ANALYSIS OF THE EARTHQUAKE RESISTANCE OF FRAME BUILDINGS TAKING INTO CONSIDERATION THE CARRYING CAPACITY OF THE FILLING MASONRY

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SUMMARY

A method is elaborated for the analysis of the earthquake resistance of multistorey buildings, taking into consideration the carrying capacity of the filling masonry.

The results are given of certain theoretical and experimental investigations of a section of a building consisting of brick masonry and a reinforced concrete frame. The carrying and deformative capacities of the masonry subjected to horizontal forces are determined. Formulae are given for the stiffness of the masonry and the distribution of the horizontal force between the frame and the masonry. The tables and pictures illustrate some of the results of the tests of walls with smaller dimensions, of full-size elements, as well as walls from old frame buildings. The results of the investigations may also be used for the earthquake analysis of frame buildings with other kinds of filling masonry or light concrete.

Frame buildings consist of a reinforced concrete frame, which takes over the vertical loading and of filling masonry which serves as isolating partition. When they are subjected to horizontal forces the frame and the masonry work jointly. This common action is determined by the small deformative capacity of the masonry. On account of this the masonry may be used for taking over the greater part of the horizontal forces, provided the dimensioning is correct.

Some of the results of the investigations on the joint work of the framework and the masonry subjected to the action of horizontal forces are presented in this report.

We assume that the size of the earthquake forces is determined according to the existing standards.

The analysis of the space frame and the masonry under the action of these forces is a difficult task; on account of this some authors consider buildings and other structures as consisting of separate diaphragms (fig.1). It is necessary that the whole horizontal force be distributed among the separate diaphragms, before the analysis of each diaphragm.

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Several methods have been worked out for the distribution of the horizontal loading among the diaphragms. A brief and simple method is that of Streletzki /1/.

This method is particularly suitable for monolithic frame buildings where the floor slabs have a high stiffness and during their horizontal displacement act as undeformable disks and distribute the horizontal loading in proportion to the stiffness of the walls beneath. The term stiffness of walls and frames denotes the force causing a unit displacement in the respective element.

If the resultant of the external forces R is symmetrical in respect to the cross section of the building (i.e. passes through the center of gravity) and if the discs are symmetrical and of equal stiffness, then the floor slab will undergo a translatory displacement and the force in any diaphragm will be determined by /1/

$$H_k' = R_H \frac{K_k}{\Sigma K_i} \qquad /1/$$

where K_k is the stiffness of the system of discs along a certain axis.

In most cases, however, the diaphragms are not symmetrically situated and have different stiffness; then the floor slab will not only undergo a translatory displacement but will also revolve around its center of rotation, which will bring about a redistribution of the forces.

In order to determine the rotation centre of the floor slab, the stiffness of the individual discs have to be defined

K_i^x - for diaphragms parallel to the x axis
K_i^y - for diaphragms parallel to the y axis

Then the coordinates of the rotation center are defined with

$$x_c = \frac{\Sigma K_i^y x_i}{\Sigma K_i^y} \qquad y_c = \frac{\Sigma K_i^x y_i}{\Sigma K_i^x} \qquad /2/$$

The coordinates of the rotation center may also be determined graphically by means of a polygon of forces and a funicular polygon, taking the stiffness of the different diaphragms as forces. The intersection of the resultants of the stiffnesses in the direction of the two axes of the structure defines the rotation center.

Expressing every stiffness through the smallest of them

$$K_n = d_n K_1 \qquad d_n = \frac{K_n}{K_1} \qquad /3/$$

the additional effort due to the rotation is

$$H_k'' = \frac{R_H \cdot \ell \cdot d_k a_k}{\Sigma d_i^2 a_i^2} \qquad /4/$$

where ℓ = excentricity of R with respect to the rotation center

a_k = distance between the respective diaphragm and the rotation center.

Then the entire force corresponding to a certain disc, due to translatory displacement and rotation, is:

$$H_k = H_k' + H_k'' \qquad /5/$$

After the force is distributed among the separate frames with masonry (diaphragms) in proportion to their stiffness, the following fundamental problems should be resolved: 1) what proportions of the force are taken over by the masonry and by the frame; 2) how to determine the stiffness of the masonry and of the frame; 3) how to calculate the masonry so that it can take over the corresponding force. These main questions, related to certain secondary ones, are solved through theoretical, photoelastic, model and full scale investigations. The latter have been carried out simultaneously and the results compared, supplemented and specified. The theoretical investigations and the full size tests of elements like the one shown on fig.1 have encountered considerable difficulties. On account of this the investigations have been carried out with elements consisting of one masonry cell surrounded by a reinforced concrete frame (fig.3). The results of the photoelastic tests determine the examination of the model of fig.3 instead of that of fig.1[1/].

