HORIZONTAL RESISTANCE OF STEEL PILES UNDER STATIC AND DYNAMIC LOADS

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ABSTRACT

A relationship between the horizontal subgrade reaction and the pile deflection which seems to be more adequate than that proposed in the past is presented. From this relation, a new method for estimating the behaviours of piles under static loads is proposed. Futhermore, in order to investigate the dynamic behaviours of piles, the alternating loading tests are carried out together with the static and dynamic loading tests. By analyzing the results of these tests and by developing the investigation of the behaviours of piles, under alternating loads, a conventional method for calculating the dynamic response of piles is suggested.

INTRODUCTION

In order to establish the method for the earthquake resistant design of pile foundations or structures, it is essential to make clear the behaviours of piles subjected to horizontal dynamic loads. However, only a few dynamic loading tests of piles have been reported so far, and an appreciable increase of the records of these tests would hardly be expected in the near future. Since it has been found that the dynamic loading test itself involves difficulties both in a test performance and in an evaluation of data. Therefore, it would be hard to draw out the general rules governing the behaviours of piles only from the dynamic loading tests. In addition, for a practical purpose, the same procedures as in case of static loads are adopted in the design calculation for dynamic loads. Hence, the authors intend to make use of the results of static and the alternating loading tests so as to investigate the dynamic behaviours of piles. A satisfactory answer to the dynamic problem of piles seems to be obtained by the approach which bases on the study of the static behaviours of piles.

BEHAVIOURS OF STEEL PILES SUBJECTED TO HORIZONTAL STATIC LOADS

Horizontal subgrade reaction

The general differential equation of a single vertical pile in a soil subjected to a static horizontal load is

$$EI\frac{d^4y}{dx^4} = -q \qquad (1)$$

where EI is the flexual rigidity of a pile, y is the deflection of a pile at the depth x, x is the depth below the soil surface and q is the horizontal subgrade reaction per unit length of a pile at the depth x. Previous investigators have made various assumptions as to the relationship between q and y. For example, Y. L. Chang(1) assumed a soil as an elastic medium and gave the relationship between q and y as follows:

$$q = E_s y \qquad (2)$$

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where E_s is the elastic modulus of a soil. He solved the equation obtained by substituting q from equation(2) into equation(1). His solution is widely adopted for analyses and designs of pile foundations or structures because of its simplicity. But as a soil is substantially an inelastic medium, it seems to give no satisfactory explanation of actual behaviours of piles under applied horizontal loads.

The authors(2) carried out many field tests on full scale prototype piles and investigated the characteristics of a horizontal subgrade reaction. As a conclusion, they obtained the relationship between q and y as follows:

$$q = k B y^{0.5} \tag{3}$$

where k is the coefficient of the horizontal subgrade reaction and B is the width of a pile. T. Shinohara and K. Kubo(3),(5) showed that the results of many systematic model tests lead to the relationship:

$$q = \bar{k} B x y^{0.5} \tag{4}$$

where $\bar{k}$ is the coefficient of the horizontal subgrade reaction. Both equations differ basically from the ones proposed in the past in the respect that these present q as a non-linear function of y.

Solution of the basic equation

When the right-hand side of equation(1) is replaced by equations(3) and (4) respectively,

$$\left.\begin{aligned} EI \frac{d^4 y}{dx^4} &= -kBy^{0.5} \\ EI \frac{d^4 y}{dx^4} &= -\bar{k} B x y^{0.5} \end{aligned}\right\} \quad (5)$$

In case of a pile with a part of its length embedded in soil, equation(5) fails to meet the boundary requirement that the subgrade reaction must be zero at the soil surface. Hence, if it is necessary to deal with it mathematically, equation(3) must be replaced by the following expression:

$$q = k' B x^m y^{0.5} \tag{6}$$

where k' is the modified coefficient of horizontal subgrade reaction and m is a positive small number, and k' and m are chosen so that q computed from equation(7) equals practically to that computed from equation(3) along the pile length.

As equations(5) and (6) are both non-linear differential equations, it is not simple to solve them. The authors solved equation(5) by using an electronic analogue computer and showed that there is a good agreement between the computed values and the measured ones. Using the results of many model tests, Shinohara and Kubo made "Standard curves", and established a general computation method of the solution of equation(6) by applying the law of similarity. In addition, they showed the computed values to be in full agreement with the measured ones. As the authors also drew up "Standard curves" in case of q expressed by equation(3), the behaviours of a pile under a horizontal load can be estimated by a method similar to that presented by Shinohara and Kubo.

Types of soils

from the standpoint of the horizontal subgrade reaction, Kubo(6) classified soils into two types. It is called "C-type soil", if the horizontal reaction of the soil is written by equation(3), while it is called "S-type soil" if it is expressed as equation(4). Both types of soil are distinguished from each other according as the distribution of N-value of standard penetration test for the depth is uniform (C-type soil) or linearly increasing with an increase of the depth (S-type soil). Necessary depth to determine the types of soils should be 5 to 10 meters. These values correspond to the depth of the first zero point of the bending moment of piles or half. In the same paper, Kubo gave examples of both types of soil. Some of the examples of "C-type soil" are the sand deposit the surface of which is compacted, and the preconsolidated clay deposit, while the sand deposit with uniform density and the normally consolidated clay deposit belong to "S-type soil".

By investigating test results and data, the authors found the relationship between k-value and N-value in case of "C-type soil" as illustrated in Fig. 1. Kubo(4) also found the relationship between k-value and N-value in case of "S-type soil" as illustrated in Fig. 2, where N-value is N-value at the depth of a meter from the soil surface.

Test results

Examples of the results of tests performed by the authors are illustrated in Fig. 3 and 4 together with the curves computed by three methods: the author's, Kubo's and Chang's. Here, E -value in Chang's method is estimated by Terzaghi's proposal(7) modified by Chang.(1) As examples illustrated in Fig. 3 and 4 are the case of "C-type soil", the values computed by the first method agree best with the measured ones. But Shinohara and Kubo showed that in case of "S-type soil" the values computed by the second methos reached a full agreement with the measured ones. In both cases, it seems that behaviours of an actual pile can not be explained by Chang's method which assumed a horizontal subgrade reaction as a linear function of pile deflection.

Effects of B and effective length of a pile

According to the Shinohara and Kubo's study, effects of the width of a pile B upon the coefficient of the horizontal subgrade reaction and the effective length of a pile are as follows: 1) The values of k and $\bar{k}$ are both practically constant for a width greater than 20 cm. 2) Let l_m be the length from the soil surface to the first zero point of the bending moment, then the effective length is about 1.5 l_m. The term "effective length" means the length, measured from the soil surface, of the pile portion which effectively resists horizontal loads. The part of a pile below it does not affect behaviours of a pile subjected to horizontal loads. In other words, it can be defined as the shortest embedment of a pile which can be regarded as one with the infinite length of embedment. In the Present paper, the authors deal only with piles of such a length of embedment.

