#### A CONSIDERATION ON EARTHQUAKEPROOF DESIGN METHOD OF

#### EARTH DAM

#### Isao Minami

In designing a dam, we must take into consideration the force of earthquakes working against the dam body, especially in Japan earth dams often damaged by earthquake. The theoretical analysis of elastic vibration of dam body by Dr. Matsumura (1), Dr. Hatanaka (2) and Minami (3) etc. is an example of a study on the stability of earth dams in earthquake. But it requires future research to get any practical result. But distribution of seismic coefficient of the dam body was approximately ascertained theoretically.

Even today we use usually the sliding circle to calculate slope stability of earth dam in earthquake, which was naturally based on the assumption that the earth dam is in plastic equilibrium. In this report too, we have studied the stability of the dam by using the sliding circle with little reference to its elasticity. And also we have tried to get the safest slope of earth dam in examining these theoretical results reference to some experiments.

We have discovered some interesting facts concerning the study of the relationship between stability of earth dam and its type in the case of an earthquake. Especially I have treated vibration character of center core earth dam.

Judging from this consideration, center core type earth dam has smaller safety against vibrations because the upstream body of earth dam is usually saturated with water and modulus of rigidity of upstream side soils is smaller than that of downstream side soil. For this reason the period of free vibration would be longer and amplitude of dam body would increase in earthquake.

In stability analysis the author proposed the formula of the most dangerous sliding circle, and by this theory, we can calculate coefficient of stability without the trial and error method, and his theory can be graphed in all kinds of Soil and dam heights.

1. Analytical Consideration of Center Core Type Earth Dam Vibration.

Analysis of vibration of earth dam had been studied by Dr. Matsumura (1), Dr. Hatanaka (2) and Minami (3) etc. on the assumption of shear vibration. But designing a dam, we should know that practical studies of earthquakeproof design method have not been done hitherto. The author proposes a practical

#### Note:

A Member of Japan Society of Civil Engineers.

Faculty of Agricultural Engineering in Kyoto University.

(Assistant Professor)

theory of earthquakeproof design formula of earth dam in this paper. Seismic coefficient in earthquake is calculated by amplitude of dam body. For this reason the analysis of earth dam vibration is important. Actually the vibration of earth dam in earthquake is unharmonic and unsteady. But the assumption of harmonic vibration of dam body is important for convenience of simpler handling of earth dam, can be understood with approximate satisfaction by harmonic vibration analysis.

 Two dimensional analysis of amplitude distribution of center core type earth dam.

I assume the species of vibration of earth dam is shearing type in general. The differential equation of shearing vibration is noted as follows. In Fig. 1 the external forces action to x direction against elementary portion ABCE are as follows: to AB plane

$$-S = -b_1 \left( G_1 \frac{\partial w}{\partial y} + \mu_1 \frac{\partial^2 w}{\partial y \partial t} \right) - b_2 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial^2 w}{\partial y \partial t} \right)$$

$$+ a_1 GD_2 = -b_1 \left( G_1 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial^2 w}{\partial y \partial t} \right)$$

$$+ a_2 GD_2 = -b_1 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial^2 w}{\partial y \partial t} \right)$$

$$+ a_3 GD_2 = -b_1 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial^2 w}{\partial y \partial t} \right)$$

$$+ a_4 GD_2 = -b_1 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial^2 w}{\partial y \partial t} \right)$$

$$+ a_4 GD_2 = -b_1 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial^2 w}{\partial y \partial t} \right)$$

$$+ a_4 GD_2 = -b_1 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial^2 w}{\partial y \partial t} \right)$$

$$+ a_4 GD_2 = -b_1 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial^2 w}{\partial y \partial t} \right)$$

$$+ a_4 GD_2 = -b_1 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial^2 w}{\partial y \partial t} \right)$$

$$+ a_4 GD_2 = -b_1 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial^2 w}{\partial y} \right)$$

$$+ a_4 GD_2 = -b_1 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial^2 w}{\partial y} \right)$$

$$+ a_4 GD_2 = -b_1 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial^2 w}{\partial y} \right)$$

