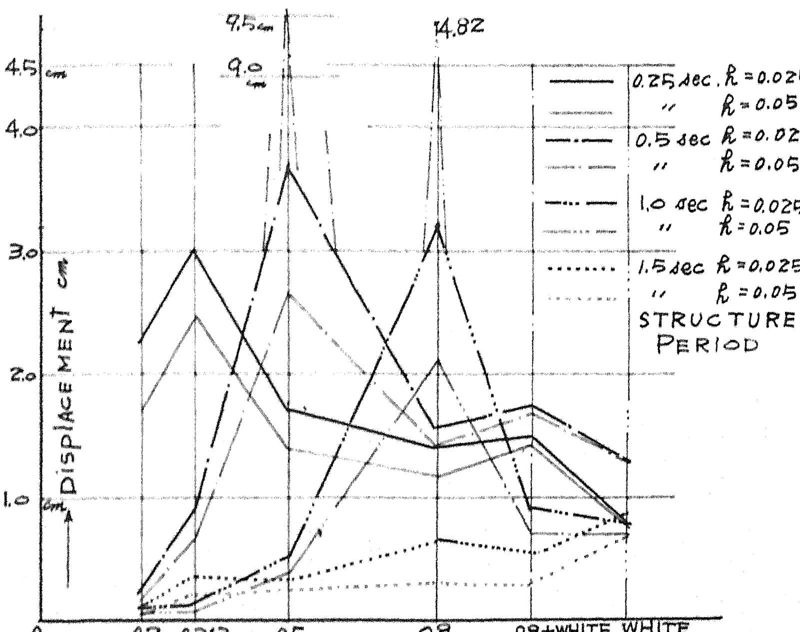


Photo.2



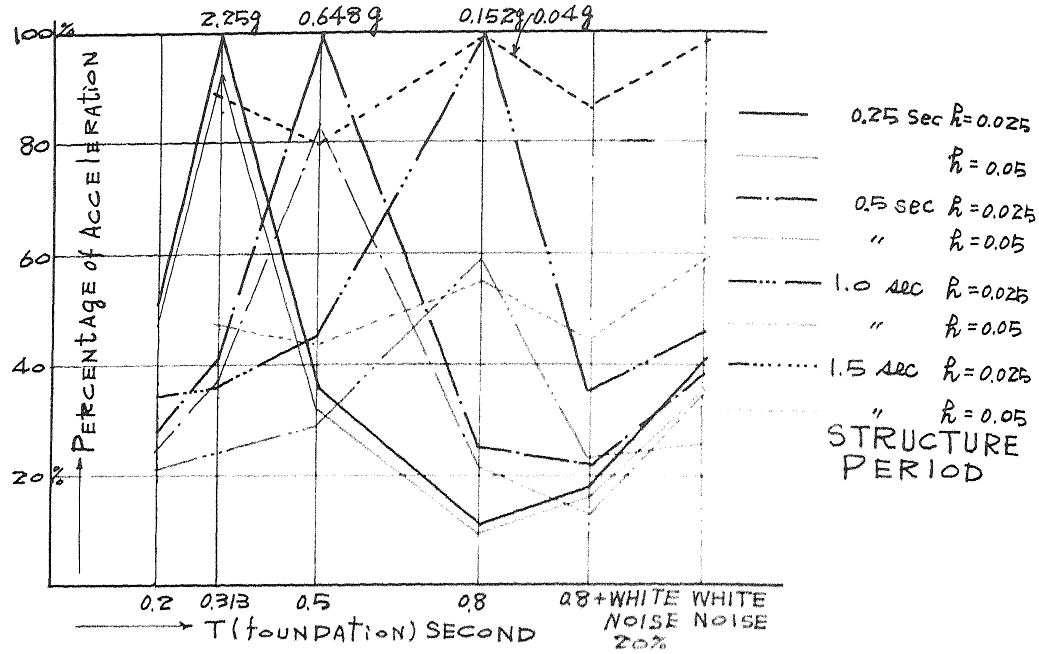
cm/sec
60
50
40
30
20
10
0
Velocity cm/sec
0.2
0.313
0.5
0.8
0.8+WHITE NOISE 20%
WHITE NOISE
T (foundation) second
0.25 sec $h = 0.025$
" $h = 0.05$
0.5 sec $h = 0.025$
" $h = 0.05$
1.0 sec $h = 0.025$
" $h = 0.05$
1.5 sec $h = 0.025$
" $h = 0.05$
STRUCTURE PERIOD