Law of similarity

Variables which enter the pile problem are the horizontal load F, the flexual rigidity of a pile EI, the free length of a pile h, the width of a pile B, the coefficient of the horizontal subgrade reaction k or $\bar{k}$, the pile deflection y, the inclination angle of a pile i, the bending moment in a pile M and the horizontal subgrade reaction q. If the length in x direction X, the length in y direction Y, the length in the perpendicular direction to x and y Z and the force Q are chosen as the fundamental dimensions, the nine variables mentioned above in case of $q = k\,B\,x^m y^n$ are given in terms of X, Y, Z and Q as follows: $F = [Q]$, $EI = [Q X^3 Y^{-1}]$, $h = [X]$, $B = [Z]$, $k = [Q X^{-(m+1)} Y^{-n} Z^{-1}]$, $y = [Y]$, $i = [X^{-1} Y]$, $M = [Q X]$, and $q = [Q X^{-1}]$. As dimensional matrix of these nine variables contains nonzero determinants of order 4, there are five dimensionless products in a complete set.

Let a pile from behaviours of which those of a prototype pile are estimated be called "the standard pile", and variables of "the standard pile" be written with a subscript s and those of a prototype pile with a subscript p. Moreover let R be the ratio of variables of a prototype pile to corresponding variables of the standard pile. The corresponding symbol of variables is attached to R as the subscript. For example,

$$R_y = y_p / y_s$$
$$R_F = F_p / F_s$$
$$\vdots$$

If ratios of dimensionless products of a prototype pile to those of "the standard pile" are equal to 1, then

$$\left.\begin{aligned}
R_F &= R_h^{\frac{m+3n+1}{1-n}}\ R_{EI}^{\frac{-n}{1-n}}\ R_k^{\frac{1}{1-n}}\ R_B^{\frac{1}{1-n}}\\
R_y &= R_h^{\frac{m+4}{1-n}}\ R_{EI}^{\frac{-1}{1-n}}\ R_k^{\frac{1}{1-n}}\ R_B^{\frac{1}{1-n}}\\
R_i &= R_h^{\frac{m+n+3}{1-n}}\ R_{EI}^{\frac{-1}{1-n}}\ R_k^{\frac{1}{1-n}}\ R_B^{\frac{1}{1-n}}\\
R_M &= R_h^{\frac{m+2n+2}{1-n}}\ R_{EI}^{\frac{-n}{1-n}}\ R_k^{\frac{1}{1-n}}\ R_B^{\frac{1}{1-n}}\\
R_q &= R_h^{\frac{m+4n}{1-n}}\ R_{EI}^{\frac{-n}{1-n}}\ R_k^{\frac{1}{1-n}}\ R_B^{\frac{1}{1-n}}
\end{aligned}\right\} \qquad (7)$$

When ratios of variables satisfy equations(7), behaviours of a prototype pile are similar to those of "the standard pile".

Conversion factors

Setting m = 0, n = 0.5 in equation(7), expressions in case of "C-type soil" are obtained as follows:

$$\left.\begin{aligned}
R_F &= R_h^{5}\ R_{EI}^{-1}\ R_k^{2}\ R_B^{2}\\
R_y &= R_h^{8}\ R_{EI}^{-2}\ R_k^{2}\ R_B^{2}\\
R_i &= R_h^{7}\ R_{EI}^{-2}\ R_k^{2}\ R_B^{2}\\
R_M &= R_h^{6}\ R_{EI}^{-1}\ R_k^{2}\ R_B^{2}\\
R_q &= R_h^{4}\ R_{EI}^{-1}\ R_k^{2}\ R_B^{2}
\end{aligned}\right\} \qquad (8)$$

Setting m = 1, n = 0.5 and $k = \bar{k}$ in equations(7), expressions in case of "S-type soil" are given as follows:

$$
\left.
\begin{aligned}
R_F &= R_h^{7}\ R_{EI}^{-1}\ R_{\bar{k}}^{2}\ R_B^{2}\\
R_y &= R_h^{10}\ R_{EI}^{-2}\ R_{\bar{k}}^{2}\ R_B^{2}\\
R_i &= R_h^{9}\ R_{EI}^{-2}\ R_{\bar{k}}^{2}\ R_B^{2}\\
R_M &= R_h^{8}\ R_{EI}^{-1}\ R_{\bar{k}}^{2}\ R_B^{2}\\
R_q &= R_h^{6}\ R_{EI}^{-1}\ R_{\bar{k}}^{2}\ R_B^{2}
\end{aligned}
\right\}
\qquad (9)
$$

R_F, R_y, R_i, R_M and R_q in equations(8) and (9) are the conversion factors in case of "C-type soil" and "S-type soil" respectively.

"Standard curves"

The pile "K-III" shown in the Table is chosen as "the standard pile" in case of "C-type soil". F - y_0 curve, F - i_0 curve, F - M_{max} curve and F - l_m curve of "the standard pile" are called "standard curves", where y_0 is the pile deflection at the soil surface, i_0 is the inclination angle of the pile at the soil surface and M_{max} is the maximum bending moment in the pile. "Standard curves" of "C-type soil" in case of the free head pile are illustrated in Figs. 5 to 8. Shinohara and Kubo chose the pile "MODEL" shown in the Table as "the standard pile" of the "S-type soil". They presented F - y_0 curve, F - y_{top} curve, F - M_{max} curve and F - l_m curve of "the standard pile" as "standard curves", where y_{top} is the top deflection of the pile. "Standard curves" are illustrated in Figs. 9 to 12 in case of the free head pile and in Figs. 13 to 16 in case of the fixed head pile.

Use of "standard curves"

The important properties for design of horizontally loaded piles such as the deflection of a pile above the soil surface, the maximum bending moment and the necessary length of a pile embedment can be calculated by using "standard curves". The procedure of calculation is as follows:

C-type soil [S-type soil]

1) Determine the corresponding k-value [$\bar{k}$-value] to the N-value [$\bar{N}$-value] of the soil from Fig. 1 [Fig. 2].
2) Compute ratios of variables R_h, R_{EI}, R_k, R_B [R_h, R_{EI}, $R_{\bar{k}}$, R_B].
3) Compute ratios of variables R_F, R_y, R_i, R_M [R_F, R_y, R_M] from equations (9) [(10)].
4) Compute F_s from the following relation:
   $\log F_s = \log F_p - \log R_F$
5) Determine the corresponding $(y_0)_s$, $(i_0)_s$, $(M_{max})_s$, $(l_m)_s$ [$(y_0)_s$, $(y_{top})_s$, $(M_{max})_s$, $(l_m)_s$] to F_s obtained in 4) by "standard curves"
6) Compute $(y_0)_p$ [$(y_0)_p$, $(y_{top})_p$] from the following relation:
   $\log y_p = \log y_s + \log R_y$
   Also compute likewise $(i_0)_p$, $(M_{max})_p$, $(l_m)_p$ [$(M_{max})_p$, $(l_m)_p$]
7) (C-type soil only) Compute $(y_{top})_p$ from the following relation:

$$(y_{top})_p = (y_0)_p + h_p \times (i_0)_p + F_p h_p^3 / 3(EI)_p$$

In the above prodedure, expressions in square brachets are adopted in computation for "S-type soil".

Thus the top deflection y_{top}, the deflection at the soil surface y_0, the maximum bending moment M_{max} and the length of the first zero point of the bending moment l_m of a given pile for a given horizontal load can be obtained. The necessary length of a pile embedment is 1.5 l_m.