I. THEORETICAL INVESTIGATIONS

The structure on fig.3 consists of two heterogeneous materials - masonry and frame; because of that the stresses arising from the horizontal force H are considered separately for the masonry and the frame, taking into account the joint work of the two elements. For this purpose the stresses arising from the interaction between the frame and the masonry have been examined.

This problem of interaction (contact problem) has been solved with the aid of the method of redundant reactions in the following manner[2/]:

1. The bond between masonry and reinforced concrete frame is substituted by joints, which take over the and τ efforts (fig.3).

1/ The results of the photoelastic tests are not presented here, since the alloted space is not sufficient for the photographs which should be appended.
2/ Experiments have been carried out to define the stresses in the masonry and the contact zone with the aid of other methods: with the method of the finite differences with the extension of the stress function over the whole disc; with a polynome of n-power in the contact zone; with orthogonal cross sections and the introduction of unknown efforts and so on.

2. The joints are cut and their action is substituted by the action of the redundant reactions - 30 in all.

3. A system of equations with 30 unknown quantities is worked out, which expresses the condition, that at the place of the cross section the sum of the mutual displacements is equal to zero.

4. The coefficients in front of the unknown quantities represent the sum of the individual displacements in the masonry and the frame.

The displacements in the masonry due to the unit loadings are defined through the integration of the stresses in the masonry, which is considered as a wall beam. The stresses are calculated after the method of the finite differences (network method). In the most common case of unit loading the function of the stresses is determined by solving a system of equations with 15 unknown quantities. In order to simplify the computation the unit loadings are broken up into symmetrical and inversely symmetrical. The deformed state of the masonry is determined with the aid of the Williott diagram.

The displacements thus determined in the wall beam and the frame are substituted in the system of equations with 30 unknown quantities, which is solved through one of the familiar methods. The unknown efforts which are thus found define the character of the stresses in the contact zone (fig.4).

II. DETERMINATION OF THE STRESSES IN THE MASONRY

In theoretical investigations the masonry is considered as an elastic element. This is done in order to use the methods of the theory of elasticity, on the one hand, and to give a solution, which may be used with filling masonry made up of different materials, on the other. The corrections taking into consideration the peculiarities of a given material are made after carrying out a series of tests for finding out to what extent the theoretical conclusions coincide with the experiments. If necessary the theoretical conclusions are corrected in accordance with the peculiarities of the material.

The solution of the problem of interaction (contact problem) indicates the way in which the horizontal force H is transmitted to the masonry.

After a series of examinations the conclusion is drawn that the law of the transfer of the loading from the frame to the masonry should be conceived as a stress function, which allows the direct determination of the stresses in the masonry.

Such a stress function has the form

$$\varphi = \varphi_1 + \varphi_2 + \varphi_3 + \varphi_4$$

where

$$\varphi_1 = \sigma_K\left[\frac{xy}{6ab}(y^2\beta^2+x^2) - \frac{1}{2}xy\beta + (x^2+y^2\beta^2)\right]$$

$$\varphi_2 = 0.5\sigma_K\left[\left(\frac{2a}{\pi}\right)^2\cos\frac{\pi}{2}\xi - \frac{0.345(x^2-a^2)(y^2-b^2)^2}{a^2(3.25a^4+3a^2b^2+5.25b^4)}\right]$$

$$\varphi_3 = 0.5\beta\sigma_K\left[\left(\frac{2b}{\pi}\right)^2\cos\frac{\pi}{2}\eta - \frac{0.345(x^2-a^2)^2(y^2-b^2)}{b^2(3.25a^4+3a^2b^2+3.25b^4)}\right]$$

$$\varphi_4 = K\sigma_K\left[\frac{xy}{6ab}(y^2\beta^2+x^2) - \frac{1}{2}xy\beta - \frac{1}{2}(x^2+\beta^2y^2)\right]$$

$$\beta = \frac{a}{b} \qquad \xi = \frac{x}{a} \qquad \eta = \frac{y}{b} \qquad W_1 = \frac{1.38(1+\beta^4)}{3.25+3\beta^2+3.25\beta^4}$$

φ_1 and φ_4 are polynoms, whereas φ_2 and φ_3 are defined after the variation method. The function φ gives the stresses along the contour of the wall beam which are similar to the ones obtained from the solution of the contact problem as well as the stresses in the wall beam (fig.4).

