SEISMIC FORCE EFFECT ON SUBMERGED BRIDGE PIERS WITH ELLIPTIC CROSS SECTIONS

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ABSTRACT

A theoretical solution of dynamic water pressure during earthquakes on submerged structures with circular or elliptic cross sections such as bridge piers and intake towers is obtained and the results of calculation are shown in diagram. Then, by using a theoretical solution of dynamic water pressures on them during their elastic vibrations, a exact solution of free vibrations of bridge piers with arbitrary loaded weights and water level are obtained, and a convenient diagram for computing the natural periods of vibrations of submerged bridge piers are shown. Furthermore, a formula giving the magnitude and the vertical distribution of added mass during vibrations is obtained, enabling to calculate the natural period of vibration of submerged structures by energy method.

INTRODUCTION

A most important matter for us in designing submerged structures is to decide the dynamic water pressure on them during earthquakes. It is already known that when the submerged structures vibrate, a part of the water around the structures acts upon them as a added mass, and for a case of a structure with a simple cross section such as a circular cylinder, a theoretical three- dimensional solution of dynamic water pressure has been obtained and verified with experiments[1]. But, for a case of a structure with a rectangular cross section or for a case of a plate, a theoretical three-dimensional solution has not yet been obtained and even in the case of two-dimensional theory, a large difference lies between the theoretical values and the experimental values.

That is to say, by analyzing two-dimensional potential flow about a rod of infinite length having a rectangular cross section and moving broade side on, Riabouchinsky[2] found that the added mass of plate was 1.05 times as large as the mass of fluid included by the cylinder having the radius of equal size of the width of the plate, and T. E. Stelson[3] confirmed it by model experiments. While, R. W. Clough[4] reported in his paper that the added mass of plate was about 1.3 times as large as the mass of the cylinder of fluid having the radius of the same size as the width of the plate.

It seems that the difference between these results of experiments depends upon various factors, such as methods of experiments and conditions of experiments. The author obtained a theoretical solution of three-dimension about the dynamic water pressure on the bridge piers and the intake towers having circular or elliptic cross sections, and furthermore, confirmed the theory by model experiments, making clear the dynamic water pressure (added mass) on submerged structures during earthquakes and elastic vibrations.

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THEORY OF THE DYNAMIC WATER PRESSURE ON BRIDGE PIERS WITH ELLIPTIC CROSS SECTION

1. Differential equation

As showh in Fig. 1, we consider anelliptic cylinder of length, ℓ, the major axis,2a, and the minor axis, 2b, standing in water of depth, h. In order to transform the rectangular coordinate, x, y, into the elliptic coordinate, ξ, η, putting as,

$$\left.\begin{aligned} x &= k \cosh\xi \cos\eta \\ y &= k \sinh\xi \sin\eta \end{aligned}\right\} \qquad (1)$$

the coordinate of the surface of the ellipse will be expressed by

$$\xi = \xi_0 = \tan^{-1}\frac{b}{a} \qquad (2)$$

and the coordinate $\xi = 0$ will express the straight line of length, 2k, the distance between both focuses of the ellipse.

The differential equations of motions of the water particles may be expressed as follows.

$$\left.\begin{aligned} \frac{w_0}{g}\frac{\partial^2 u}{\partial t^2} &= -\frac{\partial p}{\ell\partial\xi} \\ \frac{w_0}{g}\frac{\partial^2 v}{\partial t^2} &= -\frac{\partial p}{\ell\partial\eta} \\ \frac{w_0^2}{g}\frac{\partial^2 w}{\partial t^2} &= -\frac{\partial p}{\ell\partial\eta} \end{aligned}\right\} \qquad (3)$$

$$p = -E_v\left\{\frac{\partial u}{\ell\partial\xi} + \frac{\partial v}{\ell\partial\eta} + \frac{\partial w}{\partial z}\right\} + w_0(h-z) \qquad (4)$$

$$\ell^2 = \frac{k^2}{2}(\cosh 2\xi - \cos 2\eta) \qquad (5)$$

Where, w_0 is the specific weight of water, E_v the volmetric modulus of water, g the acceleration of gravity, p the water pressure, t the time, and u, v, and w the displacements of water particles towards the coordinate ξ, η, and z respectively.

Now, expressing the dynamic water pressure with σ, the differential equation about σ may be expressed by the following equation, which is obtained by using the equations (3) and (4).

$$\frac{\partial^2\sigma}{\ell^2\partial\xi^2} + \frac{\partial^2\sigma}{\ell^2\partial\eta^2} + \frac{\partial^2\sigma}{\partial z^2} - \frac{w_0}{gE_v}\frac{\partial^2\sigma}{\partial t^2} = 0 \qquad (6)$$

2. Boundary condition

Assuming the earthquake as a stationary sine form wave, $(\alpha g/\omega^2)\sin\omega t$, the boundary conditions will be as follows.