Fig. 9



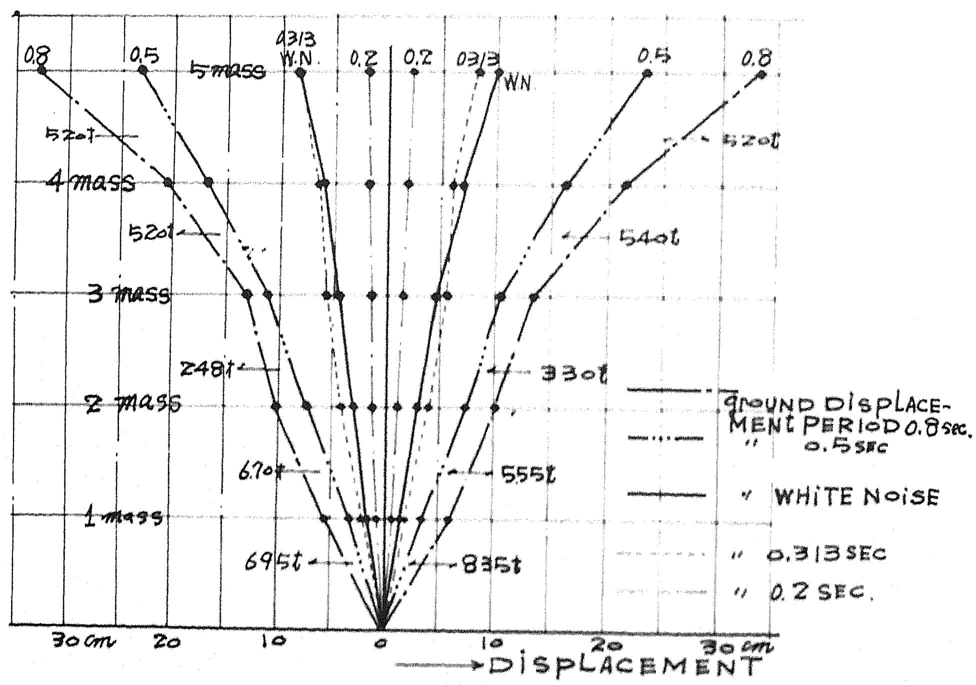
9.5 cm
9.0 cm
4.82
4.5 cm
4.0
3.0
2.0
1.0 cm
0
DISPLACEMENT cm
0.2
0.313
0.5
0.8
0.8+WHITE NOISE 20%
WHITE NOISE
T (foundation) SECOND
0.25 sec. $h = 0.025$
" $h = 0.05$
0.5 sec $h = 0.025$
" $h = 0.05$
1.0 sec $h = 0.025$
" $h = 0.05$
1.5 sec $h = 0.025$
" $h = 0.05$
STRUCTURE PERIOD

Fig. 10



2.25g
0.648g
0.152g
0.04g
100%
80
60
40
20%
PERCENTAGE of ACCELERATION
0.25 Sec h=0.025
h = 0.05
0.5 sec h=0.025
" h=0.05
1.0 sec h=0.025
" h=0.05
1.5 sec h=0.025
" h=0.05
STRUCTURE PERIOD
0.2
0.313
0.5
0.8
0.8+WHITE NOISE 20%
WHITE NOISE
T(foundation) SECOND

Fig.11



0.8
0.5
5 mass
0.313 W.N.
0.2
0.2
0.313
W.N.
0.5
0.8
520t
4 mass
520t
540t
3 mass
248t
330t
2 mass
670t
555t
1 mass
695t
835t
ground DISPLACEMENT PERIOD 0.8sec.
" 0.5sec
" WHITE NOISE
" 0.313 SEC
" 0.2 SEC.
30cm
20
10
0
10
20
30cm
DISPLACEMENT

Fig.12