BEHAVIOURS OF STEEL PILES SUBJECTED TO ALTERNATING HORIZONTAL LOADS

Alternating horizontal loading tests

Besides the horizontal static loading tests mentioned in the prrceding section, the authors carried out "the alternating horizontal loading tests" on the steel piles shown in the Table as a step to make clear the dynamic behaviours of a pile. And they measured the top deflection and the distribution of the bending strain of a pile soon after loading or unloading. "The alternating horizontal loading tests" consist of several "steps" at each of which the maximum load is increased. Each "step" consists of a few "cycles", each of which is carried out in the following order: Loading to a certain level and unloading, then loading to the opposite direction to the same level and again unloading.

Test results

As the results of the tests, the relationship between the applied load F and the top deflection of a pile y_{top} is illustrated in Fig. 17. As seen in this figure, F - y_{top} relationship has hysteretic characteristics. These hysteresis loops for the same maximum load(viz. "cycles" of the same "step")practically agree with each other except the first loading curve of the "step" and is also practically the same shape for loading to both directions.

By the measurement of the bending strain, valid results can not be obtained, therefore the top deflection of a pile y alone is the measured value. Hence, in the following discussion, the relationship between the horizontal subgrade reaction and the pile deflection at the corresponding depth is not the measured one but one determined by the way that first the relationship between F and y_{top} is computed from equation(1) with the assumed q - y relation by electronic analogue computer or "the standard curves", then F - y_{top} relation thus obtained is compared with the measured one and if they agree good the above assumed q - y relation is regarded as the repesentation of the actual one.

Determination of F - y_{top} relation

In addition to their tests, the authors collected the data of five "alternating horizontal loading tests" performed by other investigators and tried to express q - y relation analytically or graphically which lead to the same as measured F - y_{top} relation. In order to do this, F - y_{top} relation is regarded to consist of three curves, "the initial curve", the unloading curve" and "the loading curve". Here "the initial curve" means the one obtained by binding the origin and the peak points of each "step", "the unloading curve" means the one obtained by unloading and "the loading curve" means the one obtained by loading except the first loading of each "step".

First of all, the investigation on "the initial curve" leads to the following q - y relation:

$$q = k_u B y^{0.5} \quad \text{(C-type soil)} \qquad (10)$$

or

$$q = \bar{k}_u B x y^{0.5} \quad \text{(S-type soil)}$$

where k_u and $\bar{k}_u$ are both the soil constants, and are dimensionally equal to k and $\bar{k}$ respectively. Next, if F and y_{top} in F - y_{top} relation measured backward from the peak point of each "step" are called F' and Y'_{top}, all the F' - y'_{top} relation of each step are practically on the same curve as illustrated in Fig. 18. Corresponding q' - y' relation to F' - y'_{top} relation turned out to be written as follows:

$$\left.\begin{array}{ll} q' = k_d B y'^{0.5} & \text{(C-type soil)} \\ \text{or} & \\ q' = \bar{k}_d B x y'^{0.5} & \text{(S-type soil)} \end{array}\right\} \quad (11)$$

where k_d and $\bar{k}_d$ are both the soil constants and are dimensionally equal to k and $\bar{k}$ respectively, q' is the horizontal subgrade reaction measured backward from the point where the unloading begins and y' is the pile deflection at the corresponding depth measured likewise. k_u, $\bar{k}_u$, k_d and $\bar{k}_d$ are correlated with k and $\bar{k}$ respectively, and these relations are illustrated in Fig. 19. From this figure, it can be concluded that k_u and $\bar{k}_u$ are an increase of about ten per cent of k and $\bar{k}$ respectively while k_d and $\bar{k}_d$ are an increase of about forty per cent of them respectively. Finally, in order to determine "the loading curve", the following approach is made: first consider the state where the bending stress in a pile section has reached the yield value of the pile material by an appropriate load, then the depth is determined where the subgrade reaction is maximum. This maximum subgrade reaction is called q^* and the corresponding deflection at the depth above determined is called y^*. In Fig. 20 (b), A' and C' correspond to (q^*, y^*) and ($-q^*$, $-y^*$) respectively. B' is determined as the intersecting point of y axis and "the unloading curve" which passes A', and D' is determined likewise. Then two segments of straight lines $\overline{A'D'}$ and $\overline{C'B'}$ are obtained.

From the above investigations and operations a set of a loop and a curve is drawn by the following procedure:

1) Compute the yield bending moment M^* in a pile section.
2) Determine k- or $\bar{k}$-value corresponding to N-value from Fig. 1 or 2 respectively.
3) Determine k_u-, k_d-, or $\bar{k}_u$-, $\bar{k}_d$-value by k- or $\bar{k}$-value from the above mentioned relation.
4) Draw F - y_{top} relation of "the initial curve" by "standard curves" assuming that q - y relation is expressed as the equation(10).
5) Compute F^* corresponding to M^* obtained in 1) by "standard curves".
6) As illustrated in Fig. 20 (a), take the point A, the ordinate of which is F^*, on "the initial curve". Determine C likewise.
7) Determine B and D as the intersecting point of the abscissa and "the unloading curve" which passes A and C respectively.
8) Bind A and C. Also bind B and C.

Using the set of a loop and a curve (the initial curve) thus obtained, F - y_{top} relation for a given maximum load can be drawn by the following three rules:

1) In the first loading, F - y_{top} trajectory starts from the origin and moves along "the initial curve".
2) When the direction of an applied load changes, F - y_{top} trajectory which follows any curve shifts to "the unloading curve" passing the point where the direction of a load changes. Hereafter it moves along the curve.

3) If F - y_{top} trajectory which follows "the unloading curve" meets the loop, then it shifts to the loop. Hearafter it moves along the loop.

F - y_{top} relation thus obtained is illustrated in Fig. 21 together with the measured one.

"Skeleton" of the hysteresis loop

F - y_{top} relation obtained by the alternating horizontal loading tests has the characteristics of a hysteresis as pointed out in the preceding section. In order to understand the nature of this loop, it is convenient to introduce the concept of the "skeleton" of a hysteresis loop. From a hysteresis loop it is known that there are two F values corresponding to any y_{top} value smaller than the maximum deflection. The curve binding the mean value of these two F-values is called the "skeleton" of a hysteresis loop. The "skeleton" is an important index of properties of a hysteresis loop.

The characteristics of the "skeleton" of F - y_{top} relation obtained by the tests or the geometrical method presented in the preceding section are as follows: 1) The "skeleton" of a cycle belongs to a hardening-spring system. In other words, the soil behaves as if it hardens with an increase of an applied load. 2) The "skeleton" is dependent on the maximum load or the maximum deflection: an increase of the maximum load causes the "skeleton" to shift apart from the ordinate and yet it still passes through the origin. These are illustrated in Fig. 22. In this figure any "skeleton" of a, b and c shows the increasing slope with an increase of F, while by increasing the maximum load the "skeleton" moves from a to b and then to c. Thus F - y_{top} relation turns out to be dependent on the maximum deflection. Therefore when the maximum deflection of a "step" is designated as Y, F - y_{top} relation are written in the form: $F = F(y, Y)$

BEHAVIOURS OF STEEL PILES SUBJECTED TO HORIZONTAL DYNAMIC LOADS

Dynamic loading tests

The authors performed two kinds of dynamic loading tests on the same pile used in the alternating loading tests. One is the free vibration tests and the other is the forced vibration tests. Measured quantities are the distribution of the bending strain along a pile, the top deflection of a pile, the deflection of a pile and the acceleration destribution along a pile. But no valid relationship between the horizontal subgrade reaction q and the pile deflection at the corresponding depth y computed from the measured bending strain can be obtained.