In case of the earthquake in the direction of the minor axis of the elliptic cylinder;

$$\left.\begin{aligned}
&\text{(i)}\ \left(\frac{\partial\sigma}{\partial z}\right)_{z=0}=0 \qquad \text{(ii)}\ (\sigma)_{z=h}=0\\
&\text{(iii)}\ (\sigma)_{\eta=0}=0 \qquad \text{(iv)}\ \left(\frac{\partial\sigma}{\partial\eta}\right)_{\eta=\frac{\pi}{2}}=0\\
&\text{(v)}\ \left(\frac{\partial\sigma}{l\partial\xi}\right)_{\xi=\xi_0}=-\frac{w_0}{g}\frac{\partial^2 u}{\partial t^2}=\alpha w_0\sin\theta\sin\omega t\\
&\text{i.e.}\\
&\qquad \left(\frac{\partial\sigma}{\partial\xi}\right)_{\xi=\xi_0}=\alpha w_0 a\sin\eta\sin\omega t\\
&\text{(vi)}\ \left(\frac{\partial\sigma}{\partial\xi}\right)_{\xi=\xi_1}=0 \qquad (\xi_1\gg\xi_0)
\end{aligned}\right\} \qquad (7)$$

In case of the earthquake in the direction of the major axis of the elliptic cylinder;

$$\left.\begin{aligned}
&\text{(i)}\ \left(\frac{\partial\sigma}{\partial z}\right)_{z=0}=0 \qquad \text{(ii)}\ (\sigma)_{z=h}=0\\
&\text{(iii)}\ \left(\frac{\partial\sigma}{\partial\eta}\right)_{\eta=0}=0 \qquad \text{(iv)}\ (\sigma)_{\eta=\frac{\pi}{2}}=0\\
&\text{(v)}\ \left(\frac{\partial\sigma}{\partial\xi}\right)_{\xi=\xi_0}=\alpha w_0 b\cos\eta\sin\omega t\\
&\text{(iv)}\ \left(\frac{\partial\sigma}{\partial\xi}\right)_{\xi=\xi_1}=0
\end{aligned}\right\} \qquad (8)$$

3. Solution of the differential equation

The solution of the differential equation (6) in case of the earthquake in the direction of the minor axis may be obtained as follows.

$$\sigma=\sum_{m=0}^{s-1}\frac{4\alpha w_0 a(-1)^m B_1^{(1)} S_1(\xi,\bar{q}_m')\,se_1(\eta,\bar{q}_m')\cos\lambda_m z\cdot\sin\omega t}{S_1'(\xi_0,\bar{q}_m')\left\{\sum_{r=0}^{\infty}\left(B_{2r+1}^{(1)}\right)^2\right\}(2m+1)\pi}$$

$$+\sum_{m=s}^{\infty}\frac{4\alpha w_0 a(-1)^m A_1^{(1)} S_1(\xi,-q_m)\,se_1(\eta,-q_m)\cos\lambda_m z\cdot\sin\omega t}{S_1'(\xi_0,-q_m)\left\{\sum_{r=0}^{\infty}\left(A_{2r+1}^{(1)}\right)^2\right\}(2m+1)\pi} \tag{9}$$

Where,

$$S_1(\xi,q_m')=Se_1(\xi,q_m')-\frac{Se_1'(\xi_1,q_m')}{Ge_1'(\xi_1,q_m')}Ge_1(\xi,q_m')$$

$$S_1(\xi,-q_m)=Se_1(\xi,-q_m)-\frac{Se_1'(\xi_1,-q_m)}{Ge_1'(\xi_1,-q_m)}Ge_1(\xi,-q_m)$$

$$q_m'=\frac{k^2}{4}(C^2-\lambda_m^2)\qquad C>\lambda_m$$

$$q_m=\frac{k^2}{4}(\lambda_m^2-C^2)\qquad \lambda_m>C$$

$$\lambda_m=\frac{(2m+1)\pi}{2h},\qquad C^2=\frac{w_0\omega^2}{gE_v}$$

$$se_1(\eta,q_m')=\sum_{r=0}^{\infty}B_{2r+1}^{(1)}\sin(2r+1)\eta$$

$$se_1(\eta,-q_m)=\sum_{r=0}^{\infty}(-1)^r A_{2r+1}^{(1)}\sin(2r+1)\eta$$

and $se_1(\xi,q_m')$ and $se_1(\xi,-q_m)$ are Mathieu functions, and $Se_1(\xi,q_m')$, $Se_1(\xi,-q_m)$ $Ge_1(\xi,q_m')$ and $Ge_1(\xi,-q_m)$ are modified Mathieu functions.

Therefore, the resultant force of the dynamic water pressure in the direction of earthquake per unit length of the elliptic cylinder may be expressed by the next formula.

$$P=4\int_0^{\pi/2}\sigma\, l\, d\eta\sin\theta=\alpha w_0\pi a^2 F$$

$$F=\sum_{m=0}^{s-1}\frac{4(-1)^m S_1(\xi_0,q_m')\{B_1^{(1)}\}^2\cos\lambda_m z\sin\omega t}{(2m+1)\pi S_1'(\xi_0,q_m')\left\{\sum_{r=0}^{\infty}\left(B_{2r+1}^{(1)}\right)^2\right\}}$$
$$+\sum_{m=s}^{\infty}\frac{4(-1)^m S_1(\xi_0,-q_m)\{A_1^{(1)}\}^2\cos\lambda_m z\sin\omega t}{(2m+1)\pi S_1'(\xi_0,-q_m)\left\{\sum_{r=0}^{\infty}\left(A_{2r+1}^{(1)}\right)^2\right\}}$$

In the same way, the resultant force of the dynamic water pressure per unit length of the elliptic cylinder in the direction of the major axis may be obtained as follows.

$$P = \alpha w_0 \pi b^2 F$$

$$F = \sum_{m=0}^{s-1} \frac{4(-1)^m C_1(\xi_0, \vartheta'_m)\{A_1^{(1)}\}^2 \cos\lambda_m z \sin\omega t}{(2m+1)\pi\, C'_1(\xi_0, \vartheta'_m)\left\{\sum_{r=0}^{\infty}(A_{2r+1}^{(1)})^2\right\}}$$

$$+ \sum_{m=s}^{\infty} \frac{4(-1)^m C_1(\xi_0, -\vartheta_m)\{B_1^{(1)}\}^2 \cos\lambda_m z \cdot \sin\omega t}{(2m+1)\pi\, C'_1(\xi_0, -\vartheta_m)\left\{\sum_{r=0}^{\infty}(B_{2r+1}^{(1)})^2\right\}} \qquad (11)$$

