DIRECT DETERMINATION OF EQUIVALENCE COEFFICIENTS OF MASSES IN THE ANTISEISMIC COMPUTATION OF STRUCTURES

by

STEFAN BALAN[1)]

MIHAIL IFRIM[2)] CORNEL PACOSTE[3)]

ABSTRACT

The paper deals with a method whereby a transition from systems with several degrees of freedom to a system with a single mass may be effected, using to this end equivalence coefficients.

The structures investigated have been classified according to their dynamic rigidity (characterized by the fundamental period of vibration), namely: rigid, semi-rigid and flexible structures.

For practical purposes tables of direct values of equivalence coefficients of masses are given.

The results obtained refer to multiple storeyed structures. The equivalence coefficients have been computed for n - storeyed structures.

The experiments which have been effected confirm the validity of the hypothesis admitted.

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1) Professor of Theoretical Mechanics at the Institute of Civil Engineering, Bucharest. Head of the Chair of Structure Mechanics. D.Sc.Fellow of the Rumanian Academy. Adress: Institute of Applied Mechanics, 15 C.Mille Street, Bucharest, Rumania.

2) Civil Engineer, Lecturer on Structure Mechanics at the Institute of Civil Engineering, Bucharest. Adress: Institute of Applied Mechanics, 15 C.Mille Street, Bucharest - Rumania.

3) Civil Engineer, Lecturer on Structure Mechanics at the Institute of Civil Engineering, Bucharest. Adress: Institute of Applied Mechanics, 15 C.Mille Street, Bucharest, Rumania.

I. INTRODUCTION

A series of problems related to civil engineering seismology have been investigated theoretically and experimentally at the vibration laboratory of the Chair Department of Civil Engineering Mechanics of the Institute of Civil Engineering, Bucharest, Rumania. An important part of the work has been devoted to the application of the mechanical equivalent on the antiseismic computation of structures.

In presenting their work to the 3rd International Conference of Engineering Seismology the authors give an account of the theoretical solution and the experimental evidence which corroborates the results obtained in connection with the direct determination of the equivalence coefficients of masses as a function of the dynamic rigidity of structures.

Knowing the numerical values of these coefficients simplifies materially the computation of the base shear.

The authors have also determined the numerical values of the distribution coefficients of the base shear for structures up to 4o - storeys high so that seismic stresses along the structure height may be obtained directly.

By applying the results obtained the designer will achieve in some cases a considerable saving of time in actual computations and will quickly master the method for the computation of seismic forces.

The experimental researches have been carried out at the Department of Civil Engineering Mechanics in collaboration with the Research Center of the Institute of Civil Engineering, Bucharest. Further experiments with the same object in view are being effected.

II. VARIANTS FOR THE COMPUTATION OF SEISMIC FORCES

Two variants are offered for the determination of seis-.c forces acting on a structure during an earthquake. Both riants are based on a dynamic computation which takes into count the elastic characteristics of the structures and the namic properties of the seismic motion.

In the first variant of the computation the floor seisic forces (acting along the structure height) are determined, while in the second variant the base-shear acting on the structure at the foundation level is first considered, and then distributed over the height of the structure.

Refering to fig. 1 and 2 a brief account of the method for the computation of seismic forces by applying the two variants will be given.

1. Direct computation of the seismic force

The inertia force acting on a structure at a certain floor x may be determined from the dynamical theory of elastic systems subjected to any perturbation of the earthquake type which appears at the base of the structure. Thus:

a.- In the case of structures with distributed mass (fig.1) the earthquake force at the floor x corresponding to the vibration mode i, is

$$F_x(i) = C(T_i) \cdot a_x(i) \cdot q_x \tag{1}$$

where

$$q_x = \bar{m}_x g$$

$$a_x(i) = \frac{\int_0^H \bar{m}_x y_x(i)dx}{\int_0^H \bar{m}_x y_x^2(i)dx} y_x(i) = \frac{\int_0^H q_x y_x(i)dx}{\int_0^H q_x y_x^2(i)dx} \tag{2}$$

b.- In the case of structures with concentrated masses (fig.2) the earthquake force at the floor k, corresponding to the vibration mode i, is:

$$F_k(i) = C(T_i) \cdot a_k(i) \cdot Q_k \tag{3}$$

where

$$Q_k = m_k \cdot g$$

$$a_k(i) = \frac{\sum_{k=1}^{n} m_k y_k(i)}{\sum_{k=1}^{n} m_k y_k^2(i)} y_k(i) = \frac{\sum_{k=1}^{n} Q_k y_k(i)}{\sum_{k=1}^{n} Q_k y_k^2(i)} y_k(i) \tag{4}$$

In these expressions the gravitational load corresponding to the floors x and k has been noted by q_x and Q_k respectively, and the ordinates of the normal vibration mode i corresponding to the floors x and k, by y_x (i) and y_k (i) respectively.

$C(T_i)$ represents the earthquake spectrum coefficient whose value is dependent on the natural vibration period T_i and which corresponds to a one-mass oscillating system. The variation of the coefficient $C(T_i)$ as a function of the period T_i represents the fundamental earthquake spectrum the variation of which (fig.3) agrees with the analyses effected by M.A.Biot and E.C.Robison 5 .

The earthquake spectrum coefficient $C(T_i)$ may be expressed by means of the velocity spectrum S_v as follows:

$$C(T_i) = \frac{1}{T_i}\,\frac{2\pi}{g}\,S_v = \frac{\alpha}{T_i}$$

in which α is a numeric coefficient, and T_i the natural period corresponding to the i normal mode.

This variant for the computation of earthquake forces is used in the URSS design regulations for structures located in earthquake zones 6 .

2. Computation of earthquake forces by means of the base shear

In this case the computation is effected in two steps, namely: the base shear is determined in the first step while in the second step the base shear is distributed over the height of the structure. This approach to the problem has been included in the american code for the State of California and has been later adopted also by other countries [5][7]. The base shear and the earthquake forces are of the nature of inertia forces. Their expression is a problem of theoretical dynamics and will be given below.

a.- In the case of structures with distributed mass (fig.1) the base shear corresponding to the vibration mode i, is:

$$F(i) = C(T_i)\cdot Q_e(i) \qquad (5)$$

where $Q_e(i)$ represents the equivalent load obtained by transforming the real system with an infinite number of degrees of freedom in a single mass (one-mass) system:

$$Q_e(i) = \mu(i)\,Q\,; \qquad \mu(i) \leq 1 \qquad (6)$$

In the above formula the equivalence coefficient corresponding to the vibration mode i has been noted by $\mu(i)$, and the total load on the structure by Q, i.e.:

$$Q = \int_0^H q_x\,dx$$

In the third part of the paper more will be said about the way for determining the equivalence coefficient $\mu(i)$.

