Some New Problems of Seismic Vibrations of a Structure.

By

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Abstract

The idea of the multiple reflection problem of waves in an elastic layer is applied to the problem of seismic vibration of a structure. The theoretical results by using a simple formula obtained here, in which the form of seismic motions at the lower boundary of a structure is able to get from that at the top of it, are compared with the observational results of the actual buildings as well as dams.

It is concluded that the problem of seismic vibration of a structure should be treated as the multiple reflection phenomena of waves.

Finally, the methods of design of a resistant earthquake structure were investigated by the present idea.

1. Introduction

The problem of vibrations of a structure due to seismic waves has been studied by many researchers. But the standpoints, on which these researchers based their theories are not theoretically rigorous, as their methods of treatment are mainly to apply approximately the solution of the equation of motion of materials or to neglect certain boundary conditions. Indeed, the exact calculation of the complicated structure is, in general, very difficult.

However, the vibration problem of a structure still seems worth while to investigate in some way further because of the fact that we are now in a position, owing to its importance in earthquake-proof constructions, to re-examine several points which were not adopted in the observational results.

Recently, we had an occasion to study the problem of behaviour contained in this question. In the present investigation, we shall first deal theoretically with the multiple reflection problem of waves (1) in a structure, and then compare the theoretical results with the observational ones.

2. Vibration of a structure treated by a wave problem

The multiple reflection problem of waves in a structure is treated rather analytically in the present investigation.

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If the incident waves arriving at the foundation of a structure, $z=0$, be of the type:

$$w_0 = F(t), \qquad (1)$$

the transmitted waves in a structure, w_1, at $z=0$ and $z=H$ are expressed by

$$\left.\begin{aligned} w_{1z=0} &= \gamma F(t), \\ w_{1z=H} &= \gamma F(t-\frac{H}{V}), \end{aligned}\right\} \qquad (2)$$

where γ is the transmission coefficient of waves from the ground to the structure and V the apparent transmission velocity of waves in that structure.

If the assumptions are made that the reflection coefficients of waves at the free surface, $z=H$, and the lower boundary, $z=0$, in a structure are 1 and β, respectively, and the attenuation of waves in that structure is negligibly small, the expression for the resulting motions at $z=0$ and $z=H$, as influenced by the multiple reflection of waves in that structure can be written by an infinite series, as follows*:

$$\begin{aligned} w_{z=0}(t) &= \gamma F(t) + \{\gamma F(t-\frac{2H}{V}) + \gamma\beta F(t-\frac{2H}{V})\} \\ &\qquad + \{\gamma\beta F(t-\frac{4H}{V}) + \gamma\beta^2 F(t-\frac{4H}{V})\} + \cdots\cdots \\ &= \gamma[F(t) + \{F(t-\frac{2H}{V}) + \beta F(t-\frac{2H}{V})\} \\ &\qquad + \{\beta F(t-\frac{4H}{V}) + \beta^2 F(t-\frac{4H}{V})\} + \cdots\cdots], \end{aligned} \qquad (3)$$

$$\begin{aligned} w_{z=H}(\tau) &= 2\gamma F(t-\frac{H}{V}) + 2\gamma\beta F(t-\frac{3H}{V}) + 2\gamma\beta^2 F(t-\frac{5H}{V}) + \cdots\cdots \\ &= 2\gamma[F(t-\frac{H}{V}) + \beta F(t-\frac{3H}{V}) + \beta^2 F(t-\frac{5H}{V}) + \cdots\cdots]. \end{aligned} \qquad (4)$$

It is possible to obtain the next relation by modifing Eq. (4), that is,

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* When a seismic waves of a purely plane type propagated vertically upwards in a semi-infinite elastic medium is partly transmitted through the bottom boundary of the superficial elastic layer, the transmission coefficient, γ, and the reflection one, β, become $\gamma = 2/(\alpha + 1)$ and $\beta = (\alpha - 1)/(\alpha + 1)$, respectively, in which, $\alpha = \rho_1 V_1 / \rho_2 V_2$ and ρ_1, ρ_2 ; V_1, V_2 are the densities and the velocities of the surface layer and the semi-infinite elastic medium, respectively.

$$\frac{1}{2\gamma}\{w_{z=H}(\tau+\frac{H}{V})+w_{z=H}(\tau-\frac{H}{V})\}=\{F(t)+\beta F(t-\frac{2H}{V})+\beta^2F(t-\frac{4H}{V})+\cdots\cdots\}$$
$$+\{F(t-\frac{2H}{V})+\beta F(t-\frac{4H}{V})+\beta^2 F(t-\frac{6H}{V})+\cdots\cdots\cdots\cdots\}. \quad (5)$$

From Eqs. (3) and (5), a simple relation between the resulting motions at $z=H$ and $z=0$ can be obtained as follows:

$$\frac{1}{2}\{w_{z=H}(\tau+\frac{H}{V})+w_{z=H}(\tau-\frac{H}{V})\}=w_{z=0}(t)+\beta^n F(t-\frac{\overline{2n+1}H}{V}), \quad (6)$$

in which $\tau=t-H/V$, and $\tau=0$ and $t=0$ correspond to the arrival time of waves at $z=H$ and $z=0$, respectively.