Where,

$$C_1(\xi, \vartheta'_m) = Ce_1(\xi, \vartheta'_m) - \frac{Ce'_1(\xi_1, \vartheta'_m)}{Fe'_1(\xi_1, \vartheta'_m)} Fe_1(\xi, \vartheta'_m)$$

$$C_1(\xi, -\vartheta_m) = Ce_1(\xi, -\vartheta_m) - \frac{Ce'_1(\xi_1, -\vartheta_m)}{Fe'_1(\xi_1, -\vartheta_m)} Fe_1(\xi, -\vartheta_m)$$

F is the function of b/a, h/2a and z/h, showing the ratio of added mass on the elliptic cylinder to the mass of fluid included by the circular cylinder having the diameter of length, 2a.

4. Result of computation

Fig. 2 shows the values of F computed by using the formula (10) or (11), by changing the ratio b/a and h/2a variously. As is seen from the figure, in case the ratio h/2a is small, the smaller the ratio b/a becomes, the larger the value of F becomes, but in case the ratio h/2a is large (i. e. for slender cylinder), the value of F approaches to 1 with no relation to the value of the ratio b/a. This phenomenon is different from the analytical results by Riabouchinsky and the experimental values by Clough.

Arranging the value of F into one formula by using the ratio b/a, h/2a and z/h as factors, it may be expressed by the next formula.

$$F = C\left(1 - \frac{z}{h}\right)^{\beta} \qquad (12)$$

Where, C and β are the functions of the ratio b/a and h/2a, and are shown in Fig. 3.

By using the equation (12), the shearing force S and the bending moment M due to dynamic water pressure at a arbitrary point (z_0) of the elliptic cylinder, may be calculated by the next formula.

$$S = \int_{z_0}^{h} p\, dz = \alpha w_0 \pi a^2 C h \frac{(1 - z_0/h)^{\beta+1}}{\beta+1} \qquad (13)$$

$$M = \int_{z_0}^{h} p(z - z_0)\, dz = \alpha w_0 \pi a^2 C h^2 \frac{(1 - z_0/h)^{\beta+2}}{(\beta+1)(\beta+2)} \qquad (14)$$

(I) Exact solution

1. Differential equation of free vibration

Expressing the lateral displacement of the pier with y, cross sectional area of the pier with A, specific weight of the material of the pier with w, and flexural rigidity with EI, the differential equation of free vibration of the pier will be as follows.

$$EI\frac{\partial^4 y}{\partial z^4}+\frac{wA}{g}\frac{\partial^2 y}{\partial t^2}=-P(z,t) \tag{15}$$

Where, $P(z,t)$ is the resultant force of the dynamic water pressure per unit length of the pier in the direction of vibration, and will be expressed by the next formula by assuming that the pier will vibrate as $y = Y\sin nt$.

In case, $z \geqq h$. $P=0$

In case, $h \geqq z \geqq 0$, for the vibration in the direction of minor axis,

$$\begin{aligned}P=&\frac{w_0 n^2}{gh}\pi a^2\sum_{m=0}^{s-1}\frac{2S_1(\xi_0,q_m')\{B_1^{(1)}\}^2\int_0^h Y\cos\lambda_m z\,dz\cdot\cos\lambda_m z\cdot\sin nt}{S_1'(\xi_0,q_m')\left\{\sum_{r=0}^{\infty}\left(B_{2r+1}^{(1)}\right)^2\right\}}\\&+\frac{w_0 n^2}{gh}\pi a^2\sum_{m=s}^{\infty}\frac{2S_1'(\xi_0,-q_m)\{A_1^{(1)}\}^2\int_0^h Y\cos\lambda_m z\,dz\cos\lambda_m z\sin nt}{S_1'(\xi_0,-q_m)\left\{\sum_{r=0}^{\infty}\left(A_{2r+1}^{(1)}\right)^2\right\}}\end{aligned} \tag{16}$$

and for the vibration in the direction of major axis,

$$\begin{aligned}P=&\frac{w_0 n^2}{gh}\pi b^2\sum_{m=0}^{s-1}\frac{2C_1(\xi_0,q_m')\{A_1^{(1)}\}^2\int_0^h Y\cos\lambda_m z\,dz\cos\lambda_m z\cdot\sin nt}{C_1'(\xi_0,q_m')\left\{\sum_{r=0}^{\infty}\left(A_{2r+1}^{(1)}\right)^2\right\}}\\&+\frac{w_0 n^2}{gh}\pi b^2\sum_{m=s}^{\infty}\frac{2C_1(\xi_0,-q_m)\{B_1^{(1)}\}^2\int_0^h Y\cos\lambda_m z\cdot dz\cos\lambda_m z\sin nt}{C_1'(\xi_0,-q_m)\left\{\sum_{r=0}^{\infty}\left(B_{2r+1}^{(1)}\right)^2\right\}}\end{aligned} \tag{17}$$

2. Natural period and mode of vibration

Now, expressing the mode of vibration of a bridge pier in the air having a loaded weight W at the top with $\Phi_\mu(k_\mu z)$ ($\mu = 1, 2, 3, \cdots\cdots$), the deflection of the pier under water may be assumed as follows, because not only the natural frequency but also the mode of vibration will change during the vibration under water.

$$Y=\sum_{\mu=1}^{\infty}A_\mu\Phi_\mu(k_\mu z) \tag{18}$$

Where, A_μ is unknown constant, $k_\mu l$ eigen value.