$$+ a_4 GD_2 = -b_1 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial^2 w}{\partial y} \right)$$

$$+ a_4 GD_2 = -b_1 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial w}{\partial y} \right)$$

$$+ a_4 GD_2 = -b_1 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial w}{\partial y} \right)$$

$$+ a_4 GD_2 = -b_1 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial w}{\partial y} \right)$$

$$+ a_4 GD_2 = -b_1 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial w}{\partial y} \right)$$

$$+ a_4 GD_2 = -b_1 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial w}{\partial y} \right)$$

$$+ a_4 GD_2 = -b_1 \left( G_2 \frac{\partial w}{\partial y} + \mu_2 \frac{\partial w}{\partial y} \right)$$

$$S + \frac{ds}{dy} dy = b \left( G_1 \frac{\partial u}{\partial y} + \mu_1 \frac{\partial^2 u}{\partial y \partial t} \right) + \frac{\partial}{\partial y} \left( b G_1 \frac{\partial u}{\partial y} + b_1 \mu_1 \frac{\partial^2 u}{\partial y \partial t} \right) \\ + b_2 \left( G_2 \frac{\partial u}{\partial y} + \mu_2 \frac{\partial^2 u}{\partial y \partial t} \right) + \frac{\partial}{\partial y} \left( b_2 G_2 \frac{\partial u}{\partial y} + b_2 \mu_2 \frac{\partial^2 u}{\partial y \partial t} \right)$$
(2)

Because the summation of external forces is equal to inertia force, kinematic equation of ABCD is as follows:

$$(\underline{\beta}b_{1} + \beta b_{1}) \frac{\partial^{2}w}{\partial t^{2}} = \frac{\partial}{\partial y} (b_{1}G_{1}\frac{\partial w}{\partial y} + b_{1}\mu_{1}\frac{\partial^{2}w}{\partial y\partial t}) + \frac{\partial}{\partial y} (b_{2}G_{2}\frac{\partial w}{\partial y} + b_{2}\mu_{2}\frac{\partial^{2}w}{\partial y\partial t})$$
 (3)

In fundamental triangle, since b is shown by next equation

$$b_1 = m_1 y \quad , \qquad b_2 = m_2 y \quad . \tag{4}$$

next equation is gained

$$\frac{\partial^{2}w}{\partial t^{2}} = \frac{1}{(m_{1}\rho_{1} + m_{2}\rho_{2})y} \frac{\partial}{\partial y} \left[ y(m_{1}G_{1} + m_{2}G_{2}) \frac{\partial w}{\partial y} + y(m_{1}\mu_{1} + m_{2}\mu_{2}) \frac{\partial^{2}w}{\partial y\partial t} \right] 
= \frac{m_{1}G_{1} + m_{2}G_{2}}{m_{1}\rho_{1} + m_{2}\rho_{2}} \left( \frac{\partial^{2}w}{\partial y^{2}} + \frac{1}{y} \frac{\partial w}{\partial y} \right) + \frac{m_{1}\mu_{1} + m_{2}\mu_{2}}{m_{1}\rho_{1} + m_{2}\rho_{2}} \left( \frac{\partial^{2}w}{\partial y^{2}\partial t} + \frac{1}{y} \frac{\partial^{2}w}{\partial y\partial t} \right)$$
(5)

$$C_{s}^{2} = \frac{m_{i}G_{1} + m_{2}G_{2}}{m_{i}\rho_{i} + m_{2}\rho_{2}}, \qquad C_{s}^{2} = \frac{m_{i}\mu_{i} + m_{2}\mu_{2}}{m_{i}\rho_{i} + m_{2}\rho_{2}}$$
the differential equation of shear vibration

$$\frac{\partial^2 w}{\partial t^2} = C_0^2 \left( \frac{\partial^2 w}{\partial y^2} + \frac{1}{y} \frac{\partial w}{\partial y} \right) + C_1^2 \left( \frac{\partial^2 w}{\partial y^2 \partial t} + \frac{1}{y} \frac{\partial^2 w}{\partial y \partial t} \right) \tag{7}$$

and we showed of the ground vibration

$$W_{\bullet} = d \cos \frac{2\pi}{T} \pm \tag{8}$$

$$w = \int_{(y)} e^{ipt} , p = \frac{2\pi}{T}$$
 (9)