Test results

Free vibration tests: 1) A decrease in the amplitudes leads to an increase of the natural frequency of a pile as illustrated in Fig. 23. 2) With an increase of N-value of a soil the natural frequency of a pile also increases.

Forced vibration tests: 1) The resonance curves shown in Fig. 24 seem to be those of the hardening-spring system. 2) The relationship between N - value of a soil and the resonance frequency is the same as mentioned in the results of the free vibration tests.

These test results would be completely explained, if it is assumed that the horizontal subgrade reaction against dynamic loads is similar to that against alternating loads.

Vibration system of a pile

The equation of motion of a pile is generally a non-linear partial differential equation, therefore it is not easy to solve the equation. Hence the authors adopted the physical pendulum illustrated in Fig. 25 as a simplified model of an actual pile and investigated the vibration characteristics of a pile. The physical pendulum has the same mass distribution as a prototype pile, the same length as that from the top to the first zero point of the deflection of a prototype pile and the hinged end. In case of an actual pile, the first zero point of the pile deflection moves during vibration. This movement, however, is practically negligible. Here the forst zero point of the deflection of a pile is about 1/2 to 1/3 l_m. As the relationship between the restoring force and the displacement of the spring system attached to the top of the pendulum the F - y_{top} relation of a prototype pile under dynamic loads should be adopted.

This simplification involves many assumptions described above, hence it is necessary to examine whether the characteristics of the period of vibration in case of the pendulum represent those of a prototype pile or not. The authors computed the natural period of the pendulum by the prescribed method and that of a prototype pile by the static-deflection method in case of a linear subgrade reaction by varying the free length of the pile, the length of embedment of the pile, the mass at the pile top and the end condition. The two computed values approximately agreed with each other. Especially, in case of the length of pile embedment more than 8 meters, they completely agreed with each other. It seems to be reasonable to consider that the relationship between those two natural periods in case of a non-linear subgrade reaction is similar to that of linear one.

Equation of motion of the pendulum

The equation of motion of the physical pendulum is given as follows:

$$\Phi \frac{d^2 y}{dt^2} + \bar{F}\left(y, \frac{dy}{dt}, Y\right) = G(t) \qquad (12)$$

where $\Phi = I_0 / H_0^2$, I_0 is the moment of inertia of a pile around the hinged end, H_0 is the length of the pendulum, y is the top displacement of the pendulum, Y is the maximum displacement at the top of the pendulum, $\bar{F}$ (y, dy/dt, Y) is the restoring force of the spring and the function, which is the F - y_{top} relation of a prototype pile for dynamic loads, of y, dy/dt and Y, and G (t) is the exciting force applied at the top of the pendulum.

$\bar{F}$ (y, dy/dt, Y) has not been obtained, so instead of it, F - y_{top} relation obtained by the alternating loading tests is adopted. Then, $\bar{F}$ (y, dy/dt, Y) will be written approximately in the form:

$$\bar{F}\left(y, \frac{dy}{dt}, Y\right) = c\frac{dy}{dt} + F(y, Y) \qquad (13)$$

where c is the viscous damping coefficient. Substituting $\bar{F}$ (y, dy/dt, Y) from equation(13) into equation(12), the equation of motion is reduced to the following expression:

$$\Phi \frac{d^2y}{dt^2} + c\frac{dy}{dt} + F(y, Y) = G(t) \tag{14}$$

Solution of stationary vibration

In case of the stationary vibration such as the forced vibration tests, $G(t)$ in equation(14) is written in the form: $G(t) = P_0 \sin\omega t$. When this expression of $G(t)$ is substituted into equation(14), then

$$\Phi \frac{d^2y}{dt^2} + c\frac{dy}{dt} + F(y, Y) = P_0 \sin \omega t \tag{15}$$

The solution of equation(15) may be written as follows[8]:

$$y = Y \sin(\omega t - \varphi) \tag{16}$$

where φ is the phase difference between the exciting force and the displacement of the pendulum.

To determine Y and φ in equation(16), $F(y, Y)$ is resolved into two elements as illustrated in Fig. 26: the "skeleton" of the hysteresis loop $f(y, Y)$ and a half of the vertical cord length of the hysteresis loop $g(y, Y)$. Then $F(y, Y)$ is expressed with these two elements as follows:

$$F(y, Y) = f(y, Y) \pm g(y, Y) \quad \left(\frac{dy}{dt} \gtrless 0\right) \tag{17}$$

If $F(y, Y)$ is expanded into Fourier series and only the fundamental harmonics is taken into consideration, then

$$F(Y\sin(\omega t-\varphi), Y) = F'(Y)\sin(\omega t-\varphi) + \frac{W(Y)}{\pi Y}\cos(\omega t-\varphi) \tag{18}$$

where $F'(Y) = 1/\pi \cdot f(Y \sin\theta, Y) \sin\theta \, d\theta$, θ is the dummy angle, and $W(Y)$ is the area contained by the hysteresis loop and represent the amount of the energy dissipation during a cycle. Substituting equations(16) and (18) into equation(15), the following two equations for determining Y and φ are obtained:

$$\left.\begin{aligned} F'(Y) &= \Phi\omega^2 Y \pm \sqrt{P_0^2 - \left\{c\omega Y + \frac{W(Y)}{\pi Y}\right\}^2} \\ \sin\varphi &= \left\{c\omega Y + \frac{W(Y)}{\pi Y}\right\} \Big/ P_0 \end{aligned}\right\} \tag{19}$$

$F'(Y)$ in equations(19) is determined by making use of Klother's method[8]. By substituting Y and φ determined by equations(19) into equation(16), the solution of equation(15) for a given exciting frequency ω is obtained.

Investigation on the pendulum system

Examples of the resonance curves thus obtained in case of $c = 0$ in equation(15) are illustrated in Fig. 24 together with the measured ones. In this figure, the circles and the crosses show measured values obtained by increasing and decreasing revolution of the vibrator respectively. As shown in Fig. 24 both values computed and measured show a good agreement not only on the resonance frequency but also on the amount of damping.

This fact would lead to the conclusion that the physical pendulum with the same mass distribution as a prototype pile, with the spring determined by $F - y_{top}$ relation for the alternating loads at its top, with the length equivalent to that from the top to the first zero point of the deflection of a prototype pile and with the hinged end represents a prototype pile in respect to the resonance curve. And besides the energy absorption by $F - y_{top}$ relation itself, it is unnecessary to consider any other damping term such as $c \cdot dy/dt$. Furthermore these facts show that the horizontal subgrade reaction against the alternating load is similar to that against the dynamic load. To conclude $F - y_{top}$ relation for a given pile and soil can be obtained by the way suggested in the preceding section. Then the resonance characteristics of a given pile can be estimated by the above mentioned method without making any test.

Nature of the damping force

From the above discussion, it would be concluded that as to the amount of damping only the amount equivalent to the area contained by the hysteresis loop which is obtained by the alternating loading tests should be taken into account. So the authors investigated the nature of such an area, and found that the relationship between the area W(Y) and the maximum deflection Y was expressed in the form:

$$W(Y) \propto Y^{\alpha} \qquad (1 < \alpha < 2)$$

This relation is illustrated in Fig. 27. α seems to be not dependent on the soil condition but dependent on the condition of the boundary confinement at the pile top. In case of a free head pile $\alpha = 1.7$ and in case of a fixed head pile α takes a value between 1 and 1.7 according to the degree of boundary confinement at the pile top. In general, the amount of the viscous damping is proportional to $Y^2 (\alpha = 2)$ and that of the solid friction is proportional to Y $(\alpha = 1)$. Then the above discussion would lead to the conclusion that the viscous damping force and the solid frictional force act simultaneously on a pile in vibration.