$$\sigma_y = \sigma_K\left\{\xi\eta(1+K) - 0.5\left[\cos\frac{\pi}{2}\xi + (3\xi^2-1)(\eta^2-1)^2W_1\right] + 1 - K\right\}$$

$$\sigma_x = \sigma_K\beta^2\left\{\xi\eta(1+K) - 0.5\left[\cos\frac{\pi}{2}\eta + (\xi^2-1)^2(3\eta^2-1)W_1\right] + 1 - K\right\} \qquad /7/$$

$$\tau_{xy} = -\sigma_K\beta\left\{\left(\frac{\xi^2+\eta^2}{2} - \frac{1}{2}\right)(1+K) - 2\xi\eta(\xi^2-1)(\eta^2-1)W_1\right\}$$

The contour stress σ_K is defined by the condition of equilibrium of external and internal (contact) forces.

$$\sigma_K = \frac{N}{1.46ad\beta} \qquad /8/$$

III. DETERMINATION OF THE CARRYING CAPACITY OF THE MASONRY

Taking into consideration that during the tests the first fissure (i.e. the ultimate state) appears in the middle of the masonry in the direction of the compression diagonal, we come to the conclusion that destruction is due to the principal tensile stresses.

$$\sigma_p = \frac{\sigma_x+\sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2} \qquad /9/$$

Therefore the ultimate state on the basis of which the carrying capacity of the masonry has to be determined is

$$\sigma_p \leq R_o \qquad /10/$$

for the middle of the wall beam.

By putting x = 0, y = 0 in (7) and substituting in (9) and (10) the results obtained for the stresses, and expressing σ_K through (8), we obtain the force which causes the first diagonal fissure in the masonry

$$N_m = \frac{3ld(\beta-0.5\beta^2)}{F}R_o \qquad /11/$$

valid for $\beta < 2.5$

where $F = 1 + \beta^2 - \sqrt{\beta^4 - 0.8\beta^2 + 1}$

l and d length and thickness of the masonry

The use of this method for the determination of the carrying capacity of a masonry with openings encounters considerable mathematical difficulties and the expressions obtained are rather long and unfit for practical use. On account of this the carrying capacity of a masonry with openings is determined with the aid of that without openings by introducing a "factor of the opening" k.

$$N_{m.o} = K_1 \frac{3ld(\beta - 0.3\beta^2)}{F} R_0 \qquad /12/ \qquad K_1 = \left(1 - \frac{2a_1}{3l} - \frac{3}{5}\frac{b_1}{h}\right)$$

where l = length of the masonry, a_1 length of the opening

d = thickness,

R_0 = tensile strength.

When there is no opening (a_1=o, b_1=o) from (12) we obtain (11). When there is no masonry (a_1=l, b_1=h) we obtain N=0.

IV. DETERMINATION OF THE DEFORMATIONS OF THE MASONRY

On the basis of the stresses obtained (7) for the coordinate system xy (fig.4) the stresses for x^1y^1 are determined as well.

$$\sigma_y' = \frac{a^2}{a^2+b^2}\sigma_x + \frac{b^2}{a^2+b^2}\sigma_y + \frac{2ab}{a^2+b^2}\tau_{xy} \qquad /13/$$

The specific shortening of the compression diagonal is expressed with

$$\varepsilon_s = \frac{2}{l}\int_0^{l/2} \frac{\sigma_y'^2}{\gamma E} dy' \qquad /14/$$

It is assumed that the stresses from x=0, y=0 to x=a, y=b change lineary

$$\sigma_y = \sigma_{y_0} + 2\frac{\sigma_{ya b} - \sigma_{y0}}{l} y \qquad /15/$$

After determining σ_y from (15) and (13), and from (8) and substituting them in (14) we obtain

$$\varepsilon_s = \frac{0.2 h^2 N^2 c}{E_m d^2 (h^2 + l^2)^2} \qquad /16/ \text{ where } c = 5.28\left(\beta^4 + \frac{1}{\beta^4}\right) - 7.66\left(\beta^2 + \frac{1}{\beta^2}\right) + 11.76$$

The "factor of the opening" k_1 is introduced for masonry with openings.

$$\varepsilon_{s.o.} = K_1 \frac{0.2 h^2 N^2 c}{E_m d^2 (h^2 + l^2)^2} \qquad /16^a/$$