Since, differential equation of harmonic vibration is as follows:

$$\frac{\partial^2 f}{\partial y^2} + \frac{1}{y} \frac{\partial f}{\partial y} + \frac{p^2}{C_o^2 + iC_i^2 p} = 0$$
 (10)

Earthquake-proof Design Methods of Earth Dam

Boundary condition is

$$y=0 , \frac{\partial f}{\partial y} = 0.$$

$$y=b , f=d$$
(11)

The solution of dam vibration is as follows:

$$f = d \frac{1}{J_o(mb)} J_o(my)$$
 (12)

where

$$m = \frac{1}{\sqrt{2}} \left[ A + \sqrt{A^2 + B^2} \right]^{\frac{1}{2}}, \quad A = \frac{p^2 C_o^2}{C_o^4 + p^2 C_o^4}$$

$$n = \frac{1}{\sqrt{2}} \left[ -A + \sqrt{A^2 + B^2} \right]^{\frac{1}{2}}, \quad B = \frac{1}{2} \frac{C_o^4 + C_o^4 p^3}{C_o^4 + C_o^4 p^3}$$
Two dimensional amplitude analysis of homogeneous type earth

Two dimensional amplitude analysis of homogeneous type earth dam is treated with above consideration in the case of  $C_o = \frac{G}{\rho}$ ,  $C_i^2 = \frac{H}{\rho}$  instead of eq. (6).

ii) Three dimensional vibration analysis of earth dam.

Analytical formula is introduced from the differential equation of harmonic shear vibration for three directions in Fig. 3.

$$\frac{\partial^2 w}{\partial x^2} = C_o^2 \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) + C_i^2 \left( \frac{\partial^2 w}{\partial x^2 \partial t} + \frac{\partial^2 w}{\partial y^2 \partial t} \right) + C_o^2 \frac{1}{y} \frac{\partial w}{\partial y} + C_i^2 \frac{1}{y} \frac{\partial^2 w}{\partial y \partial t}$$
(14)

When the base ground has vibration of dampt, the deformation of some point of earth dam is shown in the following formula.

Where boundary condition is as follows:

$$y=0$$
,  $\frac{\partial w}{\partial y}=0$ ,  $y=b$ ,  $w=d\sin pt$  (15)

When we analyse eq. (14) from the boundary condition eq. (15).

$$w = d \left[ \sum_{s=1}^{\infty} \frac{2}{b^2 \left[ J_s(x_0) \right]^2} \left\{ \cos m \cdot x \cosh n \cdot x + \right. \right]$$

$$\frac{(\cosh n.a - \cos m.a)(\sin m.a \sin m.z \sinh n.x + \sinh n.x \sinh n.a \cos m.x)}{\sin^2 m.a \cosh^2 n.a + \sinh^2 n.a \cos m.a} J_{o(\frac{\lambda 2}{b})} \lambda d\lambda \quad (16)$$

$$-\frac{4}{\pi} \frac{Aim \frac{n_a \pi}{a} \times \{J_{a(m_a y)} J_{a(n_a y)} + 2(-J_{a(m_a y)} J_{a(n_a y)} + \cdots)\}}{J_{a(m_a b)} J_{a(m_a b)} + 2\{-J_{a(m_a b)} J_{a(n_a b)} + J_{a(m_a b)} J_{a(n_a b)} + \cdots\}\}}$$

where

$$m_{1} = \frac{1}{\sqrt{2}} \left( A_{1} + \sqrt{A_{1} + B_{1}} \right)^{\frac{1}{2}}, \qquad A_{1} = \frac{p^{2} C_{0}^{2}}{C_{0}^{2} + p^{2} C_{1}^{2}} - \left( \frac{\lambda s}{b} \right)^{2}$$