It may not be easy to understand why the viscous damping force acts when the static loads like alternating loads are applied. This seems to be explained by the fact that in the alternating load tests the loading speed is finite. Actually the speed is about one tenth of the maximum load per two seconds.

CONCLUSION

The present paper gives the method of calculating the significant guantities for the design of vertical piles under static loads and that of estimating the vibration characteristics of piles. These results and their underlying concept would offer the basis for a proper method of designing the earthquake resistant pile structures and foundations. However, the authors treated herein only the dynamic response of piles subjected to the harmonic excitation, so the problem on piles under the random excitation remains untouched. In order to have the exact knowledge on the remaining problems and to make clear the whole situation, it seems to be necessary that the many extensive research works in this field will be carried out.

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TABLE Description of piles, soils and loads

	Test No.	Flexual rigidity EI kg-cm	Free length h m	Width B cm	Type of pils	Soil condition Soil	Soil condition Type	Load
T.T.R.I.	K-I	4.5×10^{10}	1.0	30.5	H	Sand	S,C	Static
	K-II	4.5×10^{10}	0.5	30.5	H	Sand	S	Static
	K-III,IV	4.5×10^{10}	0.5	30.5	H	Sand	C	Static
	L-II	1.5×10^{10}	0.5	30.0	H	Sand	S,C	Static
	L-IV	1.5×10^{10}	0.5	30.0	H	Sand	C	Static
	E-I	4.5×10^{10}	3.0	30.5	H	Sand	S,C	Dynamic
	E-II	4.5×10^{10}	0.5	30.5	H	Sand	S	Dynamic
	E-III,IV	4.5×10^{10}	0.5	30.5	H	Sand	C	Dynamic
	F-II	4.5×10^{10}	2.0	30.5	H	Sand	S,C	Dynamic
	F-III,IV	4.5×10^{10}	2.0	30.5	H	Sand	C	Dynamic
	N-II	4.5×10^{10}	0.5	30.5	H	Sand	S,C	Alternating
	N-IV	4.5×10^{10}	0.5	30.5	H	Sand	C	Alternating
P.H.T.R.I.	KAWASAKI	8.8×10^{10}	9.0	50.0	Pipe	Silt	C	Static Alternating Dynamic
	MODEL	1.2×10^{7}	0.4	40.0	Plate	Sand	S	Static Alternating
Y.&H.	BIWAKO 1	1.8×10^{12}	10.0	120.0	Pipe	Silty clay	S	Alternating Dynamic
	BIWAKO 2	3.5×10^{12}	10.0	150.0	Pipe	Silty clay	S	Alternating Dynamic
	KITAKAMI	3.4×10^{12}	1.0	150.0	Pipe	Silt	C	Alternating

T.T.R.I. : Transportation Technical Research Institute
P.H.T.R.I. : Port and Harbour Technical Research Institute
Y.& H. : Yawata Iron & Steel Co. Ltd. and Hazama Gumi Co. Ltd.

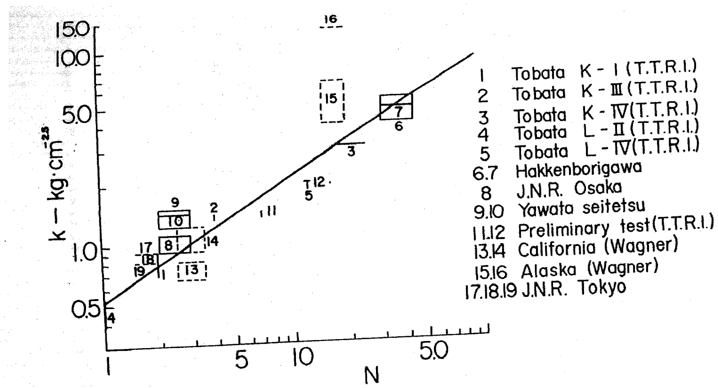

Fig. 1 · Relationship between N-value of the standard penetration test and the coefficient of horizontal subgrade reaction k.

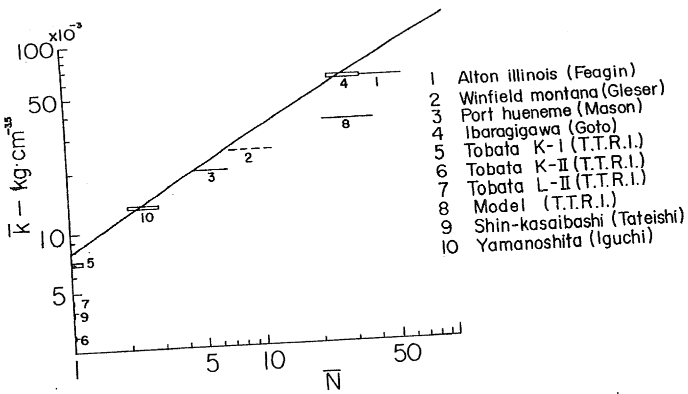

Fig. 2 Relationship between $\bar{N}$-value which is N-value at a meter depth from the soil surface and the coefficient of horizontal subgrade reaction $\bar{k}$.

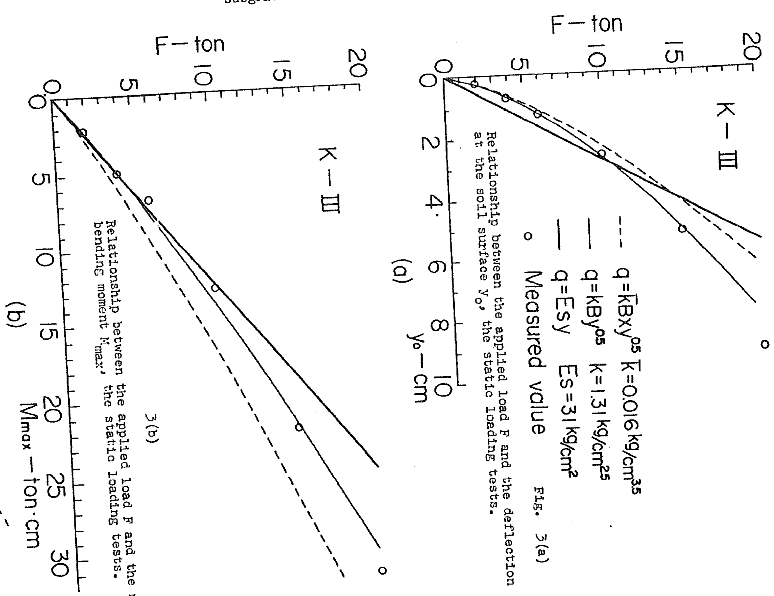

Fig. 3(a) Relationship between the applied load F and the deflection at the soil surface y_0, the static loading tests.

3(b) Relationship between the applied load F and the maximum bending moment M_{max}, the static loading tests.