The earthquake coefficient $C(T_i)$ in expression (5) has t the meaning previously given and whose variation mode is shown in fig. 3.

The floor earthquake force is obtained by distributing the base shear over the height of the structure; the distribution is proportional to the ordinates of the vibration mode $\underline{i}$, i.e.:

$$(7) \qquad F_x(i) = \frac{q_x y_x^{(i)}}{\int_0^H q_x y_x^{(i)} dx} F(i)$$

b.- In the case of structures with concentrated masses (fig.2) the base shear is computed again by means of formula (5) where

$$Q = \sum_{k=1}^{n} Q_k$$

and expression (7) becomes

$$(8) \qquad F_k(i) = \frac{Q_k y_k^{(i)}}{\sum_{k=1}^{n} Q_k y_k^{(i)}} F(i)$$

From (7) and (8) there follows that the base shear is distributed over the height of the structure in direct proportion to the product of the floor gravitational load Q_k by the corresponding ordinate of the curve which characterizes the normal vibration mode $y_k(i)$.

Therefore the fraction which multiplies the base shear F(i) may be looked upon as a distribution coefficient of the total earthquake force which will be noted by $\underline{d}$.

In this case expression (7) becomes:

$$(9) \qquad F_x(i) = d_x(i) . F(i)$$

where

$$(10) \qquad d_x(i) = \frac{q_x y_x^{(i)}}{\int_0^H q_x y_x^{(i)} dx}$$

and expression (8) becomes:

$$(11) \qquad F_k(i) = d_k(i) . F(i)$$

where

(12)
$$d_k(i) = \frac{Q_k\, y_k(i)}{\sum_{k=1}^{n} Q_k\, y_k(i)}$$

3. Utilization of the equivalent earthquake spectrum

For immediate design purposesit is advisable to compute the equivalent load Q_e by means of a single quivalent coefficient $\bar{\mu}(i)$ average or limiting) obtained through appr.oximate calcúlation. In that case the equivalent load is

$$Q_e(i) = \bar{\mu}(i) \cdot Q$$

and the base shear:

(13)
$$F(i) = C(T_i) \cdot \bar{\mu}(i) \cdot Q$$

By putting:

$$\bar{C}(T_i) = C(T_i) \cdot \bar{\mu}(i)$$

expression (11) becomes:

(14)
$$F(i) = \bar{C}(T_i) \cdot Q$$

The variation of the earthquake coefficients $\bar{C}(T_i)$ represents the equivalent earthquake spectrum (see fig.4).

It is quite obvious that the use of the single equivalent coefficient $\bar{\mu}(i)$ is fairly approximate. The value of the coefficient $\mu(i)$ depends both on the number of storeys of the structure and on the dynamical rigidity of the latter, whereas $\bar{\mu}(i)$ is single.

However, in order to simplify to a large extent the computation of earthquake forces some regulations take into consideration single equivalence coefficients thus implicating as a computation basis the equivalent earthquake spectrum $C(T_i)$. In the wake of these simplifying assumptions it may be also considered that the distribution of the base shear over the structure height is effected by assuming that the first vibration mode varies linearly. In accordance with these assumptions the exact determination of the normal mode of vibration is excluded while the fundamental period is computed by means of empirical formulas. Thus the computation of earthquake forces is effected directly but the computation itself is howerer rather approximate. Therefore the distribution coefficients (lo) and (12) may be written:

$$\text{(15)}\qquad \begin{cases} d_x(i) = \dfrac{q_x \cdot x}{\int_0^H q_x \cdot x\,dx} \\ d_k(i) = \dfrac{Q_k h_k}{\sum_{k=1}^{n} Q_k h_k} \end{cases}$$

III. GENERAL EXPRESSIONS OF THE EQUIVALENT MASS AND EQUIVALENCE COEFFICIENTS

The passage from a system with several degrees of freedom to a system with a single degree of freedom (one-mass), has been achieved by applying the fundamental theorems of dynamics. It has been namely considered that for the two equivalent systems the impulse variation and the variation of kinetic moments are equal. Similar results are obtained by applying the principle of the conservation of potential and kinetic energies of the two systems analyzed [1],[4].

1. Systems with an infinite number of degrees of freedom (fig.1)

The general equivalent relations between the system with an infinite number of degrees of freedom (real) and the one-mass system (equivalent) corresponding to oscillations in the vibration mode i, are established by:

- equating the impulse variations

$$\text{(16)}\qquad \dot{\vec{H}}_h(i) = \dot{\vec{H}}_e(i)$$

- equating the variations of the kinetic moments

$$\text{(17)}\qquad \dot{\vec{K}}_h(i) = \dot{\vec{K}}_e(i)$$

may be expressed as functions of the speed variation namely:

$$\dot{\vec{H}}_h = \int_0^H \bar{m}_x \frac{d\vec{v}_{x,t}(i)}{dt}\,dx$$

$$\dot{\vec{H}}_e(i) = m_e(i) \frac{d\vec{v}_{e,t}(i)}{dt}$$

$$\dot{\vec{K}}_h(i) = \int_0^H y_x(i)\, \bar{m}_x \frac{d\vec{v}_{x,t}(i)}{dt}\, dx$$

$$\dot{\vec{K}}_e(i) = y_e(i)\, m_e(i) \frac{d\vec{v}_{e,t}(i)}{dt}$$

For the wave corresponding to the vibration mode i assumed to be in a steady state, one may consider a solution of the Fourier type, with separate space and time variables, i.e.:

$$y_{x,t}(i) = y_x(i) \cdot \phi_t(i)$$

and

$$y_{e,t}(i) = y_e(i) \cdot \phi_t(i)$$

Hence, onemay write

$$\frac{d\vec{v}_{x,t}(i)}{dt} = y_x(i) \frac{d^2\phi_t(i)}{dt^2}$$

and

$$\frac{dv_{e,t}(i)}{dt} = y_e(i) \frac{d^2\phi_t(i)}{dt^2}$$

By substituting these expressions in the general equivalence equations (16) and (17) one obtains