$$m_{1} = \frac{1}{\sqrt{2}} \left( -A_{1} + \sqrt{A_{1} + B_{1}} \right)^{\frac{1}{2}}, \qquad B_{1} = \frac{C_{1}^{2} p^{3}}{C_{0}^{2} + p^{2} C_{1}^{2}}$$

$$m_{2} = \frac{1}{\sqrt{2}} \left( A_{2} + \sqrt{A_{2} + B_{2}} \right)^{\frac{1}{2}}, \qquad A_{2} = \frac{p^{2} C_{0}^{2}}{C_{0}^{2} + p^{2} C_{1}^{2}} - \left( \frac{n \pi}{a} \right)^{\frac{1}{2}}$$

$$m_{3} = \frac{1}{\sqrt{2}} \left( -A_{2} + \sqrt{A_{2} + B_{2}} \right)^{\frac{1}{2}}, \qquad B_{2} = \frac{C_{1}^{2} p^{3}}{C_{0}^{2} + p^{2} C_{1}^{2}}$$

2. Distribution of Seismic Coefficient.

The accerelation acting on the some point of earth dam is calculated by next equation in harmonic vibration.

$$\alpha' = -\frac{\partial^2 w}{\partial z^2} \qquad , \qquad k = \alpha' / q \qquad (18)$$

Since w are given eq. (12)

$$\alpha' = -p^2 \frac{d}{T_{colo}} J_{cong} \sin pt \tag{19}$$

and maximum accerelation is at ainpt=! .

$$O'_{max} = -p^2 \frac{d}{J_{n(mb)}} J_{n(my)}$$
 (20)

Fig. 3 shows the distribution of %. But % distribution must be calculated by eq. (20). The author proposed next assumption for practical design.

Now at crest

$$\frac{d'_{q}}{dt} = -\frac{p^{2}d}{qJ_{o}(mb)} \tag{21}$$

at the base of earth dam

and specific amplitude is  $\frac{W}{W_0} = \frac{J_0(mg)}{J_0(mb)}$ 

3. Experimental Consideration.

The vibration box is used for an experiment to clarify earth dam vibration problem. The property of vibration box has 0.8 meter width, 1.8 meter length and 0.5 meter depth, and available period of vibration is in the range of 0.05---0.18 second. Height of model is 30 cm, and acceration was messured on the dam body. At the crest the several fold amplitude of base vibration was seen. At elastic vibration, eq. (20) is available. On calculation of &, the value of modulus of rigidity G and coefficient of viscosity are not clarified in Otani Earth Dam (N. B. paragraph 6). But G-value is given in general as  $50 \text{ kg/cm}^{--}$  280 kg/cm, here G =  $50 \text{ kg/cm}^{2}$  is assumed by the experiment, but coefficient of viscosity is not clear. In the case where the amplitude of the model is small, the eq. (20) shows good approximation. But at larger amplitude, the coefficient of viscosity would be increased. The rupture of earth dam is limited to certain places of dam but most parts of dam are still safe. used the vibration analysis eq. (20) approximately. Shape of sliding surface is varied according to slope angle of dam, shearing strength of soil (cohesive soil or noncohesive soil) and type of earth dam.

i) Effect of slope angle on the shape of sliding surface:

There are two mechanical types of rupture by earthquake, and it due to slope angle of dam. In authors experiment at the slope angle 45°, the tension crack was found at first on the slope. But at the slope angle 35° it could not find tension crack but sliding crack at first near the dam crest. Of course

these phenomena depend on also mechanical property of soil.

For that reason, we assumed that the fundamental equation of earth dam vibration is approximately that of shear vibration types.

ii) Effect of shearing strength of soil.

According to my theoretical consideration, in noncohesive soil the depth perpendiculer to slope surface of sliding circle is zero it is proved by the experiment, this property was found and sliding line was straight. In cohesive soil, according to my theoretical consideration and experiments, the sliding surface would be circular type. Using these theories as a basis, the author deduce the analytical formula of earth dam slope by sliding circles, which is as follows.

iii) Effect of shape of the earth dam.

For the present I classify the shape of earth dam into the following three types.