V. DETERMINATION OF THE STIFFNESS OF THE MASONRY AND THE DISC

The horizontal displacement in the masonry is determined on the basis of the specific deformation in the compression diagonal (fig.5).

$$\Delta h = \frac{h^2 c N^2}{l(l^2 + h^2) d^2 \gamma E_m} \qquad /17/$$

Proceeding from the definition of the stiffness of the masonry (the force causing a unit displacement) we obtain

$$K_m = \frac{2 \cdot 2d}{h}\sqrt{\frac{2(d^2+h^2)}{c}\gamma E_m} = d\sqrt{\frac{BE_m}{c}} \qquad /18/$$

$$K_{m0} = K_1 d\sqrt{\frac{BE_m}{c}} \qquad /18^a/$$

The stiffness of the reinforced concrete frame in a given floor can be determined through (19), deduced on the condition, that the beams of the frame together with the slabs have an infinitely great moment of inertia

$$K_f = \frac{12E_c}{h^3}\Sigma J_k \qquad /19/$$

Then the stiffness of the system of diaphragms in a given floor is

$$K_d = K_f + \Sigma K_m \qquad /20/$$

VI. DISTRIBUTION OF THE FORCE BETWEEN THE FRAME AND THE MASONRY

The whole force which the disc has to take up is to be distributed between the frame and the masonry according to their stiffness. On the basis of the condition, that the deformations of the masonry and those of the frame are equal an expression is obtained which determines the part of the force taken over by the masonry (21)

$$N_m = \sqrt{A^2 - 2AH} - A \qquad /21/ \qquad A = \frac{K_m^2}{2K_f}$$

There remains for the frame

$$N_f = H - N_m$$

With the aid of the above formulae the analysis of the earthquake resistance of a frame building can be carried out in the following sequence: 1) The stiffnesses of the discs situated along the respective axes of the building are determined by means of (20), (19), (18) and (18°); 2) The rotation center of the building is determined through (2); 3) The horizontal force is distributed among the discs in a given floor by means of (1) or (5); 4) The force, which corresponds to a system of discs is distributed between the frame and the masonry by means of (21); 5) Through (11) it is verified whether the masonry is able to take over its share of the load; 6) The reinforced concrete frame is dimensioned with the familiar methods after determining the size of the forces in the remaining floors as well.

In order to solve these problems 26 diaphragms of different characteristics and dimensions were tested (table 1). The tests were carried out with a static horizontal force (as on fig.3).

The following more essential conclusions were drawn from the testing of the discs:

1. When loading the disc with a horizontal force one of the diagonals is subjected to compression and the other to tension. As a result of the tension the first fissure in the masonry without opening appears in the middle of the disc in the direction of the compression diagonal (fig. 6). The increase of the loading leads to an increase of the fissures or to the appearance of other fissures parallel to the first one. The further increase of the loading leads to the occurrence of fissures in the concrete frame as well - first on the internal and then on the external side of the reinforced concrete frame at a distance of 1/5 to 1/2 l or h measured from the non-compression corner. The sequence of the occurrence of fissures in the discs with opening is seen on fig.7.

2. The deformations which give rise to diagonal fissures are almost identical for masonries made with mortars of different strength.

3. The deformation of the masonry is small until the appearance of the diagonal fissure, on account of which the reinforced concrete frame takes up a smaller part of the horizontal force (5 - 20%).

4. The nature of the deformations in the contact zone corresponds to the theoretically defined law.

5. The carrying capacities of the discs turned out to be 20-30% higher than the ones calculated according to (11).

6. The stiffness of the masonry depends on the type of bricks as well as on the composition and strength of the mortar.

7. The openings of the masonry decrease substantially its carrying capacity (2-5 times and more).

VII. FULL SCALE INVESTIGATIONS

The full scale investigations were designed to check the results of the theoretical and model analysis of full-size discs.

The first series of full scale tests were carried out with the walls of an old five-storey frame building with a statical horizontal force. Three 3,5 x 3,0 m walls (fig.8) were tested, two of which 30 cm thick and the third - 15 cm.

The second series of full scale tests were made on brick walls with usual dimensions (table 2, fig.9).

The third series of tests were performed with discs of brick masonry with openings (fig.10).

The fourth series of tests were carried out with discs of lightweight concrete filling with or without openings (fig.11).

Some of the results of the tests are presented in table 2. The following inferences are made from the full-size tests:

1. The nature of the deformations and the fissures is the same as in the model investigations.

2. The carrying capacity determined by way of full-size tests is close to the one determined by calculation with (11).

3. The specific carrying capacity (for a unit cross section) of the full-size masonry is lower than the specific carrying capacity of the masonry obtained through model investigations. This fact can be explained by the different scale modulus of the elements tested in both cases. Therefrom the conclusion can be drawn, that the model investigations cannot be used for full-size elements without taking into consideration the scale modulus.