(18) $$\int_0^H \bar{m}_x\, y_x(i) \frac{d^2\phi_t(i)}{dt^2}\, dx = m_e(i)\, y_e(i) \frac{d^2\phi_t(i)}{dt^2}$$

(19) $$\int_0^H \bar{m}_x\, y_x^2(i) \frac{d^2\phi_t(i)}{dt^2}\, dx = m_e(i)\, y_e^2(i) \frac{d^2\phi_t(i)}{dt^2}$$

or

(20) $$\int_0^H \bar{m}_x\, y_x(i)\, dx = m_e(i)\, y_e(i)$$

(21) $$\int_0^H \bar{m}_x\, y_x^2(i)\, dx = m_e(i)\, y_e^2(i)$$

In these two equations the elements of the equivalent system appear as unknown quantities, namely: the equivalent mass $m_e(i)$ and the elongation $y_e(i)$. Eliminating $y_e(i)$ from these two equations one obtains the general expression of the __equivalent mass__ corresponding to the vibration mode __i__:

$$(22) \qquad m_e(i) = \frac{\left[\int_0^H \bar{m}_x \, y_x(i) \, dx\right]^2}{\int_0^H \bar{m}_x \, y_x^2(i) \, dx}$$

The ratio of the equivalent (or reduced) mass to the total mass of the real system, shall be noted by $\mu(i)$ and be called equivalent or mass reducing coefficient.

Therefore $$\frac{m_e(i)}{m} = \frac{m_e(i)}{\int_0^H \bar{m}_x dx}$$

and taking into account (22), one obtains the general expression of the equivalent coefficients

$$(23) \qquad \mu(i) = \frac{\left[\int_0^H \bar{m}_x \, y_x(i) dx\right]^2}{\left[\int_0^H \bar{m}_x dx\right] \cdot \left[\int_0^H \bar{m}_x \, y_x^2(i) dx\right]} \quad ; \quad \mu(i) \leq 1$$

It is obvious that the formulas (22) and (23) are greatly simplified when the mass of the system is distributed uniformly

$$\bar{m}_x = \bar{m} = \text{constant}$$

In that case the equivalent mass becomes

$$(24) \qquad m_e(i) = \frac{\left[\int_0^H y_x(i) \, dx\right]^2}{\int_0^H y_x^2(i) \, dx} \bar{m}$$

and the equivalent coefficient:

$$(25) \qquad \mu(i) = \frac{\left[\int_0^H y_x(i) \, dx\right]^2}{H \int_0^H y_x^2(i) dx}$$

2. __Systems with n degrees of freedom__ (fig.2)

The general expressions of the reduced mass and of the equivalent coefficients corresponding to systems with n degrees of freedom are obtained *directly from* the preceding case by transforming the integrals into sums. Hence,

- for the equivalent mass

(26)
$$m_e(i) = \frac{\left[\sum_{k=1}^{n} m_k\, y_k(i)\right]^2}{\sum_{k=1}^{n} m_k\, y_k^2(i)}$$

- for the equivalent coefficients

(27)
$$\mu(i) = \frac{\left[\sum_{k=1}^{n} m_k\, y_k(i)\right]^2}{\left[\sum_{k=1}^{n} m_k\right]\cdot\left[\sum_{k=1}^{n} m_k\, y_k^2(i)\right]}$$

In the particular case when the masses of the real system are equal (a case which is fairly frequently met with in practice)

$$m_1 = m_2 = \ldots = m_k = \ldots = m_n = m$$

the expressions (26) and (27) become:

(28)
$$m_e(i) = \frac{\left[\sum_{k=1}^{n} y_k(i)\right]^2}{\sum_{k=1}^{n} y_k^2(i)}\, m$$

(29)
$$\mu(i) = \frac{\left[\sum_{k=1}^{n} y_k(i)\right]^2}{\sum_{k=1}^{n} y_k^2(i)}\ ; \qquad \mu(i) \leq 1$$

3. General case of systems with distributed mass and concentrated masses

When an elastic system is loaded simultaneously with a distributed mass and concentrated masses, the computation formulas for the reduced mass and the equivalent coefficients are obtained by applying the same general equivalent relations (16) and (17) used previously. Effecting now the operations *indicated one obtains finally:*

(30)
$$m_e(i) = \frac{\left[\int_0^H \overline{m}_x\, y_x(i)\, dx + \sum_{k=1}^{n} m_k\, y_k(i)\right]^2}{\int^H \overline{m}_x\, y_x^2(i)\, dx + \sum_{k=1}^{n} m_k\, y_k^2(i)}$$

(31) $$\mu(i)=\frac{\left[\int_0^H \bar{m}_x\, y_x(i)\, dx+\sum_{k=1}^{n} m_k\, y_k(i)\right]^2}{\left[\int_0^H \bar{m}_x\, dx+\sum_{k=1}^{n} m_k\right]\left[\int_0^H \bar{m}_x\, y_x^2(i)\, dx+\sum_{k=1}^{n} m_k\, y_k^2(i)\right]}$$

For the particular case of a uniformly distributed mass and of concentrated masses of equal magnitude the expressions become

(32) $$m_e(i)=\frac{\left[\bar{m}\int_0^H y_x(i)\, dx+m\sum_{k=1}^{n} y_k(i)\right]^2}{\bar{m}\int_0^H y_x^2(i)\, dx+m\sum_{k=1}^{n} y_k^2(i)}$$

(33) $$\mu(i)=\frac{\left[\bar{m}\int_0^H y_x(i)\, dx+m\sum_{k=1}^{n} y_k(i)\right]^2}{\left[\bar{m}H+nm\right]\cdot\left[\bar{m}\int_0^H y_x^2(i)\, dx+m\sum_{k=1}^{n} y_k^2(i)\right]}$$

Note: if in all the expressions obtained the mass is replaced by loads

$$\bar{m} = q_x/g \quad ; \quad m_k = Q_k/g$$

one obtains corresponding formulas for reduced or equivalent load and for the equivalent coefficients of the loads.

IV. DIRECT DETERMINATION OF EQUIVALENT COEFFICIENTS.

With a view of establishing practical computation formulas for the equivalence coefficients of the masses μ, but taking into account the elastic properties of the structures, the authors have worked out a method whereby a direct evalution of the coefficients, approaching reality, can be effected.

The theoretical basis of the method which allows a direct numerical computation toghether with the results obtained, is given below.