- homogeneous type,
- (ii) center core type,
- (iii) type with impervious board on the upstream slope surface.

  In types (i) and (ii), the sliding surface was circular in cohesive soil and straight in noncohesive soil, and upstream slope was weaker than the downstream slope in earthquake when fully reservoired. In type (iii), the definite type of sliding had not been found. In many experiments, upstream slope was stronger than the down stream slope. The author treated in this paper the stability of slope in types (i) and (ii) against vibration.
- 5. Stability Analysis of Earth Dam Slope.

Sliding circle method is one of the most practical for designing of earth dam slope. But because Taylor, Fellenius and Patterson's sliding circle method, hitherto used, and it requires trial and error method for calculation and much labour must be put into safety analysis of slope. Especially when the seismic coefficient is assumed in eq. (20), trial and error method requires greater time and labour consumption.

To save this labour, the author deduced a new method, it follows.

In this method, the conception of the most dangerous sliding circle is used, and most dangerous sliding circle can be calculated in only one step (without trial and error method).

In Fig. 4 area of sliding body ABED is as follows;

$$F = R^{2} (\varphi - \cos \varphi \sin \varphi) \tag{22}$$

From geometrical property of sliding circle next formula is gained.

$$\frac{H}{\sin\alpha} = 2R \sin\phi \tag{23}$$

$$f_1 = R(1 - \cos\phi)$$

If we neglect the p from eq. (22) and eq. (23)

$$R = \frac{4 n^2 + \left(\frac{H}{A \ln \alpha}\right)^2}{8 n^2} \tag{24}$$

and radius of sliding circle can be shown by h only. From eq. (23)  $\varphi$  is shown as follows;

$$g = \sin^{-1} \frac{4h(\frac{H}{4m\alpha})}{4h^2 + (\frac{H}{4m\alpha})^2}$$
In Fig. 4 the following relation is seen generally. (25)

$$h \ll \frac{H}{Aimcd} = L \tag{26}$$

Eq. (25) could develope convergent series as follows;

$$9 = \frac{4h \frac{H}{ain\alpha}}{4h^{2} + (\frac{H}{ain\alpha})^{2}} + \frac{1}{2 \cdot 3} \left\{ \frac{4h(\frac{H}{ain\alpha})}{4h^{2} + (\frac{H}{ain\alpha})^{2}} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5} \right\}^{5} + \cdots$$
 (27)

and

$$F = \left\{ \frac{4 \hat{h}^2 + \left(\frac{H}{A \sin \alpha}\right)^2}{8 \hat{h}} \right\}^2 \left\{ \frac{4 \hat{h} \left(\frac{H}{A \cos \alpha}\right)}{4 \hat{h}^2 + \left(\frac{H}{A \sin \alpha}\right)^2} - \frac{4 \hat{h} \frac{H}{A \cos \alpha} \left(\frac{H^2}{A \sin^2 \alpha} - 4 \hat{h}^2\right)}{\left(4 \hat{h}^2 + \frac{H^2}{A \sin^2 \alpha}\right)} \right\}$$
(28)

The angle between resultant force line and vertical axis hetais shown as follows:

$$\theta = \tan^{-1} \frac{k_4}{1 \pm k_4} = \tan^{-1} k_4 \tag{29}$$

own weight of sliding body is

$$\Psi = \rho F \tag{30}$$

The length between center of sliding circle and center of gravity Q.

$$0Q = \frac{4}{3} \frac{R \sin^3 \varphi}{2\varphi - \sin^2 \varphi} = \frac{H^3}{12 F \sin^2 \alpha}$$
 (31)

moment of sliding out

$$\frac{M_{-}}{1+d^{2}} = W \cdot Q \cdot Aim (\alpha + \theta) = \frac{\rho H^{2}}{12} \frac{Aim (\alpha + \theta)}{Aim^{2} \Omega}$$
(32)