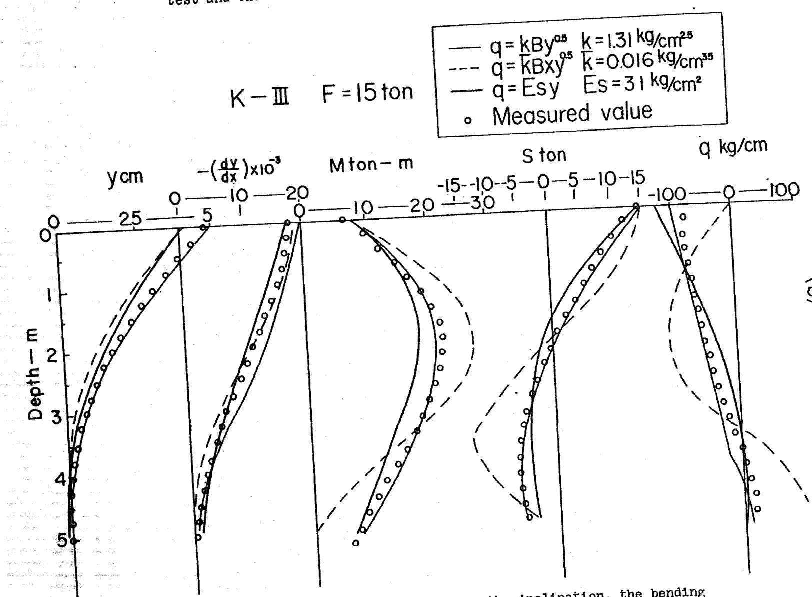

Fig. 4 Distribution of the deflection, the inclination, the bending moment, the shearing force and the subgrade reaction of the pile, the static loading tests.

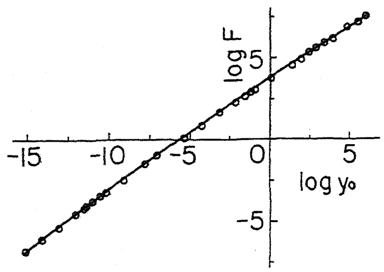

Fig. 5

Standard curve for F - y_o relationship, free head pile. (C-type soil)

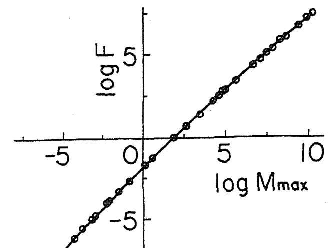

Fig. 7

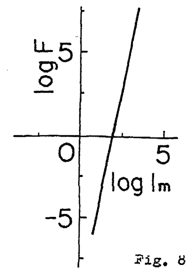

Fig. 8

Fig. 7 Standard curve for F - M_{max} relationship, free head pile. (C-type soil)

Fig. 8 Standard curve for F - l_m relationship, free head pile. (C-type soil)

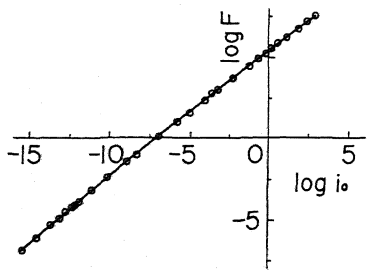

Fig. 6

Standard curve for F - i_o relationship, free head pile. (C-type soil)

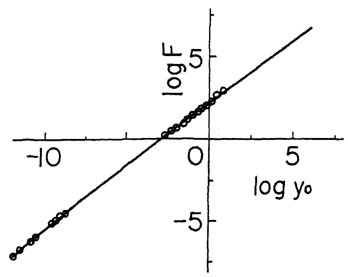

Fig. 9

Standard curve for F - y_o relationship, free head pile. (S-type soil)

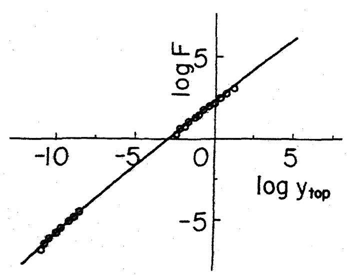

Fig. 10

Standard curve for F - y_{top} relationship, free head pile (S-type soil)

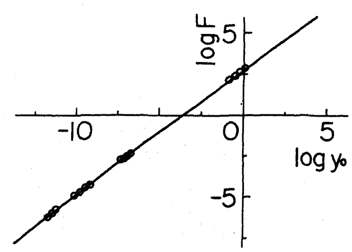

Fig. 13

Standard curve for F - y_o relationship, fixed head pile. (S-type soil)

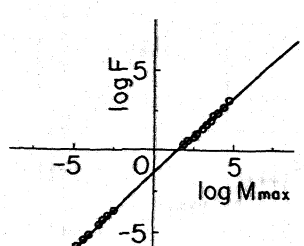

Fig. 11

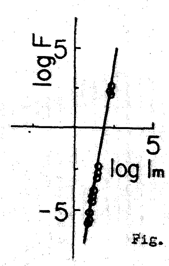

Fig. 12

Fig. 11 Standard curve for F - M_{max} relationship, free head pile. (S-type soil)

Fig. 12 Standard curve for F - l_m relationship, free head pile. (S-type soil)

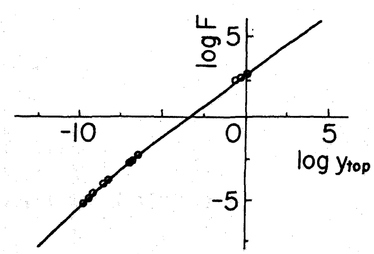

Fig. 14

Standard curve for F - y_{top} relationship, fixed head pile. (S-type soil)

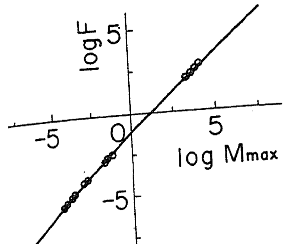

Fig 15

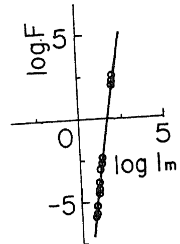

Fig. 16

Fig. 15 Standard curve for F - M_{max} relationship, fixed head pile. (S-type soil)

Fig. 16 Standard curve for F - l_m relationship, fixed head pile. (S-type soil)

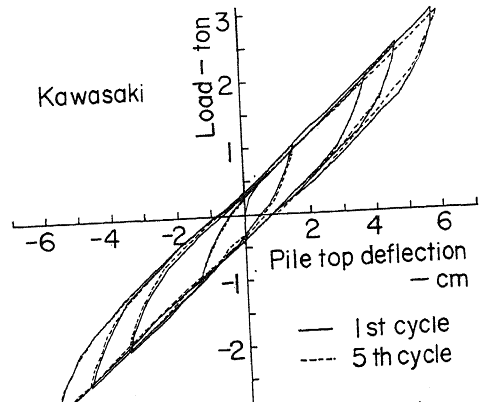

Fig. 17 1/2

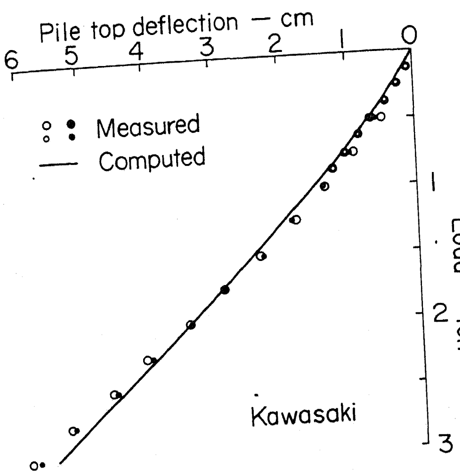

Fig. 18 1/2

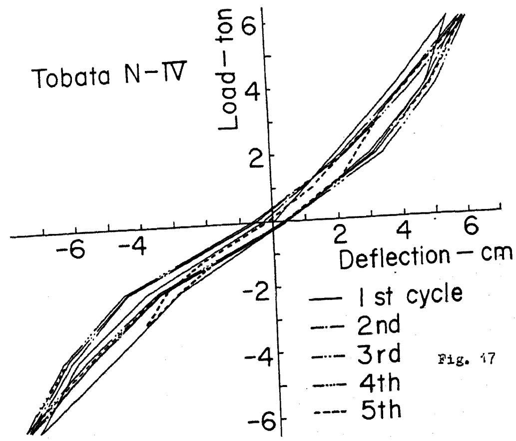

Fig. 17 2/2

Fig. 17 F - y_{top} relationship, the alternating loading tests.