1. Computation assumptions

In working out the computations, a series of simplifying assumptions have been made which are generally admitted in earthquake engineering, namely:

- the masses are considered to be concentrated at the level of each floor and to be uniformly distributed over a vertical direction;
- there are no marked abrupt changes in the variation of the relative rigidity between floors;
- the first normal vibration mode is considered to be predominant over the dynamic response for all types of structures;
- the floors of the structure are equally spaced;
- the oscillations are linear being in the range of small displacements.

Account has been taken also of the structure type, characterized by the dynamic rigidity of the latter.

2. Classification of structures from a dynamic point of view

The equivalence coefficients have been computed taking into account the behaviour of the structure from a dynamic point of view. In a dynamic state the behaviour is characterized by the fundamental vibration period T_1.

A classification of structures according to their dynamic behaviour when acted on by ground forces, is suggested, namely:

- rigid structures, for which

$$T_1 \leqslant 0{,}3 \text{ sec.},$$

- semi-rigid structures, for which

$$0{,}3 < T_1 \leqslant 1{,}2 \text{ sec.}$$

- flexibles structures, for which

$$T_1 > 1{,}2 \text{ sec.}$$

As may be seen in fig. 3 and 4, in the interval corresponding to rigid and flexible structures, the values of the seismic spectrum coefficients are practically constant. It is only in the range of semi-rigid structures that the response spectrum shows a hyperbolic variation in the sense stated by M.A.Biot.

The fundamental vibration period may be found by applying the classical methods of the dynamics of structures or by other approximating methods such as those due to Rayleigh, Dunkerley, Vianello-Stodola, Holzer, a.o.

Starting from the empirical formula

$$T_1 = 0.09 \frac{H}{\sqrt{B}}$$

which has been adopted in many standards and regulations, and from the conditions mentioned above, one may make an overall characterization of structures from a dynamic point of view, by considering the geometrical dimensions over a vertical line (H) and a horizontal line (B) in the direction of the vibration considered. One obtains thus the following criteria of classification:

- <u>rigid structures:</u>

$$H \leqslant 3{,}3 \sqrt{B}$$

- <u>semi-rigid structures</u>

$$3{,}3 \sqrt{B} < H \leqslant 13{,}2 \sqrt{B}$$

- <u>flexible structures</u>

$$H > 13{,}2 \sqrt{B}$$

In these relations H and B are expressed in meters. The authors consider that the above relations characterize to a satisfactory degree the dynamic rigidity of a structure as compared with other proposed relations in which the classification of structures is based on the ratio H/B.

3. <u>Equations suggested for the variation of the first normal vibration mode</u>

The equations proposed for the description of the fundamental forme of vibration have been obtained through a statical computation. Thus, it has been assumed that the fundamental form corresponds to the elastic line of a bar loaded with a static load and having a linear variation, nil at the base and maximum at the end, p_o (fig.6). The determination of the elastic line has been effected by considering the preceding dynamic classification. It should be pointed out that loading with a linearly varying load approaches very nearly the real state of things during an earthquake and that therefore the elastic line will deviate only slightly from the fundamental form of vibration.

In determining the elastic line, due consideration has been given to the fact that in rigid structures the predominant strains are caused through shear, in semi-rigid structures through shear and bending moment, while in flexible structures the strains are caused only through the prevailing bending moment. The equations obtained for the elastic lines and for the fundamental modes are as follows:

a) For rigid structures

Considering that the displacements are caused only through shear, the following expression has been obtained for the real displacement in a horizontal direction y_x (see fig.6):

$$dy_x = \frac{k T_x}{GA} dx \tag{34}$$

or

$$y_x = \int \frac{k T_x}{GA} dx + C = \frac{k}{GA} \int T_x dx + C \tag{35}$$

Since $$\frac{dM}{dx} = T_x$$

expression (35) becomes

$$y_x = \frac{k M_x}{GA} + C \tag{36}$$

In these formulas the following notation has been used:

T_x - shearing force in the current cross section x,

M_x - bending moment in the current cross section x,

G - transverse modulus of elasticity of material,

A - area of cross-section,

k - form coefficient of cross-section A,

C - integration constant depending on the conditions under which the bar is supported.

By introducing the relative coordinate:

$$\zeta = x/H$$

the expression of the bending moment M_x may be written:

$$M_x = -\frac{1}{6} p_0 H^2 (1-\zeta)^2 (2+\zeta) \tag{37}$$

and the displacement y_x :

$$y_x = -\frac{k p_0 H^2}{6 GA} (1-\zeta)^2 (2+\zeta) + C$$

The integration constant is obtained from the support condition at the base cross-section. Thus, for

$x = 0$ or $\zeta = 0$, $y_x = 0$,

hence $$C = \frac{k p_0 H^2}{3GA}$$

The general expression of the actual displacements becomes:

(38) $$y_x = \frac{k p_0 H^2}{6GA}\left[2-(1-\xi)^2(2-\xi)\right]$$

In the case of rigid structures (38) and assuming a configuration similar to that of the static deformation, the variation of the first normal mode of vibration has been considered to be of the form:

(39) $$y_x^R \text{ or } y_x^R = 2-(1-\xi)^2(2+\xi)$$

where:

$\xi = \frac{x}{H}$ in the case of a distributed mass

and

$\xi = \frac{h_k}{H} = \frac{k}{n}$, $k = 1.2, \ldots n$, in the case of concentrated masses.

b) Semi-rigid constructions

In semi-rigid structures the deformations are caused both by the shearing force and by the bending moment. It is found by computation that in that case the variation of the horizontal displacements approaches very nearly a straight line. The authors consider that this assumption, accepted by numerous investigators may characterize the fundamental form of vibration. The equation corresponding to the first mode of vibration will be therefore

(40) $$y_x^{SR} = \xi$$

in the case of a distributed mass and

(41) $$y_k^{SR} = \frac{k}{n}; \quad k = 1,2,\ldots n$$

in the case of concentrated masses.

c) Flexible structures

In the computation of the elastic line of flexible structures only the deformations caused by the bending moment have been considered, starting from the differential equation:

(42) $$\frac{d^2 y_x}{dx^2} = -\frac{M_x}{EI}$$

where E is the longitudinal modulus of elasticity (Young's modulus), and

I is the principal moment of inertia of the transverse cross section.