Resistant moment by cohesive force C

$$M_c = 2R^3 \rho C \tag{33}$$

Resistant moment by internal friction angle  $\phi$ 

$$\frac{Mf}{1+E} = W \cos (\alpha + \theta - d\theta) \tan \phi R$$
Small angle  $d\theta$  between OD line and OE lines
$$d\theta = -\frac{ds}{d\theta} = -\frac{d$$

$$d\theta = \frac{ds}{R} \ll 1 \tag{35}$$

and

$$\cos (\alpha + \theta - \frac{ds}{R}) = \cos (\alpha + \theta) \cos \frac{ds}{R} + \sin (\alpha + \theta) \sin \frac{ds}{R}$$
 (36)

From geometrical property of Fig. 4.

$$\frac{R}{\sin(\pi - \theta - \alpha)} = \frac{QE}{\sin d\theta} = \frac{QE}{\sin(\alpha + \theta - \theta)}$$
 (37)

From eq. (37)

$$\frac{OQ}{\sin(d+\theta-d\theta)} = \frac{R}{\sin(\pi-\theta-\alpha)}$$
 (38)

$$\sin(\alpha + \theta - d\theta) = \frac{QQ}{R} \sin(\pi - \theta - d) = \sin(\alpha + \theta) \cos d\theta - \sin d\theta \cos(\alpha + \theta)$$
 (39)

$$\cos \frac{ds}{R} = 1 - \frac{1}{2} \left( \frac{ds}{R} \right)^2 + \dots = 1$$
 (40)

$$aim d\theta = tan (\alpha + \theta) - \frac{H^2}{12 sin^2 \alpha} \frac{Aim (\pi - \theta - \alpha)}{coo (\alpha + \theta)} \frac{1}{FR}$$
 (41)

$$\cos(d+\theta-d\theta) = \left\{ \cos(d+\theta) + \sin(d+\theta) \tan(d+\theta) - \frac{H^2}{12 \sin^2 d} \sin(\pi-d-\theta) \right\}$$

$$\times \tan(d+\theta) \frac{1}{FR}$$

where

$$\frac{Mf}{1+l^2} = \left\{ \cos(\alpha+\theta) + \sin(\alpha+\theta) \tan(\alpha+\theta) \right\} p \tan \phi FR$$

$$-\frac{H^3}{12 \sin^3 \alpha} \sin(\pi-\alpha-\theta) \tan(\alpha+\theta) p \tan \phi$$
(43)

Total resistant moment

 $M_{+} = M_c + M_f$ 

Safety coefficient against sliding

$$\gamma = \frac{M_1 + M_2}{M_-} \tag{44}$$

Position of most dangerous slip circle on earth dam slope is calculated by function h, the depth of sliding circle, and h can be get followings

$$\frac{\partial \eta}{\partial h} = 0 \tag{45}$$

where

$$\frac{\partial \gamma}{\partial k} = \frac{1}{M_{\perp}} \frac{\partial}{\partial k} (M_{\downarrow} + M_{c}) = \frac{1}{M_{\perp}} \left[ \frac{\partial M_{\downarrow}}{\partial k} + \frac{\partial M_{c}}{\partial k} \right]$$
(46)

where

$$\frac{\partial \eta}{\partial k} = \frac{H}{2 \sin \alpha k^2} \left[ \left\{ \cos (\alpha + \theta) + \sin (\alpha + \theta) \tan (\alpha + \theta) \right\} (1 + k^2) \tan \phi g k^3 - c k^2 - \frac{CH^3}{4 \sin^2 \alpha} \right]$$
(47)

Now h, is led by next formula

$$\{\cos(\alpha+\theta) + \sin(\alpha+\theta)\tan(\alpha+\theta)\}(1+\beta^2)\tan\phi\rho_{\text{loss}}^2 + c_{\text{loss}}^2 - \frac{cH^2}{4\sin^2\alpha} = 0$$
 (48)

where we set as follows: and depth of most dangerous sliding circle is calculated by next formula

$$\hat{A}_{M} = \frac{1}{3Z_{M}}(\alpha - 3Z_{M}^{2}) - \frac{\alpha'}{3} \tag{49}$$

Now I got the depth of sliding circle from the slope surface. Thus I could gain the most dangerous safety factor by next equation.