Introducing the equation (18) into the equation (15) and integrating about z from 0 to ℓ after multiplying $\phi_\nu(k_\nu z)$ on both sides of above equation, the next relation will be obtained by using the orthogonality of eigen function.

$$\left(1-\frac{n_{o\nu}^2}{n^2}\right)A_\nu+\frac{w_0\pi a^2}{wA}\sum_{\mu=1}^{\infty}\frac{S'_{\nu\mu}+S_{\nu\mu}}{\Phi_\nu}A_\mu=0 \tag{19}$$

Where, $n_{o\nu}$ is the natural circular frequency of the ν-th order of the bridge pier in air. Φ_ν, $S'_{\nu\mu}$ and $S_{\nu\mu}$ are expressed by the next formula.

$$\Phi_\nu=\frac{1}{\ell}\int_0^\ell\{\phi_\nu(k_\nu z)\}^2dz+\frac{W}{wA\ell}\{\phi_\nu(k_\nu\ell)\}^2 \tag{20}$$

For the vibration in the direction of minor axis,

$$\left.\begin{aligned}S'_{\nu\mu}&=-\frac{1}{\ell h}\sum_{m=0}^{s-1}\frac{2S_1(\xi_0,\theta'_m)\{B_1^{(1)}\}^2\int_0^\ell\phi_\nu(k_\nu z)\cos\lambda_m z\,dz\int_0^\ell\phi_\mu(k_\mu z)\cos\lambda_m z\,dz}{S'_1(\xi_0,\theta'_m)\left\{\sum_{r=0}^{\infty}\left(B_{2r+1}^{(1)}\right)^2\right\}}\\ S_{\nu\mu}&=-\frac{1}{\ell h}\sum_{m=s}^{\infty}\frac{2S_1(\xi_0,-\theta_m)\{A_1^{(1)}\}^2\int_0^\ell\phi_\nu(k_\nu z)\cos\lambda_m z\,dz\int_0^\ell\phi_\mu(k_\mu z)\cos\lambda_m z\,dz}{S'_1(\xi_0,-\theta_m)\left\{\sum_{r=0}^{\infty}\left(A_{2r+1}^{(1)}\right)^2\right\}}\end{aligned}\right\} \tag{20'}$$

In the same way, the solution for the vibration in the direction of major axis will be obtained as follows.

$$\left.\begin{aligned}S'_{\nu\mu}&=-\frac{1}{\ell h}\sum_{m=0}^{s-1}\frac{2C_1(\xi_0,\theta'_m)\{A_1^{(1)}\}^2\int_0^\ell\phi_\nu(k_\nu z)\cos\lambda_m z\,dz\int_0^\ell\phi_\mu(k_\mu z)\cos\lambda_m z\,dz}{C'_1(\xi_0,\theta'_m)\left\{\sum_{r=0}^{\infty}\left(A_{2r+1}^{(1)}\right)^2\right\}}\\ S_{\nu\mu}&=-\frac{1}{\ell h}\sum_{m=s}^{\infty}\frac{2C_1(\xi_0,-\theta_m)\{B_1^{(1)}\}^2\int_0^\ell\phi_\nu(k_\nu z)\cos\lambda_m z\,dz\int_0^\ell\phi_\mu(k_\mu z)\cos\lambda_m z\,dz}{C'_1(\xi_0,\theta_m)\left\{\sum_{r=0}^{\infty}\left(B_{2r+1}^{(1)}\right)^2\right\}}\end{aligned}\right\} \tag{20''}$$

The equation (19) is the simultaneous equation about A_ν (ν = 1, 2, …), and from these equations, we are able to calculate the natural circular frequency n of bridge pier in water. And then, the relation between A_1, A_2, ……, will be calculated and the mode of vibration of the pier in water may be obtained.

3. Results of computation

Fig. 4-a shows the results of computation of elongation of natural period of vibration due to water in case of b/a=0.5 and W=0, by changing the values of $w_0\pi a^2/wA$, $\ell/2a$ and h/ℓ variously.

Fig. 4-b shows the results of computation of elongation of natural period of vibration due to water in case of b/a=0.5 and $w_0\pi a^2/wA$=1.0, by changing the values of $W/wA\ell$, $\ell/2a$ and h/ℓ variously.

From these figures, the next matters may be concluded.

(1) As the ratio $\ell/2a$ becomes smaller, the effect of added mass upon the natural period of vibration becomes smaller.

(2) In the vibration of the primary mode, the elongation of the natural period of vibration becomes smaller with the drop of water level, but in the vibration of the secondary mode, the change of the elongation of the natural period is small for the value of $h/\ell=0.6\sim0.8$, and the elongation of natural period becomes rapidly small for the value of $h/\ell=0.5$.

(3) In the neighbourhood of $h/\ell=1.0$, the elongation of natural period is almost equal for both the primary and the secondary modes of vibration, but in the region where the value of h/ℓ is small, the effect of water upon the natural period is greater in the secondary mode of vibration than in the primary mode of vibration.

(4) The usual method of calculation that a constant volume of water will be added to the structure during its vibration may be a mistake, because the elongation of natural period of vibration in the primary mode is much different from that in the vibration of the secondary mode. That is to say, for the purpose of explaining the elongation of the natural period by the added mass, we must consider the vertical distribution of the added mass.

(5) The mode of vibration in water varies much greater in the vibration of the secondary mode than in the vibration of the primary mode.

(6) The elongation of the natural period of vibration of the bridge pier due to surounding water may be computed easily by the next formula.

$$\frac{T_w}{T_a}=\sqrt{1+\frac{\delta}{1+4\gamma}\,\frac{C}{C_{0.5}}\,\psi} \tag{21}$$

Where,

$$\delta=w_0\pi a^2/wA\ ,\quad \gamma=W/wA\ell$$

and the value of ψ and C will be obtained from Fig. 5 and Fig. 7, respectively, and $C_{0.5}$ is the value of C for b/a=0.5. Fig. 5 is derived from Fig. 4-a and Fig. 4-b, and Fig. 7 may be explained later.