By successive integration of (42) the rotations and displacements expressions are obtained:

$$\frac{dy_x}{dx} = -\int \frac{M_x}{EI} dx + C \tag{43}$$

and

$$y_x = -\int dx \int \frac{M_x}{EI} dx + C_1 x + C_2 \tag{44}$$

Substituting the value of the bending moment (37) into (44) one obtains:

$$y_x = \frac{p_0 H^4}{120 EI}\left[5(1-\zeta)^4 - (1-\zeta)^5\right] + C_1(1-\zeta) + C_2 \tag{45}$$

The constants of integration C_1 and C_2 have been determined from the following conditions for the fixed-end cross-section:

(a) for x = o or $\zeta = 0$ $\quad \frac{dy_x}{dx} = 0$

(b) for x = o or $\zeta = 0$ $\quad y_x = 0$

Hence:

$$C_1 = \frac{p_0 H^3}{8 EI} \quad \text{and} \quad C_2 = \frac{11 p_0 H^4}{120 EI}$$

The general expression for displacements (45) becomes:

$$y_x = \frac{p_0 H^4}{120 EI}\left[11 - 15(1-\zeta) + 5(1-\zeta)^4 - (1-\zeta)^5\right] \tag{46}$$

Therefore we have considered for the fundamental vibration form corresponding to flexible structures, the following equation:

$$y_x^F \text{ or } y_k^F = 11 - 15(1-\zeta) + 5(1-\zeta)^4 - (1-\zeta)^5 \tag{47}$$

Sometimes, in design calculations, the elastic line is determined by considering a uniformly distributed load over the vertical. It is throught that the triangular loading on which the preceding equations are based brings some additional accuracy into the problem.

Note.

Note.

In determining the vibration mode of rigid structures one may consider curve (47) with the concavity reversed with respect to the straight line which characterizes the vibration form of semi-rigid structures. In that case:

$$y^R = 2y^{SR} - y^F$$

Substituting into the above relation expression (46) and expression (40), multiplied by the factor 11 representing the max. displacement as deduced from (46), one obtains:

$$(48) \qquad y_x^R \text{ or } y_k^R = 4 + 7\zeta - 5(1-\zeta)^4 + (1-\zeta)^5$$

Practically it has been found that this curve characterizes fairly well the variation of the first normal mode of vibration for rigid structures.

4. Numerical computation of equivalence coefficients.

For the fundamental mode of vibration (i = 1), formula (27) becomes:

$$(49) \qquad \mu = \frac{\left[\sum_{k=1}^{n} Q_k y_k\right]^2}{\left[\sum_{k=1}^{n} Q_k\right] \cdot \left[\sum_{k=1}^{n} Q_k y_k^2\right]}$$

The numerical value of the equivalent coefficients are given in table 1, their computation is based on the initial assumptions made and on the previously established modal equations.

It will be seen that these coefficients show a pronounced variation depending on the number of storeyes and on the dynamic rigidity of the structure (see fig.7). As the number of storeys increases the coefficients μ approach a limiting value; for this reason it is suggested that for current design purposes use shall be made of the coefficients given in table 2.

The limiting value of these coefficients which would correspond to structures with an unlimited number of storeys, have been determined by means of formula (23) corresponding to the fundamental mode:

$$(50) \qquad \mu = \frac{\left[\int_0^H q_x y_x \, dx\right]}{\left[\int_0^H q_x \, dx\right] \cdot \left[\int_0^H q_x y_x^2 \, dx\right]}$$

These limiting coefficients are given in table 2.

It is obvious that the variation of the coefficients μ is more conclusive in the range of 1 ... 15 storeys. (n = 1,2 ... ... 15).

V. DIRECT DETERMINATION OF DISTRIBUTION COEFFICIENTS

The numerical computation of the distribution coefficients d has been effected starting from relation (12) which corresponds to the first mode of vibration (i = 1) and taking into account the classification of structures in three groups depending on their dynamic rigidity:

$$d_k = \frac{Q_k y_k}{\sum_{k=1}^{n} Q_k y_k}$$

The numerical values of these coefficients obtained from previous results are given in tables both as function of the number of storeys and of the type of structure. They allow to find directly the base shear without any additional computations.

Owing to lack of space the tables could not be included in the present paper; however they will be given in full in a work to be published shortly in Rumania.

VI. EXPERIMENTAL RESULTS

Experiments were carried out intended to confirm the possibility of using the dynamic equivalent in the computation of structures and also to check the numerical value of conventional equivalence coefficients.

The main results are summed up in table 3.

The tests have been carried out under laboratory conditions on metal models. Dynamical testing of the models has been achieved by means of a vibrating table having three components and variable amplitude and frequency.

Records were obtained with a single channel cathode oscillograph.

I^st Experiment

The experiment refers to the way in which a successive passage may be effected through equivalence from a complex structure with two degrees of freedom (model I/3) to a structure with a single degree of freedom (model I/3) and finally to a one mass system (model I/1).

The experimental results are in good agreement with the theoretical data. It has been possible also to point out the effect which the mass of the elastic support consisting of the frame proper is exerting on the natural oscillation frequencies.

II[nd] Experiment

The second experiment was intended to check the equivalence coefficient for a complex structure with four degrees of freedom (model II/1).

In the determination of the equivalent load (Q_e) use has been made of the equivalent coefficient $\mu = 0{,}7087$ corresponding to the flexible structures group to which such a structure belongs and which is considered as the model of the real structure.

The same frame (model II/2) was used as an elastic support for the one-mass system. The location of the equivalent load has been determined by means of the formula

$$X_o = \frac{2n+1}{3} l$$

The preceding formula has been established by equating the base bending moments of the real structures and the one-mass equivalent system.

The following notation was used:

- x_o - distance from the base to the location of the equivalent mass
- n - number of storeys
- l - distance between two successive floors.

It may be mentioned that for the theoretical treatment of the problem an analog computer has been made use of, for determining the eigenvalues corresponding to the structures II/1.

The general system of differential equations and the coefficients have been determined by applying the principle of virtual work.

III[rd] Experiment

This experiment has confirmed the validity of the limiting value of the equivalent coefficient for flexible structures ($\mu = 0{,}6159$). Taking into account the mass of the elastic support, the results obtained are satisfactory. The location of the equivalent load (Q_e) has been found by applying the relation

$$x = 0{,}835\ H$$

obtained by equating the potential energies of the two systems.

x

x x

Owing to lack of space no details are given regarding the experiments effected under various conditions of loading and mass distribution; the results only, are presented in table 3.

In fig. 8 the oscillograms are shown which were recorded in the course of experiments carried out for obtaining the figures given in table 3. The numerical calculation have been effected by means of an electric calculating machine.