CONCLUSION

The results of some of our investigations represent an attempt to give a more exact solution of a problem notable for its complexity.

The theoretical investigations were carried out without taking account of the kind of filling mateiral. This makes it possible to use the results for every kind of material, provided certain corrections taking account of the properties of the material are made. This is also confirmed by the fact that the discs[1/] tested with a filling of monolithic slag concrete do not differ substantially from the ones with brick masonry. It is only necessary to introduce a correction factor. In this way the expressions obtained may also be used for the analysis of the earthquake resistance of certain types of prefabricated houses.

1/ The results of these investigations are not given here.

BIBLIOGRAPHY

1. Варвак П.М. Развитие и приложение метода сеток к разчету пластикок, Киев 1949 (Varvak P.M. Development and application of the method of finite differences /network method/ by the analysis of plates, Kiev 1949)

2. Върбанов Хр.П. Теория на еластичността, София 1954 (Varbanov Chr.P. Theory of elasticity, Sofia 1954)

3. Колманок А.С. Пространственная работа сборных многаэтажных зданий, Москва 1954 (Kolmanok A.S. Space work of prefabricated multistorey buildings, Moscow 1954)

4. Протукевич Я.А. Вариационные методы строительной механике, Москва 1948 (Protukevich J.A. Vatiation methods in structural mechanics, Moscow 1948)

5. Сачански С.В. Иследване на монолитни сгради за хоризонтални сили чрез използване носещата способност на зидарията, "Строителство", София 1960 (Sachanski S.V. Analysis of the earthquake resistance of monolithic buildings under horizontal loading taking into condideration the carrying capacity of the masonry, "Stroitelstvo", Sofia 1960)

6. Стрелецки. Стальные конструкции, Москва 1952 (Streletzki. Steel structures, Moscow 1952)

7. Уманский А.А. Расчет пространственные конструкции (Umanski A.A. Analysis of space structures)

8. Фрохт М.М. Фотоупругость (Frocht M.M. Photoelasticity)

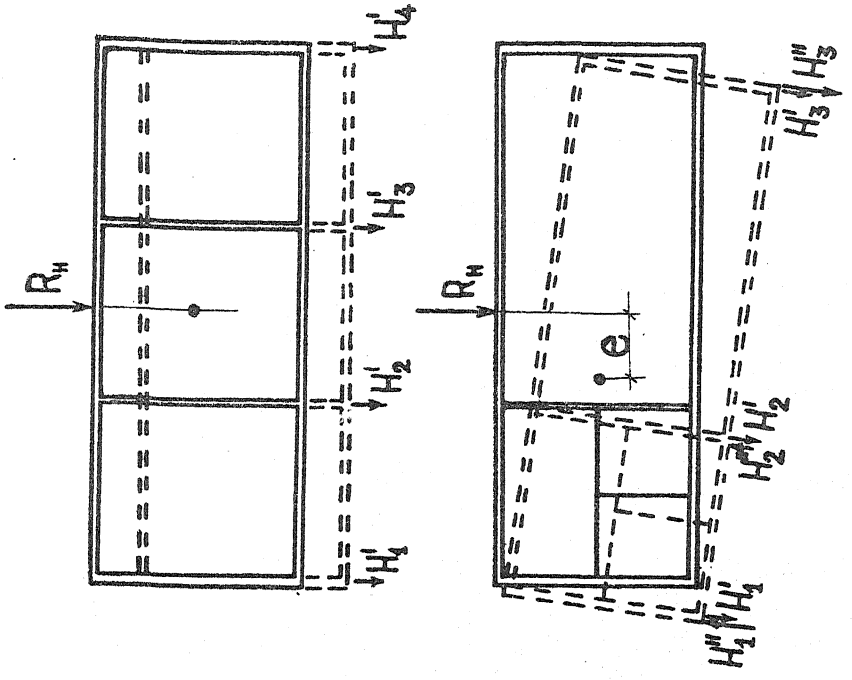

Fig.2
Displacement of the frame building under the action of horizontal forces; a - by symmetrical stiffness; b - by asymmetrical stiffness.