If we take the case in which

$$|\beta|<1, \quad (7)$$

the last term of Eq. (6) becomes as follows:

$$\beta^n F(t-\frac{\overline{2n+1}\,H}{V})\longrightarrow 0, \quad (8)$$

because Eq. (3) as well as Eq. (4) are an infinite series. Substituting Eq. (8) in Eq. (6), we obtain

$$w_{z=0}(t)=\frac{1}{2}\{w_{z=H}(\tau+\frac{H}{V})+w_{z=H}(\tau-\frac{H}{V})\}. \quad (9)$$

Another expression of Eq. (9) is

$$w_{z=0}(t-\frac{H}{V})=\frac{1}{2}\{w_{z=H}(\tau)+w_{z=H}(\tau-\frac{2H}{V})\}. \quad (9')$$

Eqs. (9) as well as (9') tell us that if only we know the value of $2H/V$ in a structure, even if we know neither the absolute values of the thickness, velocity and other constants of that structure or anything about the ground on which that structure stands, the earthquake motions at the foundation of that structure will be ascertained easily by utilizing the earthquake records obtained at the top of it.

One way of estimating the value of $2H/V$ is from the equation $T_s=4H/V$, in which T_s is the natural period of a structure.

3. Comparison of the theoretical and observational results

(i) Case of the actual buildings (2)

The seismographs installed at the buildings are SMAC type*. The way of estimating the value of $2H/V$ is from the equation $T_s = 4H/V$, in which T_s is the predominant period of the earthquake motions observed at the upper part of a structure. In order to estimate the natural periods of the buildings, period distribution curves are obtained from the seismograms recorded at the upper part of the buildings.

The final results of the theoretical study by means of Eq. (9) or (9') are illustrated in Figs. 1-4. In each figure, the uppermost curve, (a), represents the actual record of earthquake motions obtained at the upper part of each building and the second, (b), and third, (c), curves, respectively are the actual record of the lower part of that building and the theoretical result obtained by using the Eq. (9) or (9').

As will be seen from Figs. 1-4, the agreement of the observational result and the theoretical one in each building is well beyond expectation.

(ii) Case of the actual dams (3)

The actual dams treated in the present investigation are the Tsukabaru dam of gravity type and the Kamishiiba dam and Sazanamigawa dam of arch type. Now, in order to get the values of T_s of the dams, period distribution curves are obtained from the seismograms recorded at the upper part of the dams.

The final results of the theoretical study by means of Eq. (9) or (9') are illustrated in Figs. 5-7. In each figure, the uppermost curve, (a), represents the actual record of earthquake motions obtained at the upper part of each dam and the middle (b) or (b'), and the lowest, (c), curves, respectively are the actual record at the bottom of or the ground close by the dam and the theoretical result obtained by using Eq. (9) or (9').

As will be seen from Figs. 5-7, the agreement of the observational result, (b), (b'), and the theoretical one, (c), in each dam is well beyond expectation.

It seems somewhat strange that the agreement of the record at the bottom of a dam and the one at the ground surface close by it is considerably good. This however is merely the result of the fact that the wave length of the main parts of the incident seismic waves is satisfactorily large compared with the height of the dams.

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* The constants of the SMAC seismograph used here are as follows: natural period of pendulum; 0.1 sec, mechanical magnification; 16 times, recording range; 10-1,000 gals, recording speed; 1 cm/sec, time marking; every 1 sec.

It is natural to state, with the following considerations in mind, that the kind of waves transmitted in the dams are distortional waves. This is because the calculated value of the apparent transmission velocity of waves in the dams, V , using the relation $4H/T_S$ becomes about 2 km/sec and the velocity of distortional waves in the dams estimated by the velocity of dilatational waves of 3.5-4.0 km/sec and the Poisson's ratio of 1/6 is 2.0-2.5 km/sec.

(iii) Conclusion

From the results of the present investigation, we know that the vibration of a structure due to earthquake motions should be treated as the multiple reflection phenomena of waves.

It is a noteworthy fact that the absorption of vibration energy or inner damping in a structure is negligibly small, because there is no consideration of the attenuation of waves in that structure as seen in Eqs. (3) and (4). Further, the vibrational damping of a structure relates to the so-called outer damping and depends mostly on the boundary conditions between the structure and the ground, that is, β in Eqs. (3) and (4). It may be said that the most important part of the vibrational damping of a structure mentioned above is based on that at the time of an earthquake, the vibration energy of a structure dissipating into the ground again as in a wide sense the elastic waves which start from the foundation.

4. Application to a tall building

We shall apply the present idea to the vibration problem of a building of which the natural period, T_S , is considerably larger than the predominant period of ground, T_G , on which the building stands.