(II) Approximate solution by added mass

1. Added mass

In case the bridge pier has a uniform cross sectional area, the elongation of the natural period may be computed by using the equation (21), but in case the bridge pier has not a uniform cross sectional area, energy method will be used and for this purpose, it is necessary to make clear the vertical distribution of added mass.

The author calculated the dynamic water pressure which occurs when the bridge pier vibrates in the various type of mode of vibration, and the results of calculation are shown in Fig. 6. From these results, the added mass, W_a, which will produce the equivalent dynamic water pressure upon the pier may be expressed approximately by the following formula.

$$W_a = w_0 \pi a^2 C (1 - z/h)^{\beta} \tag{22}$$

The value of β and C may be obtained from Fig. 3 and Fig. 7 respectively. A more or less error takes place at the bottom of the pier, but the error at this part has little effect on the natural period of vibration. See Fig. 4. For $h/\ell < 0.4$, the effect of water may be neglected.

2. Approximate solution of natural period by the use of added mass

The black points in Fig. 3 shows the elongation of the natural period of vibration which were computed by energy method by using the added mass shown in equation (22).

It is clearly seen that the approximate values fairly coincide with the values computed by the theoretically exact solution.

FORCED VIBRATION OF BRIDGE PIER DURING EARTHQUAKE

(I) Exact solution

Assuming the earthquake as $(\alpha g/\omega^2)\sin \omega t$, the dynamic water pressure as a external force may be expressed by the equation (10) and (11). Therefore, by introducing

$$y = Y \sin \omega t \tag{23}$$

into the differential equation (15) and using the equation (18), the simultaneous equations about $A_\nu (\nu = 1, 2, \cdots\cdots)$ may be obtained as follows.

$$\left(1 - \frac{n_{0\nu}^2}{\omega^2}\right) A_\nu + \frac{w_0 \pi a^2}{w A} \sum_{\mu=1}^{\infty} \frac{S'_{\nu\mu} + S_{\nu\mu}}{\Phi_\nu} A_\mu = \frac{\alpha g}{\omega^2} \left\{ F_\nu + \frac{w_0 \pi a^2}{w A} (D'_\nu + D_\nu) \right\} \frac{1}{\Phi_\nu} \tag{24}$$

Where, Φ_ν, $S'_{\nu\mu}$ and $S_{\nu\mu}$ are expressed by the formula (20), and in the vibration in the direction of minor axis,

$$\left.\begin{aligned} F_\nu &= \frac{1}{\ell}\int_0^\ell \phi_\nu(k_\nu z)\,dz + \frac{W}{wA\ell}\{\phi_\nu(k_\nu \ell)\} \\ D'_\nu &= -\frac{1}{\ell h}\sum_{m=0}^{s-1} \frac{4(-1)^m S_1(\xi_0, \delta'_m)\{B_1^{(1)}\}^2 \int_0^\ell \phi_\nu(k_\nu z)\cos\lambda_m z \cdot dz}{(2m+1)\pi\, S'_1(\xi_0, \delta_m)\left\{\sum_{r=0}^{\infty}(B_{2r+1}^{(1)})^2\right\}} \\ D_\nu &= -\frac{1}{\ell h}\sum_{m=s}^{\infty} \frac{4(-1)^m S_1(\xi_0, -\delta_m)\{A_1^{(1)}\}^2 \int_0^\ell \phi_\nu(k_\nu z)\cos\lambda_m z\, dz}{(2m+1)\pi\, S'_1(\xi_0, -\delta_m)\left\{\sum_{r=0}^{\infty}(A_{2r+1}^{(1)})^2\right\}} \end{aligned}\right] \tag{25}$$

In the vibration in the direction of major axis,

$$\left.\begin{aligned} D_v' &= -\frac{1}{\ell h}\sum_{m=0}^{s-1}\frac{4(-1)^m C_1(\xi_0, q_m')\{A_1^{(1)}\}^2\int_0^h \Phi_v(k_v z)\cos\lambda_m z\,dz}{(2m+1)\pi C_1'(\xi_0, q_m')\left\{\sum_{r=0}^{\infty}(A_{2r+1}^{(1)})^2\right\}} \\ D_v &= -\frac{1}{\ell h}\sum_{m=s}^{\infty}\frac{4(-1)^m C_1(\xi_0, -q_m)\{B_1^{(1)}\}^2\int_0^h \Phi_v(k_v z)\cos\lambda_m z\,dz}{(2m+1)\pi C_1'(\xi_0, -q_m)\left\{\sum_{r=0}^{\infty}(B_{2r+1}^{(1)})^2\right\}} \end{aligned}\right] \tag{25'}$$

Computing A_1, A_2, ····, from these simultaneous equations, the elastic deflection Y may be calculated by equation (18). Fig. 8 shows the state of increase of the amplitude of the forced vibration of the primary mode of the pier according to the increase of the water lavel, expressing the ratio of the amplitude in water, y_w, to the amplitude in air, y_o. It is shown for the case of b/a=0.5, ℓ/2a=10, $w_0\pi a^2$/wA=1.0 and W/wAℓ=0. For the case of arbitrary values of b/a, $w_0\pi a^2$/wA and W/wAℓ, the value, y_w/y_o, may be expressed approximately by the next formula.

$$\frac{y_w}{y_o} = 1 + \frac{\delta}{1+2.54\delta}\psi' \tag{26}$$

By using the equation (26), we are able to understand the state of increase of the amplitude and the bending moment of the submerged pier during earthquakes.