Further experiments are being effected for the purpose of checking the values taken by μ over a larger range. They will be followed by checks carried out an actual structures and possibly by work in connection with corrective factors for equivalence coefficients.

VII. CONCLUSIONS

The analysis of the theoretical and experimental results leads to the following conclusions:

1. The general relations of equivalence show that the real system and the equivalent one-mass system have the same vibration pulse and implicitly the same natural period.

2. The total inertia force (equal to the base shear) is the same in both systems because the pulse variations are identical in the two situations. In this way the use of the equivalent seismic spectrum is justified in the antiseismic computation of structures. Hence, the base shear for a given structure may be determined through the equivalent one-mass system.

3. The equivalence coefficients for masses, μ, can be determined directly only as functions of the number of storeys and of the dynamic rigidity of the structure.

4. There is a certain likeness between the limiting values of the μ coefficients computed by the authors and the unique coefficients obtained by M.Ludwig [5], by considering a uniform distribution over the lenght of the bar. Thus, M. Ludwig obtained

$$\mu_R = 0.810\ ;\ \mu_{SR} = 0.712\ ;\ \mu_F = 0.613$$

and the authors

$$\mu_R = 0.8043;\ \mu_{SR} = 0.7500;\ \mu_F = 0.6159$$

The differences are as follows:

- for rigid structures + o,o715 %
- for semi-rigid structures - 5,o7 %
- for flexible structures - o,o43 %

Considering that μ_{SR} has been determined by M.Ludwig as an arithmetic mean of μ_R and μ_F and comparing his value with the arithmetic mean of the same coefficients as given in the present paper (o,71ol), there results a difference of o,o268 %.

5. The coefficients obtained by M.Ludwig and on which the Californian code is based [5], do not take into account the number of storeys, being unique. It is considered that for structures with a lower number of storeys the computation values of the equivalence coefficients play an important practical part since their differences from the limiting values are fairly great (see table 1).

6. The experimental results have confirmed the practical validity of the current equivalence coefficients. The small differences pointed out in table 3 are quite justified if one considers the inaccuracies which are bound to occur in the manufacture of models, the effect of temperature and moisture changes on the factors involved, and possible recording errors.

It should be pointed out that the assumptions which are at the base of the computations effected in the elastic range leave out a series of secondary factors whose effect is sometimes of the order of experimental errors.

The effect of axial forces and of damping on the natural frequencies of oscillations have been also left out.

x

x x

The present work is part of a langer program of investigation on engineering seismology which is carried out at the laboratories of the Chair of Structure Mechanics of the Institute of Civil Engineering, Bucharest, in collaboration with the Applied Mechanics Institute of the Rumanian Academy. The work will be completed, the effect of the following factors being in coorse of investigation:

a) Effect of two or multiple step set backs over the vertical direction of structures.

b) Effect of higher vdbration modes.

c) Effect of rotation of the foundation caused by the interaction between ground and structure.

d) Checking with instruments the behaviour of real structures.

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TABLE 1

E Q U I VA L E N C E COEFFICIENTS $\mu = m_e/m$			
n Numbers of floors	Rigid Structures $T \leqslant 0.3$ sec.	Semi-rigid Structures $0.3 < T \leqslant 1.2$ sec.	Flexible Structures $T > 1.2$ sec.
1	1	1	1
2	0,9587	0,9000	0,8075
3	0,9324	0,8571	0,7387
4	0,9128	0,8333	0,7087
5	0,9007	0,8181	0,6874
6	0,8915	0,8076	0,6729
7	0,8831	0,8000	0,6666
8	0,8773	0,7941	0,6605
9	0,8704	0,7894	0,6565
10	0,8690	0,7857	0,6521
11	0,8671	0,7826	0,6485
12	0,8637	0,7800	0,6459
13	0,8606	0,7777	0,6434
14	0,8588	0,7758	0,6414
15	0,8568	0,7741	0,6387
16	0,8552	0,7727	0,6367
18	0,8531	0,7702	0,6342
20	0,8509	0,7682	0,6329
22	0,8487	0,7666	0,6312
24	0,8487	0,7653	0,6299
30	0,8454	0,7622	0,6276
40	0,8400	0,7592	0,6245
unlimited	0,8043	0,7500	0,6159

DESIGN VALUES FOR THE EQUIVALENCE

COEFFICIENTS μ

TABLE 2

Number of floors	STRUCTURE TYPE		
	RIGID μ_R	SEMI-RIGID μ_{SR}	FLEXIBLE μ_F
1 ... 1o	According to table nr. "1" values		
11 ... 15	o,861	o,778	o,647
16 ... 2o	o,853	o,771	o,635
over 2o	o,846	o,765	o,630
unlimited	o,805	o,750	o,616