Actually, the delineation of seismic waves is very complicated but we may postulate for the sake of simplicity that the form of waves which yields the largest strain in a building is similar to the simple harmonic motion of a few train. Two cases of the relation among the period of waves T , T_G and T_S will be dealt with, that is (I) $T=T_G \ll T_S$ and (II) $T=T_S \gg T_G$

(i) Case (I); $T = T_G \ll T_S$

In this case, as a first approximation, the effect of the superposition of waves reflected at the top as well as the bottom of a building may not be taken into consideration excepting once, a reflection at the top. Consequently, the maximum displacement occurs at the top of the building and the value becomes as follow:

$$y_{max} = \frac{a}{2} \times m \times \gamma \times 2 = am\gamma \qquad (10)$$

in which a is the displacement of seismic waves at the outcrop, m is the magnification factor of the ground, γ is the transmission coefficient from the ground to the building and 2 of the last term is the effect of the reflection at the top of the building. Therefore, the maximum strain occurs at a position(1/4)(wave length) distant from the top, and the value becomes as follows:

$$\left(\frac{\partial y}{\partial z}\right)_{max} = a m \gamma \times \frac{2\pi}{VT} \tag{11}$$

in which, V is the transmission velocity of waves in the building. On the other hand, there exist the following relations, that is

$$V = \frac{4H}{T_s}, \qquad v = \frac{2\pi a}{T} \tag{12}$$

in which H is the height of the building and V represents the velocity amplitude of seismic waves at the outcrop and takes a constant value depending on both the magnitude and the hypocentral distance of an earthquake regardless of the period of waves (4).

Substituting Eq. (12) in Eq. (11), we get

$$\left(\frac{\partial y}{\partial z}\right)_{max} = \frac{m\gamma T_s v}{4H}. \tag{13}$$

Eq. (13) tells us that the maximum strain in the building caused by seismic waves is proportional to the velocity amplitude of incident waves in the bed rock (1/2 of the amplitude at the outcrop).

If we assume that the natural period of the building is proportional to the number of stories, N, that is

$$T_s = cN \tag{14}$$

and the strain distributes uniformely in the maximum strained story, the maximum value of the relative displacement between the neighbouring floors, D_{max}, becomes approximately as follows

$$D_{max} = \left(\frac{\partial y}{\partial x}\right)_{max} \times \frac{H}{N} = \frac{m\gamma c v}{4} \tag{15}$$

In obtaining the value of m, the following empirical formula may be available in the ground where the predominant period of seismic waves appears clearly (5).

$$m = 1 + \frac{\sqrt{T_G}}{0.3} \tag{16}$$

In the cases of the San Francisco earthquake of 1906 and the Kwanto earthquake of 1923 the severest damage to the buildings occurred at certain middle stories resulting in several kinds of explanations concerning the cause of this fact being made by many different researchers (6). The result of the present investigation seems to be applicable for interpreting naturally the above-mentioned fact. Some examples of the practical application of the problem are written in Table 1. Nevertheless, as seen in Table 1, the condition $T_G \ll T_s$ does not hold satisfactorily in these buildings, the agreement between the stories that suffered the most severe damage and those of the calculated largest strain seems very good. We hope to ascertain more sufficiently the feasibility of the present idea by obtaining such kinds of data for the taller buildings.

(ii) Case (II): $T = T_s \gg T_G$

In this case, as a practical approximation, the magnification of the amplitude of waves in the ground can be neglected* and the magnification factor ξ in a building should be taken into consideration. The maximum displacement occurs at the top of the building and the value may be written as follows:

$$\begin{aligned} y_{max} &= \frac{a}{2} \times \gamma \times \xi \times 2 \\ &= a \gamma \xi \end{aligned} \tag{17}$$

Consequently, the maximum strain occurs at the base of the building and the value becomes as follows

$$\left(\frac{\partial y}{\partial z}\right)_{max} = \frac{\gamma \xi T_s v}{4H} \tag{18}$$

Subsequently, the maximum relative displacement between the neighbouring floors takes on a value as follows:

$$D_{max} = \frac{\gamma \xi c v}{4H} \tag{19}$$

in which

$$\xi = \frac{1 - |\beta|^{2n}}{1 - |\beta|} \qquad [n = 1/2, 1, 3/2, \cdots\cdots] \tag{20}$$

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* Rigorously speaking, the value of magnification becomes $2/(1+\alpha_0)$, in which, α_0 is the impedance ratio of the ground to the bed rock.

where β is the reflection coefficient of waves at the bottom of the building and $2n$ is the succession number of waves in which the period is equal to the natural period of the building (7). It may be said, from Eqs. (15) and (19), that when the natural period of a building is fairly larger than the predominant period of the ground on which the building is standing, the maximum strain occurs at some considerable height in which the case where the magnification factor of ground, m, is larger than that of the building, ξ; on the contrary, it occurs at the base in the case where $m < \xi$.