EXPERIMENT

1. Experiment for the elliptic cylinder

The purpose of this experiment is to verify the theoretical solution of dynamic water pressure obtained by the author. First of all, in order to verify that the dynamic water pressure does not depend upon the value of b/a when the ratio, ℓ/2a, is infinitely large, models of elliptic cylinder were made as shown in Fig. 9 (type I) and Table 1. Hanging these models in water by plate spring, measuring the difference of natural period of vibration and damping coefficient in water and in air, the added mass W_a was obtained by the next relation.

$$\frac{W_o + W_a}{W_o} = \frac{n_o^2}{n_w^2} = \frac{T_w^2}{T_o^2}\,\frac{1-(\varepsilon_w/n_w)^2}{1-(\varepsilon_o/n_o)^2} \tag{27}$$

Where, W_o is the vibrating mass in air, n_o and n_w the natural circular frequency in air and in water, ε_o/n_o and ε_w/n_w the damping constant.

The black points, I, in Fig. 10 show the coefficients F which are computed from the added mass W_a. The experiments were carried out with period of 0.2~0.3 sec. and the amplitude of about 0.2~3 mm. and the values of ε_w/n_w were about 0.2~0.3 %.

By the author's theory, the value of F is equal to unity with no relation to the value of b/a, and the experimental value is larger than the analytical value in case of plate or of elliptic cylinder having a small value of b/a.

Such a difference between the theoretical value and the experimental value seems to occur because of the ignoring of viscosity of water in the theory. Fig. 11 shows the theoretically calculated distribution of tangential velocity of water on the surface of ellipse, which is drawn by taking the velocity of vibration of elliptic cylinder as unity.

As is seen clearly from the figure, in case of elliptic cylinder having a small value of b/a, the velocity of water on the surface of elliptic cylinder is so large that the viscous drag becomes large and the water around the cylinder cannot flow smoothly as in the theory. Therefore, a greater part of water than that obtained by the theory will be added to the cylinder.

2. Experiment for the rectangular or other sections

The size and the type of models are shown in Fig. 9 and in Table 1. The results of experiment are shown in Fig. 10. As is clearly seen from the figure, the results of experiment for type II are nearly equal to the results of experiment for type I. In the limiting case where the value of b/a becomes to 0, the results of experiments for type I, II, and III should coincide with each other. Considering these phenomena, it may be recognized that the experimental values are fairly larger than the theoretical values for small value of b/a.

The cause of this phenomenon will be the ignoring of viscosity of water in the theory, and the difference between the theoretical values and the experimental values will be larger in case of thin sections than in case of thick sections. It seems to depend not only upon the shape of the cross section, but also upon the amplitude of vibration and the roughness of the surface of the cylinder.

3. Consideration for the prototype

The difference between the theoretical values and the experimental values puzzles us in adoption of the value of F in designing a structure. In order to solve this problem, a law of similarity must be satisfied for experiment or tests for the prototypes should be performed. Though it is very difficult to satisfy a law of similarity for model experiments, it is seen that in the range of the author's experiments, the smaller the amplitudes of vibration become, the nearer the values of F approach to the theoretical values. Therefore, in case of prototype where the ratio of the amplitude to the width of the pier is small, we may be able to use the theoretical values obtained by ignoring the viscosity of water even when the value of b/a is small.

CONCLUSION

From above mentioned matters, the following items may be concluded.

(1) The dynamic water pressure on submerged structures during earthquakes are functions of the shape of the structures, the ratio of thickness to the width of the sections, b/a, and the ratio of the depth of water to the width of the sections, h/2a.

(2) In case of elliptic cylinder, for small value of h/2a, the smaller the ratio, b/a, becomes, the larger the dynamic water pressure becomes, but for large value of h/2a, it does not depend upon the ratio, b/a.

(3) For the case of small value of the ratio, b/a, of elliptic sections, or for rectangular sections, the experimental values are always larger than the theoretical values. This may be due to the viscosity of water, and we must consider a law of similarity to obtain a exact results of experiment.

(4) But, in case of prototype, the effect of viscosity may be able to be ignored and the theoretical value may be used without large error.

(5) The elongation of the period of vibration differs in accordance with the mode of vibration and it is not right to consider that a constant amount of water will be added to the structure. In order to explain the elongation of period of vibration for arbitrary mode of vibration, the longitudinal distribution of added mass should be considered.

(6) The viscous damping due to water is very small.

(7) The submerged structures are disadvantageous ones because they are subjected to larger seismic forces than those in air.

Though a stationary vibration has hitherto been dealed with, the transient phenomenon of dynamic water pressure must be considered in order to solve the response of the submerged structures during earthquakes, and this problem will be left to future studies because of the difficulty of the theoretical solution.

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TABLE — 1

No.	ℓ (mm)	2a (mm)	2b (mm)	b/a	t_0 (mm)	t_1 (mm)	t_2 (mm)
1	250	50	5	0.1	3	10	50
2	250	50	10	0.2	3	10	50
3	250	50	15	0.3	3	10	50
4	250	50	20	0.4	3	10	50
5	250	50	[illegible]	0.5	3	10	50
6	250	50	[illegible]	0.6	3	10	50
7	250	50	40	0.8	3	10	50
8	250	50	50	1.0	3	10	50