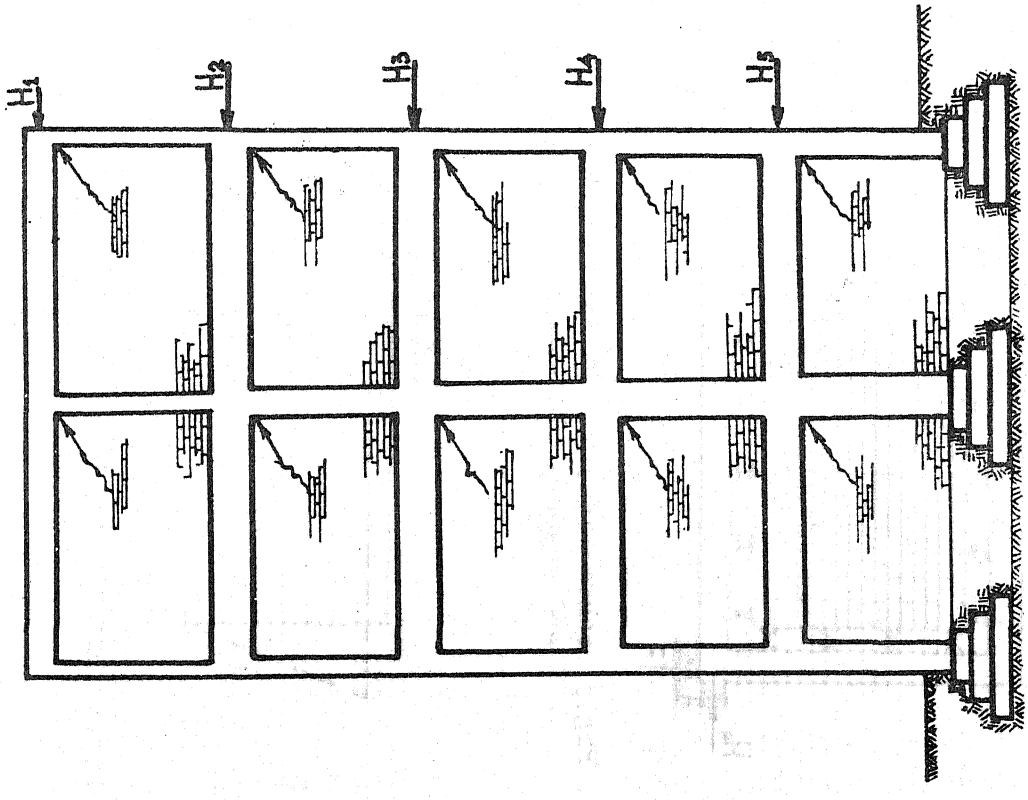

Fig.1
Transverse diaphragm in the frame building

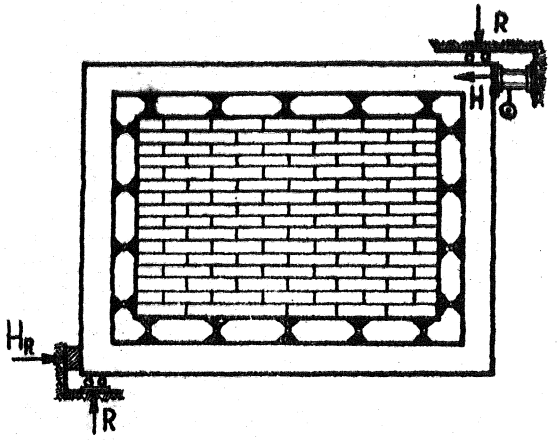

Fig.3 Schematic joint between frame and masonry. Pattern of loading

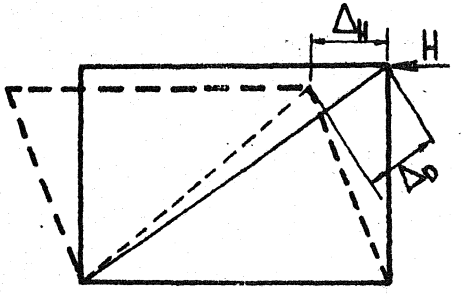

Fig.5 Diagram for the determination of the horizontal displacement