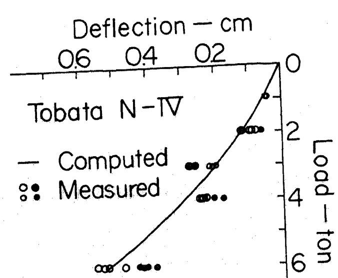

Fig. 18 2/2

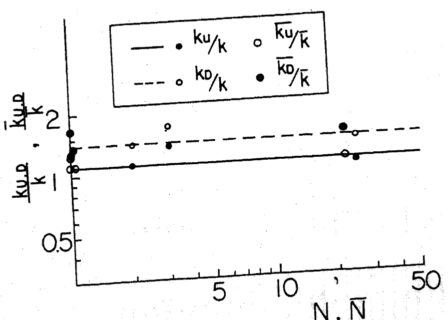

Fig. 18 "Unloading Curve", the alternating loading tests.

Fig. 19 Relationshp between N - or $\bar{N}$-value and k_u/k and k_d/k or $\bar{k}_u/\bar{k}$ and $\bar{k}_d/\bar{k}$ respectively.

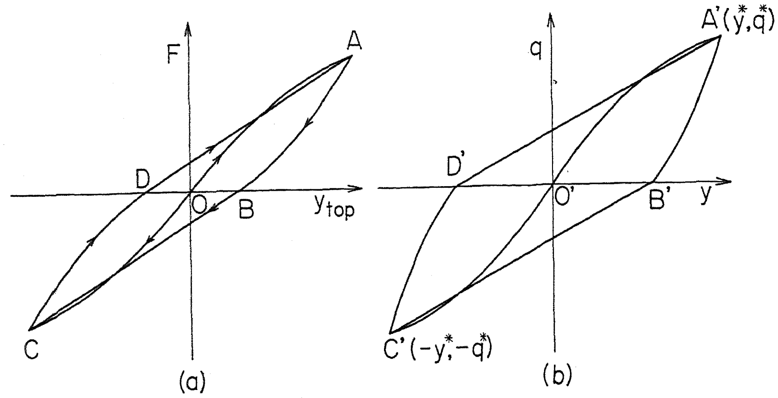

Fig. 20(a)

Illustration of a set of a loop and a curve ("the initial curve") $F - y_{top}$ relation.

(b) Illustration of a set of a loop and a curve, q - y relation.

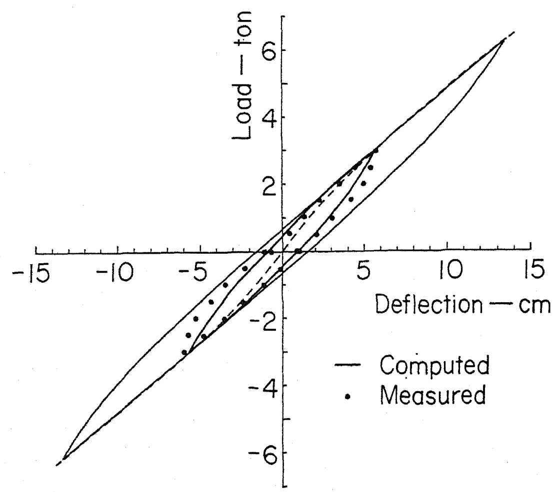

Fig. 21 Comparison of $F - y_{top}$ **relationship**, measured and computed.

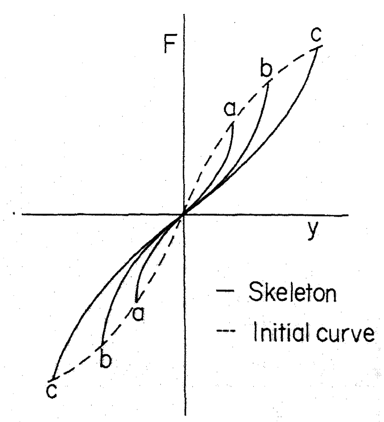

Fig. 22 Illustration of "**Skeletons**"

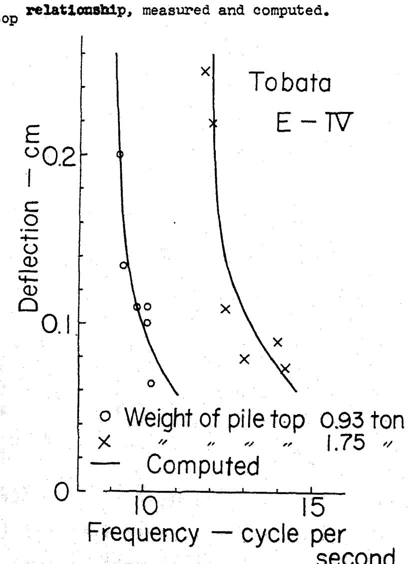

Fig. 23

Relationship between the deflection and the natural frequency, the free vibration tests.

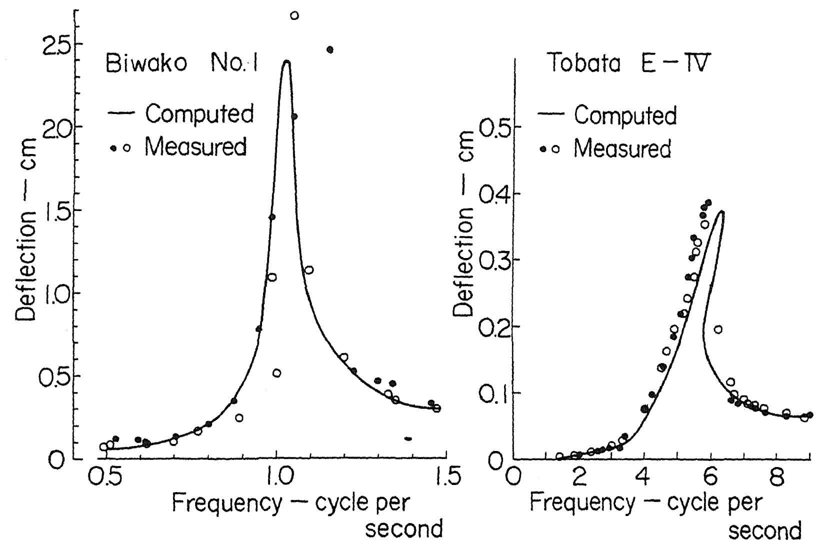

Fig 24 Resonance curve, the forced vibration tests.