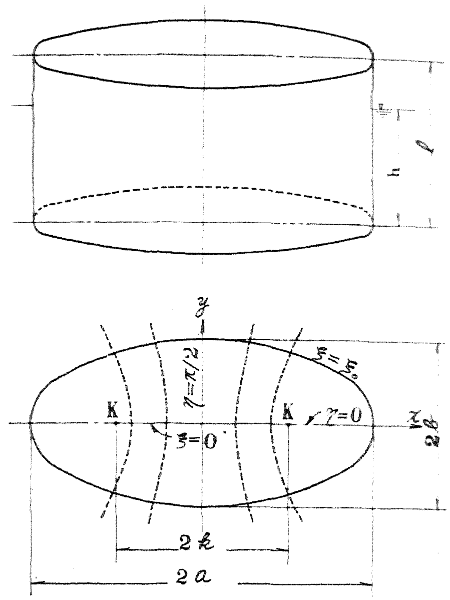

Fig. 1 Elliptic Cylinder

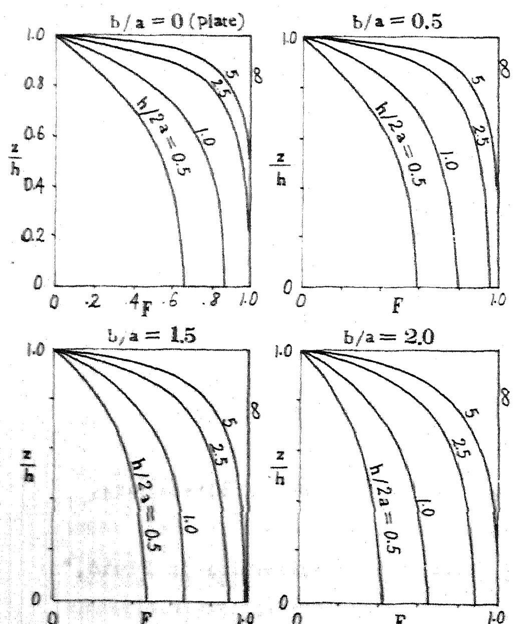

Fig. 2 Coefficient of Dynamic Water Pressure
$(P = \kappa w_0 \pi a^2 F)$

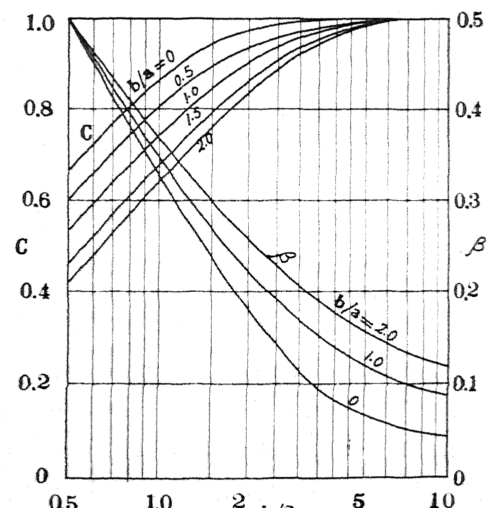

Fig. 3 Values of C and β

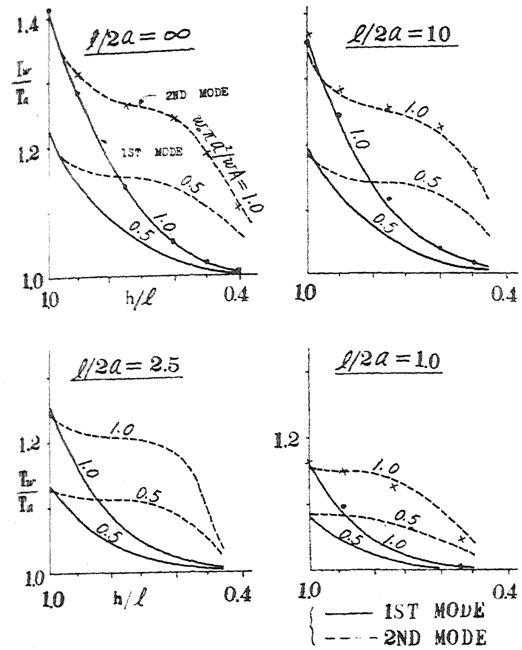

Fig. 4-a Elongation of Natural Period
($b/a = 0.5$, $W = 0$)

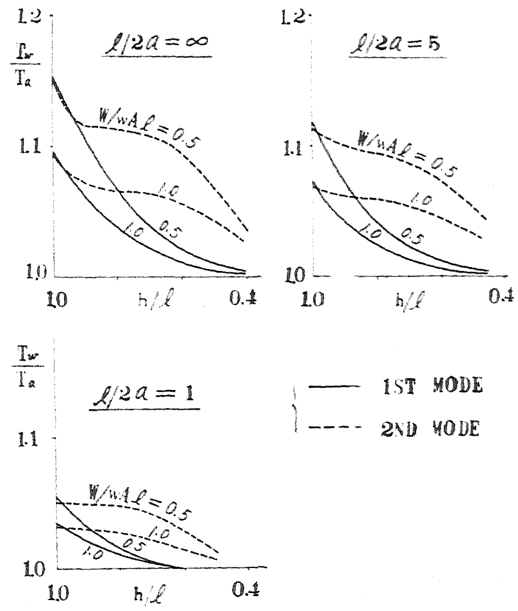

Fig. 4-b Elongation of Natural Period
($b/a = 0.5$, $w_0 \pi a^2/wA = 1.0$)