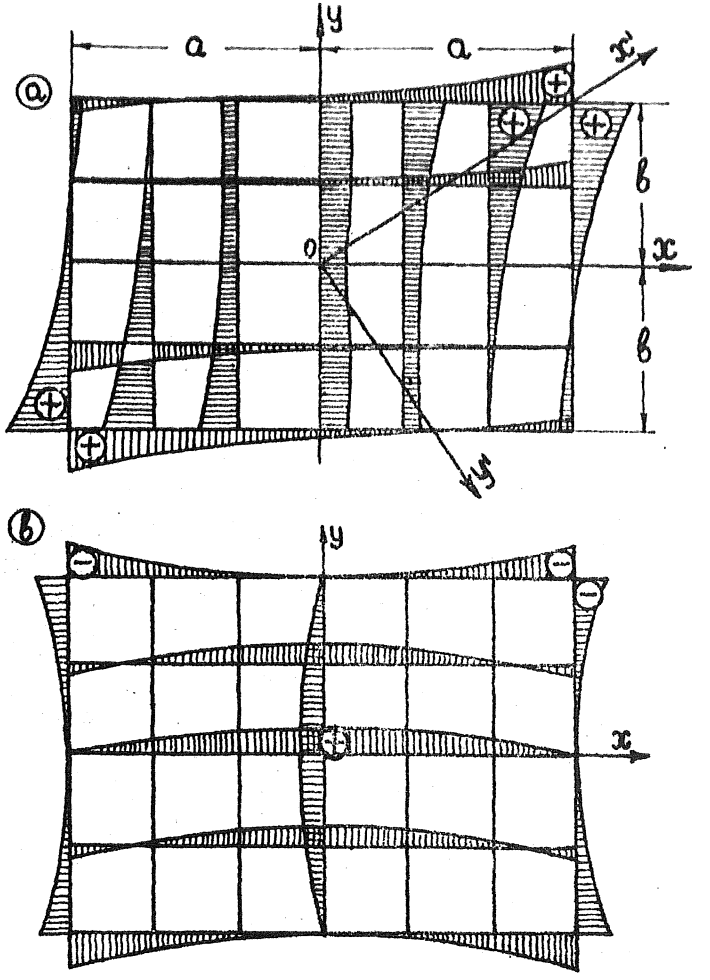

Fig.4
Normal and tangential stresses in the masonry

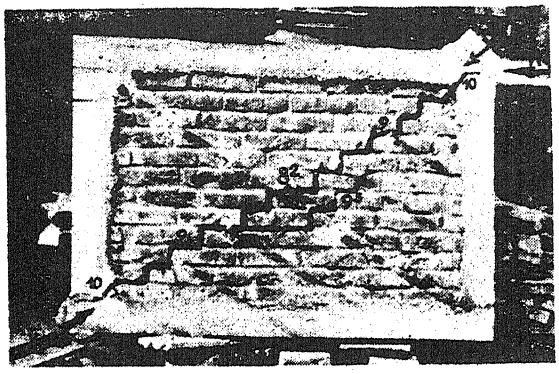

Fig.6 Diagonal fissure in one of the model discs

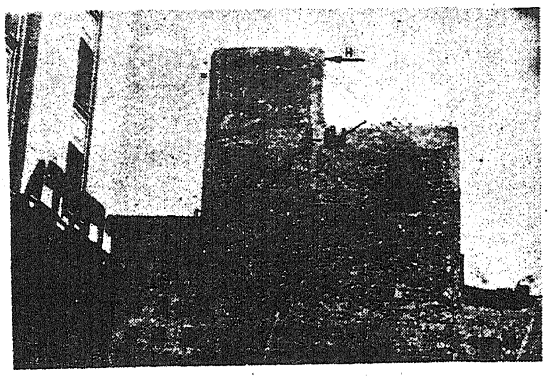

Fig.7 Testing a disc of a frame building; 1 - disc No.1 after the test; 2 - disc No.2 before the test