Rigorously speaking, it is a very difficult problem to decide the height where the stress reaches its maximum, because the boundary conditions between building and ground are not so simple. Nevertheless, the results just mentioned and those of the theoretical studies (8) based upon the idea that at the time of earthquake the vibration energy of buildings dissipates to the ground again as the elastic waves which start from the foundation do not contradict each other, and that both results agree qualitatively with the results of the statistical studies of the damage of buildings due to the past big earthquakes.

In the special case of $T = T_G = T_B$, the maximum displacement, strain and the relative displacement between the neighbouring floors become as follows:

$$y_{max} = a m \gamma \xi , \tag{21}$$

$$\left(\frac{\partial y}{\partial z}\right)_{max} = \frac{m \gamma \xi T_B v}{4H} , \tag{22}$$

$$D_{max} = \frac{m \gamma \xi c v}{4} , \tag{23}$$

and it is a matter of course that Eq. (21) appears at the top and Eqs. (22) as well as (23) take place at the base.

When the size of the plane of a building is so large as to be treated as a wave problem of a single stratified layer, the transmission coefficient, γ, and the reflection one, β, can be written as follows (9):

$$\gamma = \frac{2}{\alpha + 1} , \qquad \beta = \frac{\alpha - 1}{\alpha + 1} , \tag{24}$$

in which α is the impedance ratio of building to ground, that is, $\alpha = \rho V / \rho' V'$ and ρ, ρ' and V, V' are the densities and velocities, respectively, of the building and the ground.

Let h be the fraction of critical damping, β (9) can be expressed by

$$|\beta| = e^{-\pi h} \tag{25}$$

From Eqs. (24) and (25), we obtain

$$\gamma = 1 + e^{-\pi h} \tag{26}$$

Accordingly, if we know the value of h , we can obtain the values of γ and ξ by using Eqs. (20) and (26), respectively.

The values of the velocity amplitude of seismic waves at the outcrop, v , obtained from the following empirical formula (10) are shown in Fig. 8.

$$v = 10^{0.61M - 1.73\log X - 0.67} \tag{27}$$

in which M and X are the magnitude and the hypocentral distance of an earthquake.

In carrying out the numerical calculation of the case (I) by using Eqs. (15), (16), (26) and Fig. 8, we assume that the larger the predominant period of ground the smaller the damping of structure (11).

In practice, we shall adopt the following values, that is, h =0.02 to T_G =0.2 sec, h =0.12 to T_G =0.6 sec and the proper values of $m\,\gamma$ to the values of T_G between 0.2 sec and 0.6 sec. The results of the numerical calculation are shown in Figs. 9-12.

Next, in carrying out the numerical calculation of the case (II) by using Eqs. (19), (20), (26) and Fig. 8, we assume that the smaller the damping of a structure the larger the succession number of the waves of equal period (12). In practice, we shall adopt the following values, that is, n =2 to h =0.02, n =1 to h =0.12 and the proper values of $\gamma\,\xi$ to the values of h between 0.02 and 0.12. The results of the numerical calculation are shown in Figs. 13-16.

(iii) Conclusion

We shall summarise briefly the results of the present investigation.

(a) The strain in a building caused by an earthquake is proportional to the velocity amplitude of seismic waves.

(b) In general, on soft ground, as the magnification of amplitude in a building is relatively small because the damping of it is large and the magnification of amplitude in the ground is large, the largest damage to tall buildings is liable to take place at some considerable height.

(c) On the contrary, the damage to buildings on hard ground is liable to take place near the base, because the damping of a building is relatively small and the magnification of amplitude in the ground is not large.

(d) Reducing the value of c is very important for the design of a resistant earthquake building.

(e) The height itself of a building has a slight effect on the problem of resistant earthquake.

(f) The methods of design of a resistant earthquake building presented by the present investigation are as follows:

(1) To decide the magnitude as well as the hypocentral distance of an expectant earthquake.

(2) To decide the allowable maximum relative displacement between the neighbouring floors.

(3) To determine the natural period which corresponds to the satisfactory value of c .

(4) To determine the stiffness, weight and other factors which satisfy the natural period determined by Item (3).

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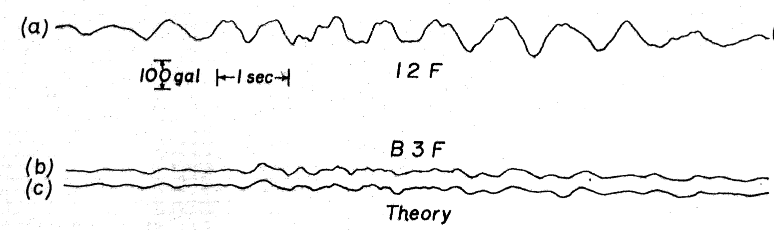

Fig. 1. 12-storied S. R. C. bldg. (Kanden Bldg.). Original x 1·1.