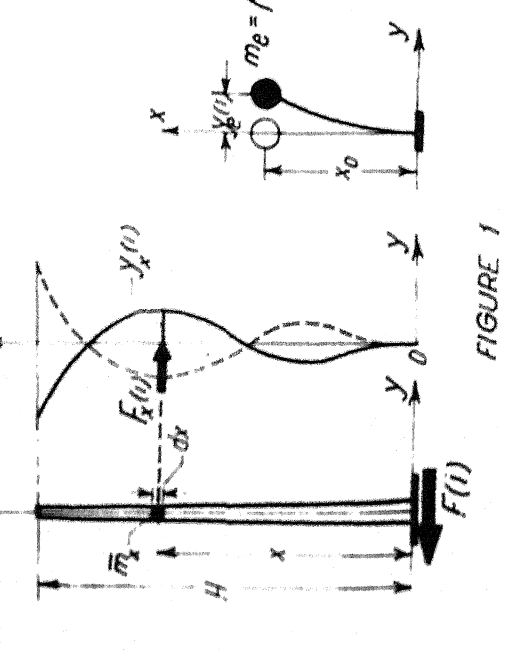


FIGURE 1

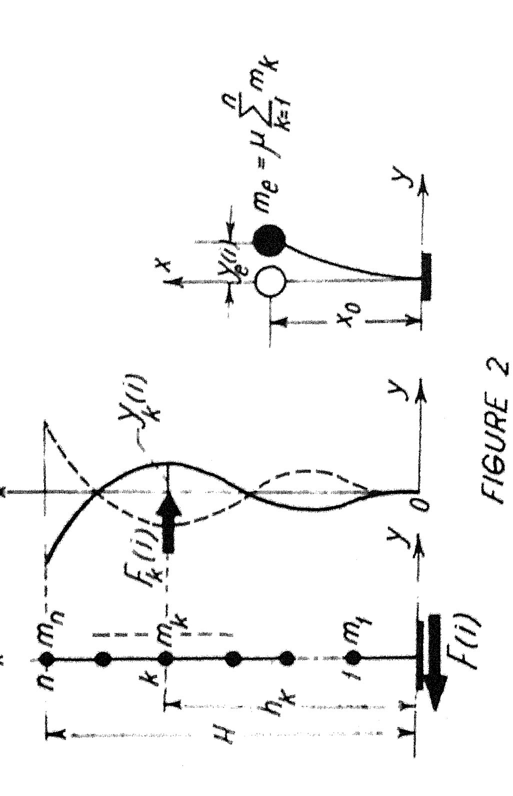


FIGURE 2

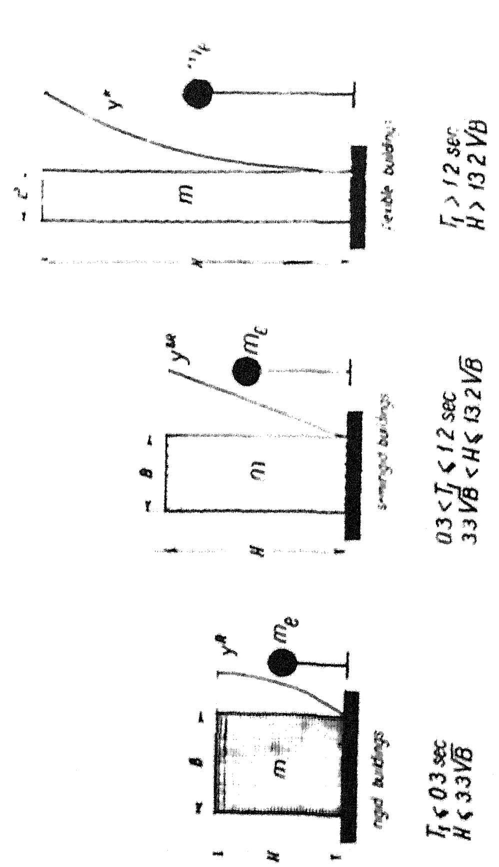


FIGURE 5

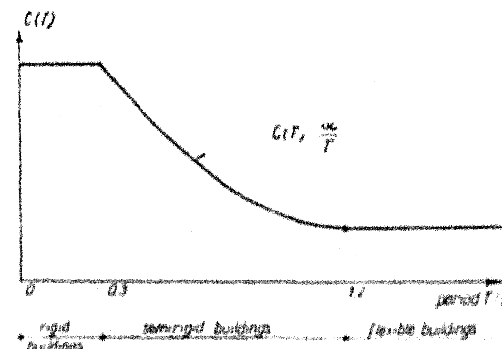


FIGURE 3

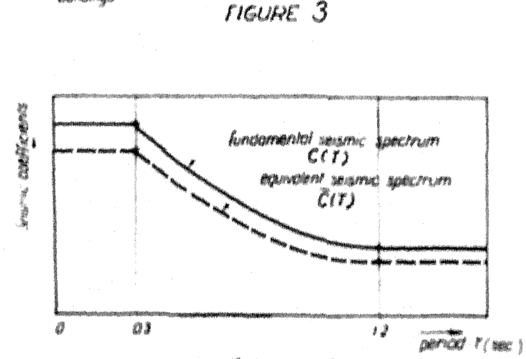
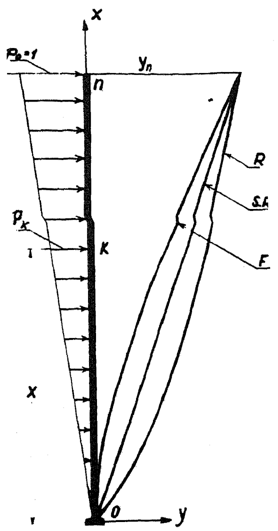


FIGURE 4



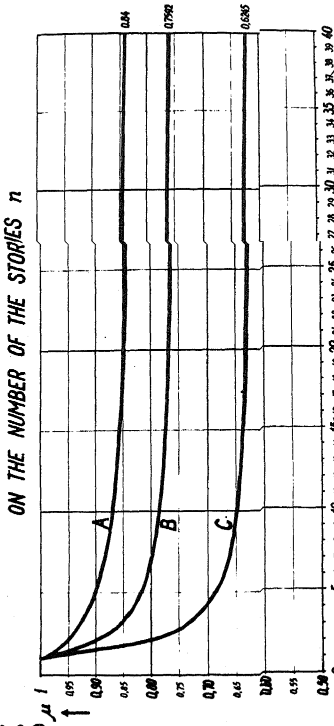
R - rigid buildings
S.R - semirigid buildings
F - flexible buildings

FIGURE 6

TABLE 3

No. Experiments	No. Models	Scheme	Frequency in Hz. Calculated	Frequency in Hz. Measured	Errors %
I	1		15,9	16	< 1%
	2		15,9	15,8	< 1%
	3		15,9	16	< 1%
II	1		11,2	10,65	- 4,9%
	2	x_0; $\mu = 0,7087$	11,3	10,95	- 3,1%
III	1	H	3,63	3,70	+ 1,9%
	2	x_0; H; $\mu = 0,6159$	3,71	3,75	+ 1,1%