$$\eta_{\min} = \frac{M_{\frac{1}{2}(f_{\min})} + M_{\epsilon}(f_{\min})}{M_{-}}$$
(50)

where

$$\frac{H}{\sin \alpha} = l, \quad \frac{c}{\rho \tan \beta} = k' \text{ and } \frac{4hml}{4hm + l^2} = \beta \qquad d = \frac{\ell^2}{4}a' \qquad (51)$$

Mf  $+gz = \{ \cos(\alpha+\theta) + \sin(\alpha+\theta) \tan(\alpha+\theta) \} p \tan \phi$ 

$${}^{x}\left\{R\right\}^{3}\left\{\sin^{3}\phi-\phi\sqrt{1-\phi^{2}}\right\}-\frac{H^{3}}{12\sin^{3}\phi}\sin(\pi-\alpha-\theta)\tan(\alpha+\theta)\rho\tan\phi$$
(52)

$$M_c = 2C(P^2)\sin^2\phi \tag{53}$$

$$M_{-} = \frac{P}{12} l^{2} \sin \left(\alpha + \theta\right) \tag{54}$$

$$a' = \frac{\kappa'}{4 \left( \cos (\alpha + \theta) + \sin (\alpha + \theta) \right)^2}$$
 (55)

$$b' = a' l^2 \tag{56}$$

$$\theta = \tan^{-1}k \tag{57}$$

$$\mathbb{Z}_{\mathsf{M}} = \left[ \left( \frac{a^{1/3}}{3} \right)^{2} - \frac{d}{2} + \sqrt{\frac{a^{1/3}}{3} - \frac{d}{2} \right)^{2} - \left( \frac{a^{1/2}}{3} \right)^{3}} \right]^{1/3}$$
 (58)

#### 6. A Research on Otani Earth Dam.

Otani Earth Dam is in Komatsucho Ehime Prefecture in Japan. In great earthquake of Nankaido in 1946, the dam body was destroyed as in Fig. 5. The scale of Otani Earth Dam is showed in table 1. And mechanical property of soil is as in table 2.

At the time of the Nankaido earthquake, the scale of earthquake is as in table 3.

The analytical results of earth dam slope were as follows:

i) Vibration analysis of Otani Earth Dam:

Because the data of soil mechanics of table 2 are gained by labolatory test, the analytical results have only approximate values. In eq. (19) the cohesive coefficient is not clear. But I assumed several values of coefficient. The result of calculation is as Fig. 3. As you see from Fig. 3, the earth dam undergoes more vibration than the foundation.

ii) Stability analysis of earth dam slope:

In no quaking condition, the safety coefficient of slope by eq. (50) was 1.5 at upstream side, and 2.3 at downstream side. The result of this calculation shows that in steady condition the slope is satisfactly safe. And I calculate the neccesary degree of vibration when the safety coefficient  $\eta$  is equal to 1. By this calculation, I presume the action of earthquake force to earth dam at total body of upstream (cracked side).

The relation between the stability factor and degree of vibration at Otani Earth Dam is as table 4.

iii) Calculation of coefficient of viscosity by presumed seismic coefficient.

Amplitude of vibration of dam body decreases in consequence of  $\mu$  value. For this reason coefficient of viscosity of Otani Earth Dam could be calculated by the use of eq. (19).

The result of calculation was as table 4. From table 4, I got the coefficient of viscosity of Otani Earth Dam which was as follows:

$$\sqrt{\frac{\mu}{\rho}} = 25 \, (\frac{m}{\text{Aec}^k})$$

## Bibliographies;

- (1) M. Matsumura: Deformation of the Earth Dams due to Earthquake Motion, Report of the Public Works Res. Inst., Ministry of Construction, No. 28, 1934, (in Japan).
- (2) M. Hatanaka: Fundamental Considerations on the Earthquake Resistant Properties of the Earth Dam, Disaster Preventation Research Institute Bulletin No. 11, December, 1955.
- (3) I. Minami: A Shape of Earthquakeproof Cross Section of Earth Dam, Second Discussion on Earthquake Engineering, 1959. (in Japan).
- (4) Geophysical Review, No. 568, December 1946, Published by the Central Meteorological Observatory.