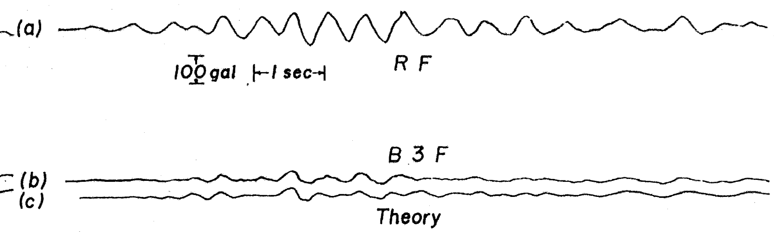

Fig. 2. 12-storied S. R. C. bldg. (Shin-sumitomo Bldg.). Original x 1·1.

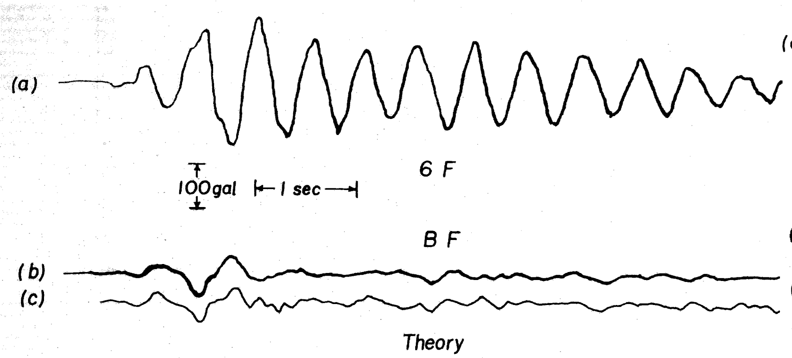

Fig. 3. 7-storied S. C. bldg. (Daimaru Bldg.). Original x 1·5.

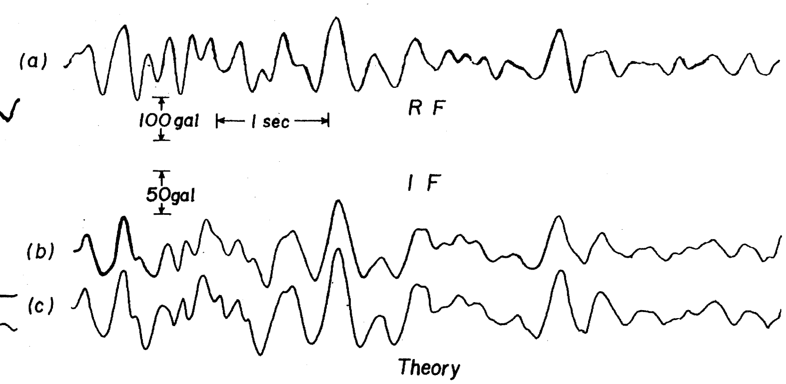

Fig. 4. 3-storied R. C. bldg. (Maibara Station Bldg.). Original x 1·7.

Table 1.

Name of the bldgs.	No. of the stories	Natural period of the bldgs. (Before Kwanto earthq.)	Predominant period of the ground	Stories of most severe damage (By Kwanto earthq.)	Stories of the largest strain (VT/4)
Kaijo	7	0.45sec	0.4sec	2 to 3	2
Maru-no-uchi	8	0.67,0.71	0.4	3 to 5	4

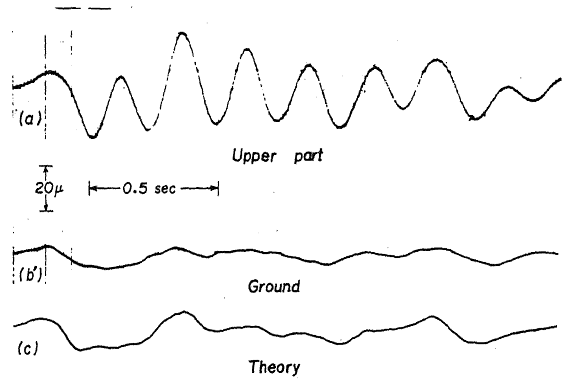

Fig. 5. Arch type dam of 110m height (Kamishiiba Dam). Original x 1/3.

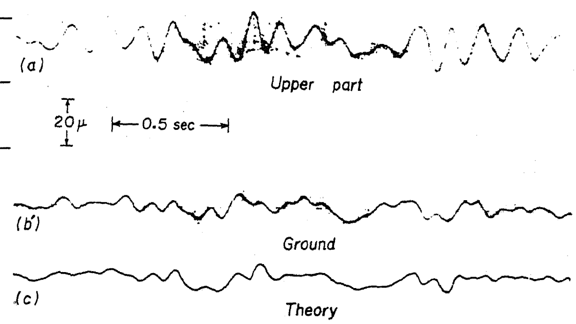

Fig. 6. Gravity type dam of 78m height. (Tsukabaru Dam). Originalx 1/3.