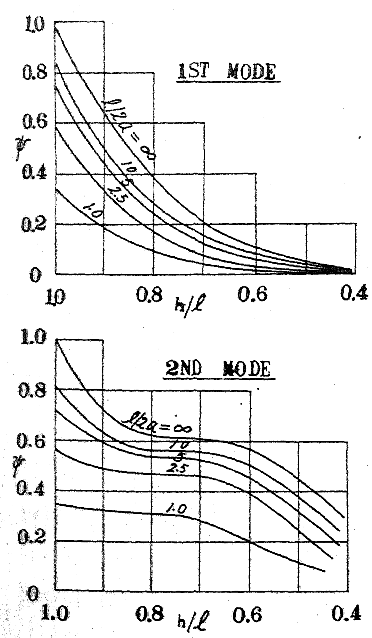

Fig. 5

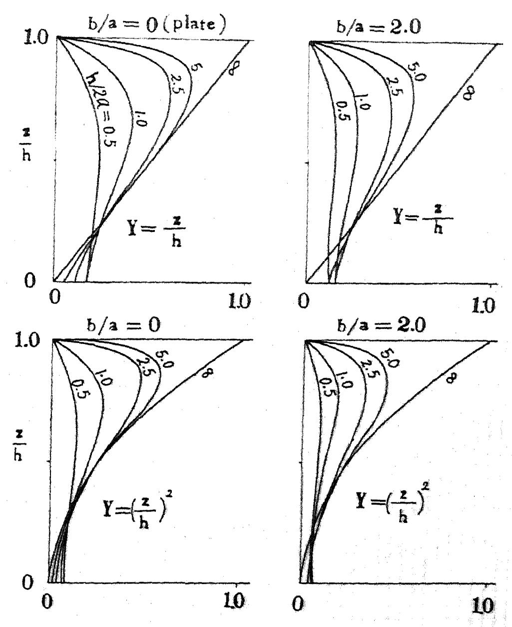

Fig. 6 Coefficient of Dynamic Water Pressure due to Elastic Vibration

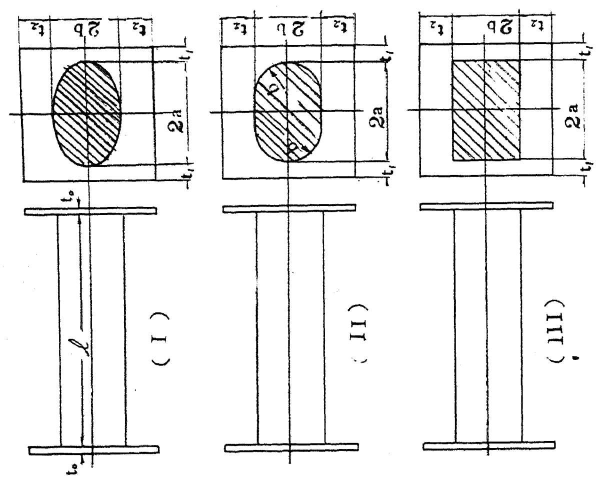

Fig. 9 Type of Models

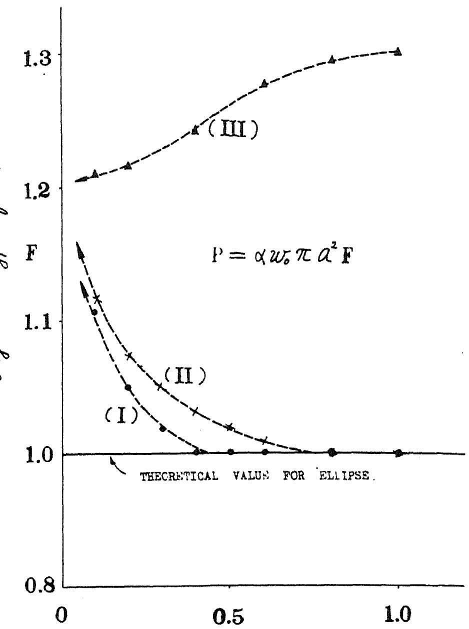

Fig. 10 Results of Experiments

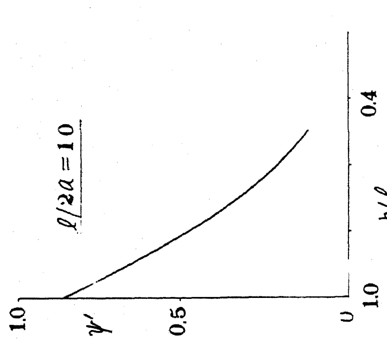

Fig. 8 Change of Amplitude of Forced Vibration due to Drop of Water Level

($b/a = 0.5$, $W/wAl = 0$, $w_0 \pi a^2/wA = 1.0$)

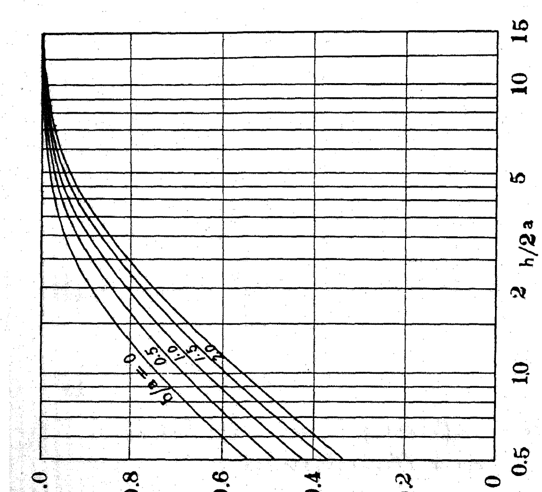

Fig. 7 Values of C for Elastic Vibration

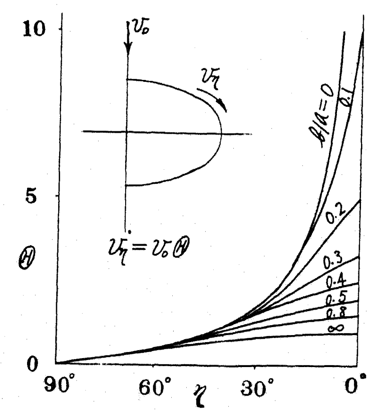

Fig. 11 Velocity of Water on the Surface of Ellipse