THE VARIATION OF THE EQUIVALENCE COEFFICIENT μ DEPENDING ON THE NUMBER OF THE STORIES n



A - for rigid buildings
B - for semirigid buildings
C - for flexible buildings

FIGURE 7

RECORDED OSCILLOGRAMS

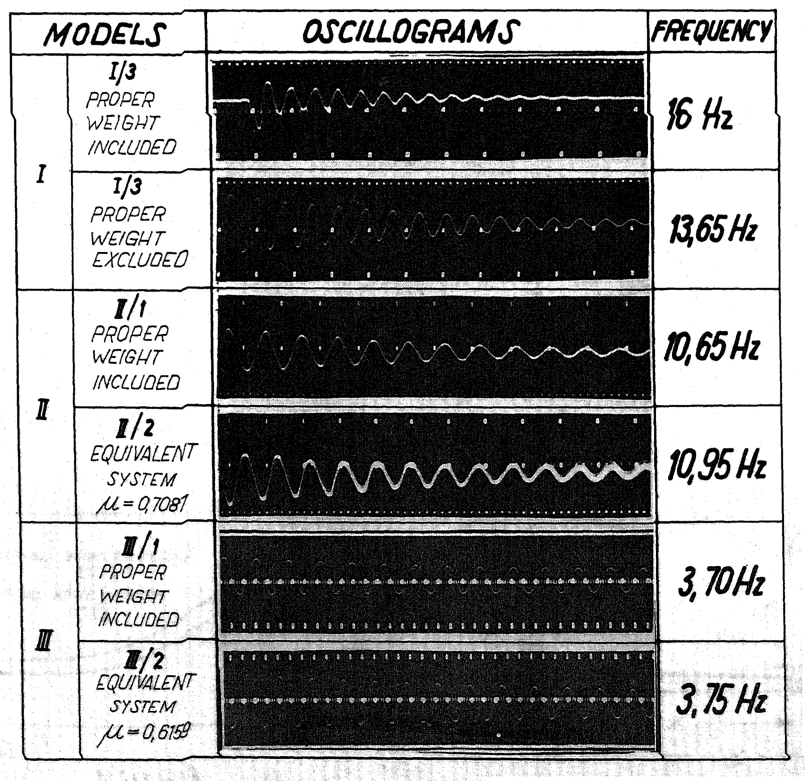


FIGURE 8

PROBLEMS RELATED TO THE DESIGN OF EARTHQUAKE RESISTANT BUILDINGS AND STRUCTURES IN RUMANIA

BY A. A. BELES*

DISCUSSION

Until 1940 the last important earthquake in Rumania was the earthquake of October 26, 1802 mentioned in history as "the great earthquake" which caused damages in the country and some collapses in Bucarest, the Capital of the country. Among the collapsed structures was the historical tower: "Turnul Coltzei". But soon its effects were forgotten and nobody cared about designing buildings to resist earthquakes.

Before 1900 buildings in Rumania were of traditional make with load-bearing brickwalls and wood floors, with ties and floorjoist anchors. Since the beginning of the century and especially after the first world war, reinforced concrete was introduced and many buildings were provided with reinforced concrete frames; but generally no horizontal forces were considered in designing.

Meanwhile some earthquakes of little importance occurred, but as the damages were insignificant, no attention was paid to them. Therefore, when in the early morning of November 10, 1940, a very strong earthquake shook a great part of the country there was a general surprise. Bucarest especially suffered severe damages. One of the most important and quite new buildings, "Carlton", collapsed completely and many other buildings with reinforced concrete frames suffered important damage.

It is interesting to note that Bucarest is situated in the open plain and is crossed by a river called "Dimbovitza" with a main bed of some hundred meters wide. In the plain, the soil is formed of a loessial clay set on a deep layer of sandy gravel, the underground water being at a depth of 5 to 10 meters. The allowable stresses of the foundation soil is of 2 to 3 kg/cm^2. In the valley of the river on account of the alluvial layers of silt, fine sand and clayey sand the allowable stresses of the soil drops to 1 to 1.5 kg/cm^2. During the earthquake of 1940 the buildings situated in the lower part of the town, that is in the main valley of the river, were by far less affected than those in the upper part of the town.

Concerning the system of building, one observed that in general, old buildings with solid load-bearing walls had less to suffer than the new ones, even those having reinforced concrete frames. So for instance, the "Carlton" building with 12 stories and reinforced concrete frames collapsed completely whilst just opposite the street, an old six storied building with load-bearing brickwalls showed no visible deteriorations.

The examination of the different damages produced in buildings and structures, showed that either a poor conception of the project or a bad

* Member of the Rumanian Academy of Sciences, Bucarest, Rumania.

execution was the starting point of the collapse or of the damage.

Soon after this earthquake, official building regulations were issued, based on the "static method of design". Horizontal accelerations of 5% and 10% g with uniform distribution along the height were adopted.

After 1945 on account of the high economical progress of the country, when new industries were developed, new factories built, new towns and especially new living houses erected the necessity of new regulations for earthquake resistant designing appeared. The new experimental regulations which were recently introduced are based on the "dynamic method" which is adopted in U.S.A., U.S.S.R., and recently in France.

The paper presented to the Congress by Prof. Dr. S. Balan, M. Ifrim and C. Pacoste : "The direct determination of equivalence co-efficients of masses in the antiseismic computation of structures" gives a method for reducing a system with several degrees of freedom, as for instance, a multistoried building to a system with a single mass. For practical purposes the equivalence co-efficients are given for several types of structures with different rigidities. This work is one of the different studies made in Rumania to facilitate the use of dynamic methods of design.

Although the design of structures to resist horizontal forces is of great importance for the safety of structures built in seismic zones, the damages produced by earthquakes in Rumania and recently in Agadir, Skopje, Anchorage and Niigata, showed that the most important causes of collapses and severe damages are due to errors in projects or to bad quality of execution.

One cannot deny the importance of calculation of structures to horizontal forces produced by earthquakes, but the best mathematical calculations give only a raw approximation of the reality. One must not forget that the mechanical characteristics of the earthquake shocks differ from one earthquake to another and consequently the numerical values accepted by the different codes may differ essentially from reality.

Also the elastic and plastic behaviour of structures is based on assumptions which differ from reality. At the 50th Anniversary of the "Deutscher Beton-Verein" it was shown that the lateral rigidity of different kinds of buildings rises from 1 to 20 and even much more if one considers the structural frame alone or one takes into account the effect of floors, walls, roofs and all the different constructive elements of the building.

Besides this, there are different elements which cannot be taken into account with sufficient precision for the designing. For instance, the evaluation of the intensity of the earthquake shock, the mechanism of the propagation and the distribution of the stresses produced by the shock, the real cause of collapse, are all elements which have not yet been cleared.

The real behaviour of structures during the earthquake shocks, depend on so many random factors that one has tried to introduce probabilistic methods of calculation. However, it must be observed that statistical data are insufficient to draw valuable conclusions.

As was already said, observation of earthquake damage has shown the prominent importance of errors in conception of buildings and structures and weak points in execution as the main causes of these damages.

These two deficiencies have been pointed out by specialists who have investigated the effects of earthquakes and many of such aspects have been issued in various publications. I find that it would be of great utility if different and illustrative examples from different earthquakes could be collected and published. Such a work could be easily done by the co-operation of Unesco and the International Association for Earthquake Engineering. In such a work, where damages produced by earthquakes with the necessary explanation would be given, the architect, the engineer, the designer and the builder could find the necessary information for designing and erecting of structures with most chances to resist earthquakes.