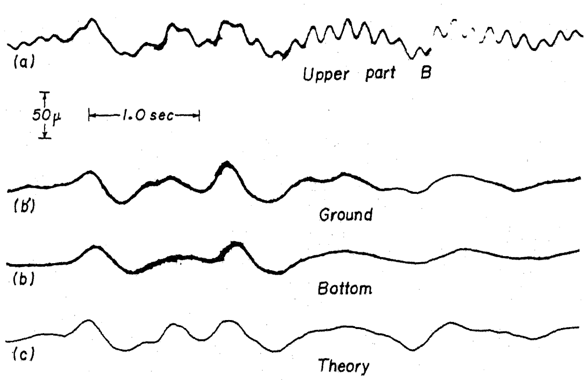

Fig. 7. Arch type dam of 67m height (Sazanamigawa Dam). Original x 1/3.

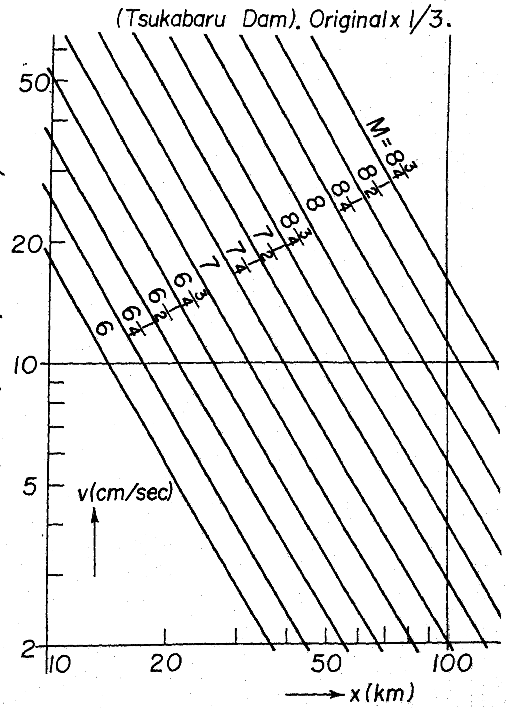

Fig. 8.

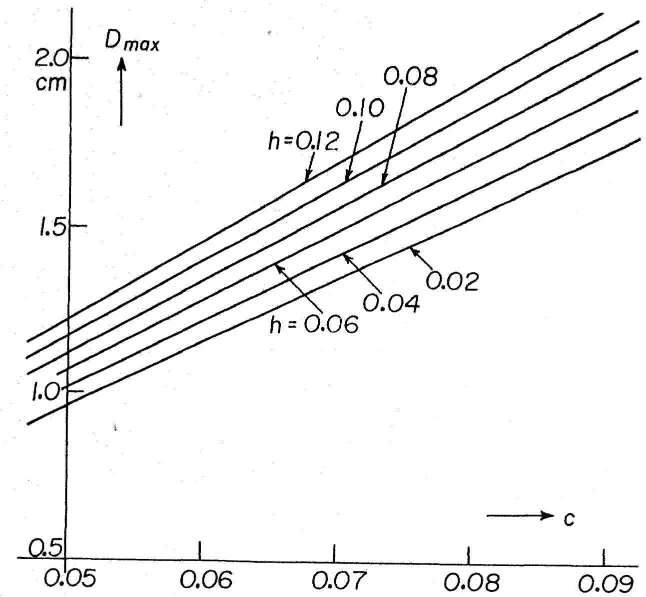

Fig. 9. $T = T_G \ll T_S$, $v = 16$ cm/sec.

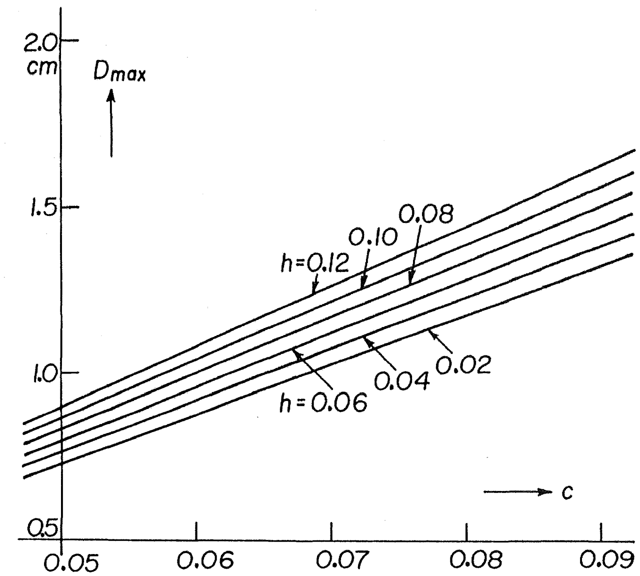

Fig. 10. $T = T_G \ll T_S$, $v = 12$ cm/sec.