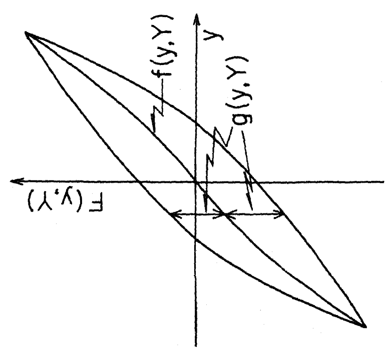

Fig. 26 Illustration of resolving a hysteresis loop into two elements.

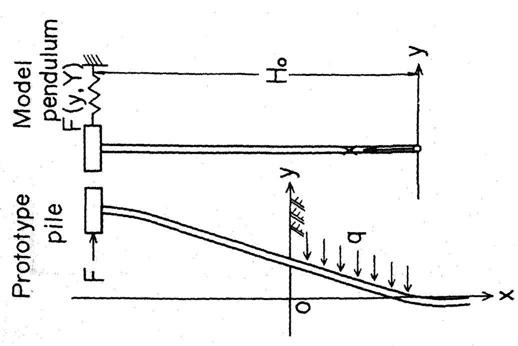

Fig. 25 Model pendulum.

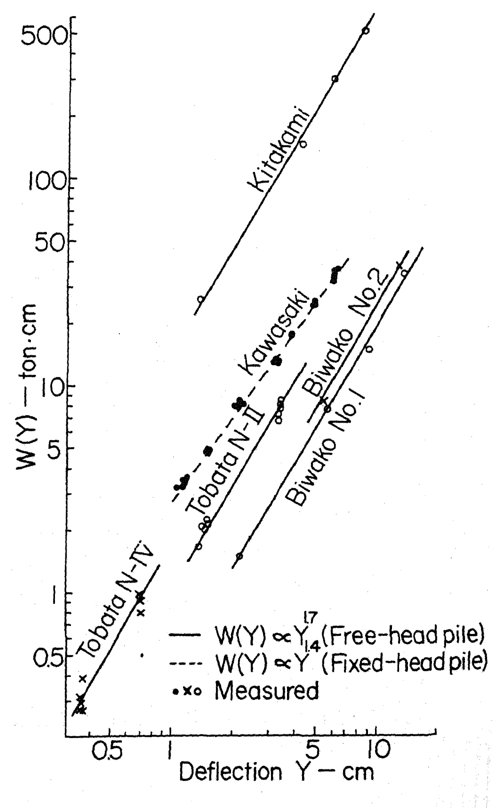

Fig. 27

Relationship between W(Y) and Y, the alternating loading tests.

ERRATA

"Horizontal Resistance of Steel Piles under Static and Dynamic Loads"

by Satoshi Hayashi, Nobuo Miyajima and Ikuhiko Yamashita

Page	Line	Error	Correction
146	footnote	Goverment	Government
147	23	(5)	(3)
"	30	(7)	(6)
"	32	(5) and (6)	(5)
"	bottom	from	From
148	16	N-value	$\bar{N}$-value
"	17	N-value is N-value	$\bar{N}$-value is N-value
"	22	E-value	E_s-value
"	27	methos	method
149	27	equation (7)	equations (7)
150	41	prodedure	procedure
"	41	brachets	brackets
"	bottom	length	depth
151	5	prreceding	preceding
152	43, 8)	A and C	A and D
155	26	Klother's	Klotter's
156	33	load	loading
"	38	guantities	quantities
162	Fig. 23	0 - 0.93 ton; X-1.75 ton	0-1.75 ton; X-0.93 ton

HORIZONTAL RESISTANCE OF STEEL PILES UNDER STATIC AND DYNAMIC LOADS

BY S. HAYASHI, N. MIYAJIMA AND I. YAMASHITA

QUESTION BY: P.W. TAYLOR - NEW ZEALAND

There is an interesting comparison between the load-deflection curves for piles, given in the paper (Fig.17) and the hysteresis diagrams for clay samples (Figs.11 & 12) in the paper presented earlier by Mr. Hughes and myself. Both show the "hardening spring" type of non-linearity. Presumably that observed in the pile tests is caused by this property of the soil.

The alternating load tests on piles described in the paper were, no doubt, conducted over an appreciable interval of time. Have the authors considered the possibility that, at earthquake frequencies, a gap may open, between the soil, and the sides of the pile?

REPLY BY: S. HAYASHI

As was pointed out by Mr. Taylor in his question, the load-deflection relation obtained from a stage of alternating loading test at a certain load level, shows characteristics of a hardening spring system. (See Fig.22 a,b,c). Since the stresses in the steel piles were always below the elastic limit throughout the tests carried out, it is undoubtedly concluded that this characteristic depends mainly on nature of soil, as Mr. Taylor and Mr. Hughes reported. Moreover, this characteristic is more evident in clay and in moist sand.

It is interesting to know that the load-deflection relation of a pile under horizontal loading and the stress-strain relation of the clay sample have another similar character as below. Mr. Taylor and Mr. Hughes stated in their paper "....the measured value of elastic modulus is markedly dependent on strain amplitude, being greatest at small amplitude". If the phrases "elastic modulus" and "strain" in their statement are replaced by "slope of load-deflection relation" and "deflection" respectively, their statement is also valid for the load-deflection relation of the pile under horizontal loading. In other words, the load-deflection relation of the pile under horizontal loading and of the stress-strain relation of the clay sample, show non-linearity of a softening spring system, as the load level (or stress level) increases. (See the dotted line connecting a, b, and c in Fig. 22).

The second point of the question was on the gap between the soil and the pile. In the tests of piles driven into moist sand, the gap between the soil and the pile was seen during the alternating loading tests and also during the dynamic loading tests with a mechanical vibration exciter. However, for piles driven into sea bottom, it was not clear whether the gap had appeared or not, during the tests, because of difficulties in the observation.

The authors understood that the point of the second question was that under some loading condition such as earthquake loading, the gap may appear even though the gap was not seen during the alternating loading tests, and that the questioner had asked the authors' opinion about differences between phenomenon in the alternating loading tests and those in earthquake loading.

Of course, the tests carried out by the authors do not cover all the situations imaginable. Therefore, the situation pointed out by the questioner may exist and even the reversed situation, namely, the gap appears in the alternating loading tests but not in the earthquake loading, may occur.

In addition to the problem of the gap, there are other points on which the same phenomenon may not occur between the alternating loading tests and the dynamic loading tests or earthquake loading. They are, for instance, additive mass during vibration, coefficient of dynamic subgrade reaction or damping force proportional to loading speed. However, it is not the main purpose of this paper, to discuss in detail the differences between the phenomena.

The procedure to evaluate the dynamic behaviour of a soil-pile system, proposed by the authors, is based on the fact that, in many tests, there was good practical agreement between the dynamic behaviour of the soil-pile system in the dynamic loading tests and the computed behaviour considering the soil-pile system as a pendulum system whose restitutive force characteristic was the same as the load-deflection relation obtained in the alternating loading tests.

A more rationalized method for evaluating dynamic behaviour of a soil-pile system, based on the actual behaviour of the system, is very desirable. The studies to improve the proposed procedure in this direction will be continued, and the authors hope that other engineers or organizations will make many studies in this field.