Fig.8 Diagonal fissure of a disc from a frame building

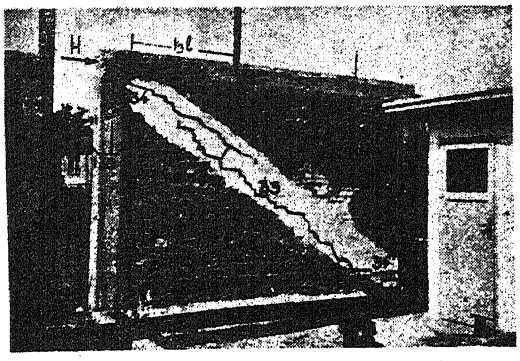

Fig.9 Diagonal fissure of a full-size disc

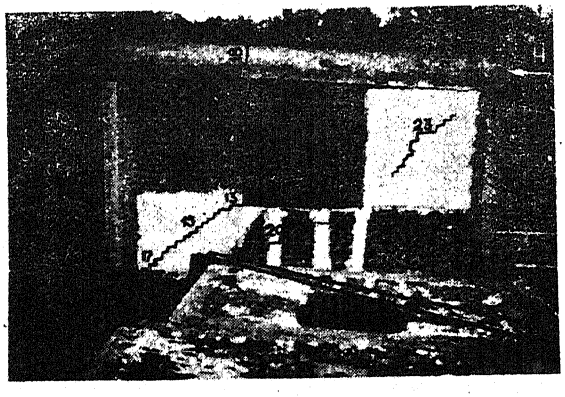

Fig.10 Fissures of a disc with an opening under the action of horizontal forces

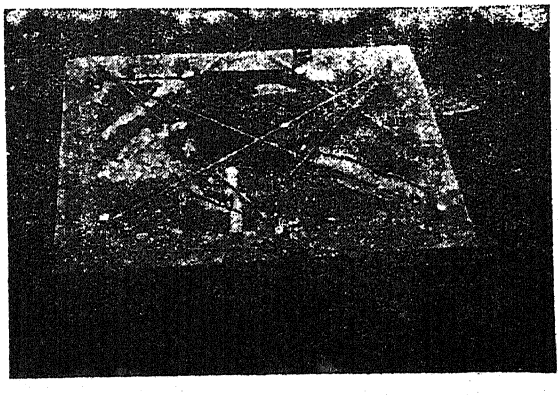

Fig.11 Fissures of a disc of lightweight monolithic concrete

Table 1

GEOMETRIC DIMENTIONS AND CHARACTERISTICS
OF THE TESTED MODEL BRICK DISCS

No	Dimensions of masonry		Dimensions of opening		Cross section of the frame		Mortar		Test		Calculated capacity
	l cm	h cm	a cm	b cm	b_1 cm	h_1 cm	compression kg/cm^2	tension kg/cm^2	First fissure t.	Collapse t.	t.
1	160	108	-	-	12	12	30	2	12.2	13.5	8.2
2	-	-	160	108	12	12	-	-	1.1	1.2	-
3	160	108	-	-	12	12	20	1.47	9.5	12.2	6
4	160	108	-	-	12	12	10	1.2	7	8.2	5
5	160	108	-	-	12	12	6	0.97	4.5	6.5	4
6	-	-	160	108	12	12	-	-	1.4	1.6	-
7	160	108	-	-	12	12	20	1.4	5.5	7.5	5.8
8	160	108	-	-	12	25	25	1.6	9.5	13.3	6.6
9	-	-	-	-	12	20	-	-	1.95	2.6	-
10	190	120	70	75	12	20	28	1.7	4.30	5.3	3.8
11	190	120	35	75	12	20	25	1.6	4.80	5.2	4.14
12	190	120	95	75	12	20	25	1.6	3.60	5.1	3.2
13	190	120	50	50	12	20	25	1.5	4.70	5.7	5.3
14	190	120	35	100	12	20	20	1.2	1.35	2.8	2.2

Table 2

GEOMETRIC DIMENSIONS AND CHARACTERISTICS OF TESTED FULL-SIZE DISCS[1/]

No	Dimensions of masonry		Dimensions of opening		Cross section of the frame		Mortar		Test		Calculated capacity
	l cm	h cm	a cm	b cm	b_1 cm	h_1 cm	compression kg/cm^2	tension kg/cm^2	First fissure t.	Collapse t.	t.
1	350	280	-	-	15	60	-	0.65	23.5	2.5	15.4
2	350	280	-	-	15	60	-	0.65	21.4	23.2	15.1
3	350	280	-	-	15	60	-	0.65	7.0	8.6	7.6
4	405	265	-	-	25	15	40	1.30	39	41	38.3
5	395	255	-	-	25	25	35	1.47	31	32	30
6	385	245	-	-	25	40	38	1,50	33	33	30.6
7	395	255	-	-	25	25	15	1.12	26.6	27	22.9
8	395	255	-	-	25	25	7.3	0.71	14.8	15.2	14.5
9	296	191	-	-	20	20	35	1.47	25	26.4	22.5
10	200	127	-	-	13	13	-	1.53	10	12	7.5
11	284	255	-	-	25	25	32	1.70	23.2	24.3	26
12	174	255	-	-	25	25	30	1.65	17.2	18.8	22
13	128	255	-	-	25	25	33	1.65	14.7	15.8	21.7
14	380	240	140	150	25	40	30	1.4	14.8	15.7	13.8
15	380	240	140	150	25	40	30	1.4	12	13.2	13.8

[1/] The results are the average of three tests.