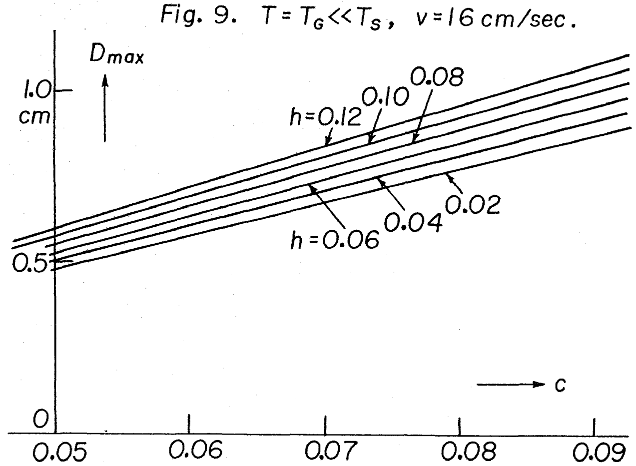

Fig. 11. $T = T_G \ll T_S$, $v = 8$ cm/sec.

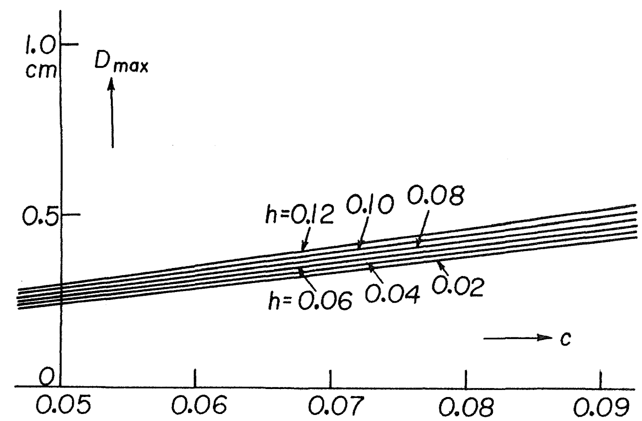

Fig. 12. $T = T_G \ll T_S$, $v = 4$ cm/sec.

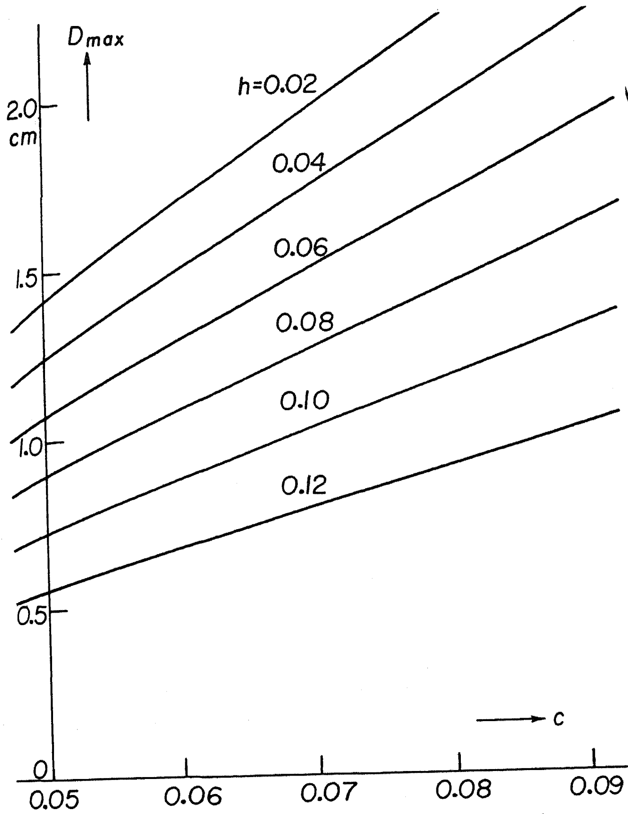

Fig. 13. $T = T_S \gg T_G$, $v = 16$ cm/sec.

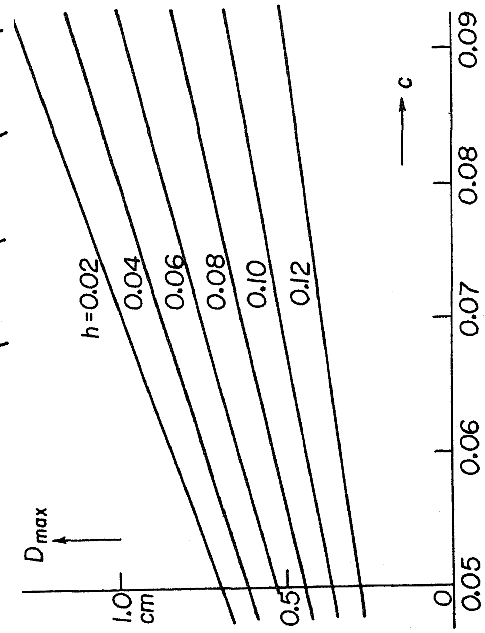

Fig. 15. $T = T_S \gg T_G$, $v = 8$ cm/sec.

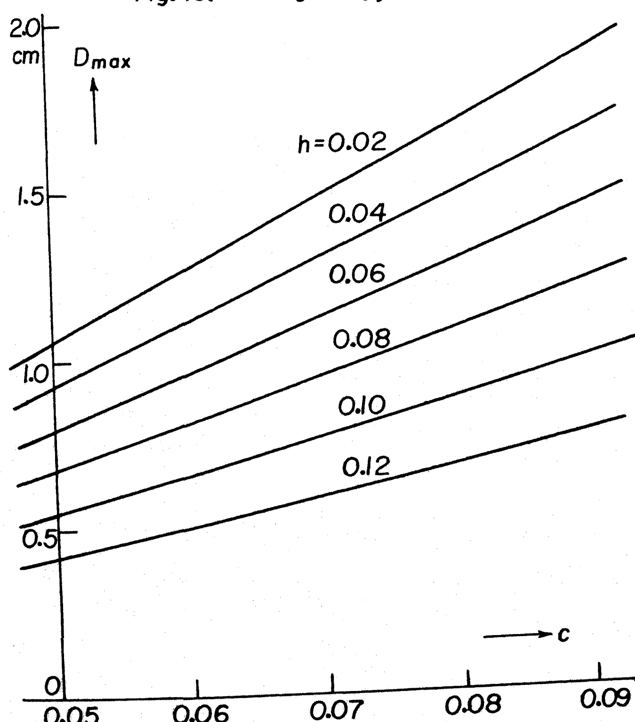

Fig. 14. $T = T_S \gg T_G$, $v = 12$ cm/sec.

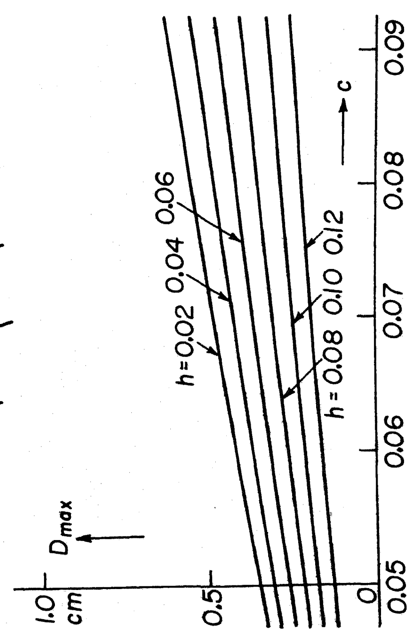

Fig. 16. $T = T_S \gg T_G$, $v = 4$ cm/sec.

SOME NEW PROBLEMS OF SEISMIC VIBRATIONS OF A STRUCTURE
BY K. KANAI

QUESTION BY: T.KATAYAMA - JAPAN

Referring to page 5 the loading applied to a tall building - I think that this theory is more rigorous than reasonable, but when we have to predict the dynamic response of a proposed building, or when we have to predict the displacement e.g. for the equation given on page 5, we have to assume several coefficients and I think the practical difficulty is in how to assume these coefficients and how correctly we do it. When a coefficient is given by the product of 3 factors a, m and γ and if there is an error of 50% or 100% in estimation of γ the max. displacement also contains 100% error, but if we use the conventional dynamic response, even if we make a real error in assuming the damping constant, the maximum displacement is not much in error.

AUTHOR'S REPLY: i) I think the theory adopted here is reasonable rather than rigorous, because the agreement of the theoretical and the observational results is very good indeed as shown in Figs. 1-7 in spite of adopting some assumptions in a theoretical treatment.

ii) As a first approximation, the relation between γ and the fraction of critical damping, h, can be written as equation (26), and then an error of 50% or 100% in estimation of γ will be improbable.

QUESTION BY: D.G. ELMS - NEW ZEALAND

Professor Kanai is to be congratulated for one of the most interesting papers of this session. Of particular importance is his conclusion, based on measurements of several types of building, that the structural damping in a building is of negligible importance compared with that due to the dissipation of energy into the soil, which in turn depends on the nature of the foundation. This suggests that the current discussions on the need for maximum structural damping and on methods of construction best able to achieve this are less important than was thought hitherto, and that more work should now be put into a thorough investigation of foundation design for optimum damping. The paper did not make it clear, however, whether the results given in Figures 1 - 4 were for buildings which remained mostly in the elastic range, or whether there was structural damage indicating that the buildings had in fact yielded. It is, of course, important to know this when trying to assess Professor Kanai's conclusions. If the buildings were in fact mostly in the elastic range, this would explain why the theory was only able to produce a qualitative correlation in Table 1; and it would be hoped that the addition of wave attenuation into the theory would make this correlation quantitative as well as qualitative.

AUTHOR'S REPLY: The results given in Figs. 1-4 were for the buildings which remained in the elastic range. On the other hand, the result presented by a slide at the session room was for a building which suffered little damage by the Niigata earthquake of June 16, 1964.

The result of the latter case showed also the good agreement of the theoretical and observational results, and will be published in the forthcoming Bulletin of the Earthquake Research Institute, University of Tokyo.