#### I. Minami

### Symbols;

```
w. : deformation of foundation in earthquake (horizontal).
  : horizontal deformation of dam by shearing vibration.
we: horizontal deformation of dam crest,
   : modulus of rigidity of soil (average),
   : unit weight of soil (average),
   : period of vibration in earthquake,
      period of free vibration of earth dam,
T.
   : height of earth dam,
   : amplitude of foundation in earthquake.
     coefficient of viscosity against shearing strain (average),
t
     time.
x, y, z : coordinate,
m, m: : cotangent of slope angle to vertical axis, suffix l
is upstream and suffix 2 is down stream (m_1 = \tan \alpha_1, m_2 = \tan \alpha_2),
i : imaginary number,
g : accerelation of gravity,
a': accerelation on dam body in earthquake,
J. : Bessel function,
     center of gravity of sliding body,
   : length of surface of the sliding surface,
   : depth from slope surface to bottom of sliding circle,
h
œ
   : angle of slope against horizontal plane,
   : angle between OA line and OQ line,
   : area of sliding body.
R : radius of sliding circle,
  : angle between resultant force line and vertical axis,
  : seismic coefficient at dam body.
M.: moment of sliding out,
M; : resistant moment by internal friction,
M.; resistant moment by cohesion,
   : cohesion of soil,
: internal friction angle,
ds: arc length of ED,
b, , b, : width of element in Fig. 1 (1; upstream, 2; down
stream).
G, G, : modulus of rigidity of soil (1; upstream, 2; down
stream).
#, #2 : coefficient of viscosity of soil (1; ups. 2; dow.).
      : unit weight of soil (1; ups. 2; dow.).
β, ρ : u
p : 2π/T,
e : base of natural logarithm.
b ; longitudinal length of earth dam,
k: horizontal seismic coefficient.
k. : vertical seismic coefficient,
W : weight of sliding body ( = pF),
d9 : angle between OD and OE.
? : coefficient of slope stability,
S : shearing force.
```

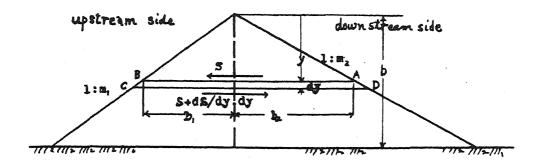
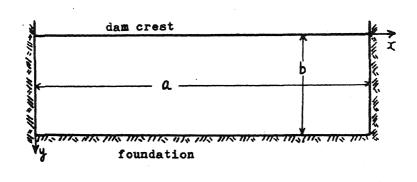


Fig. 1 : Cross section of center core type earth dam.



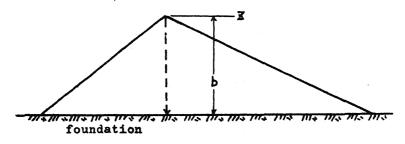


Fig. 2 : Model of 3-dimensional center core type earth dam.

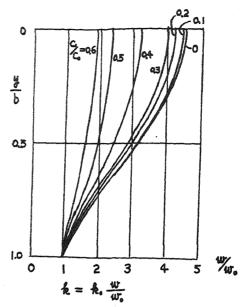


Fig.  $\mathfrak{Z}$ : Specific amplitude distribution of earth dam and its practical assumptions.

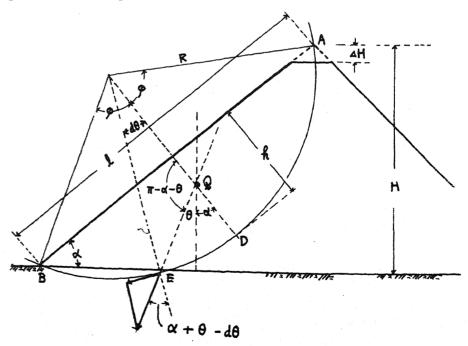


Fig. 4 : Model of circular sliding circle of the earth dam slope.

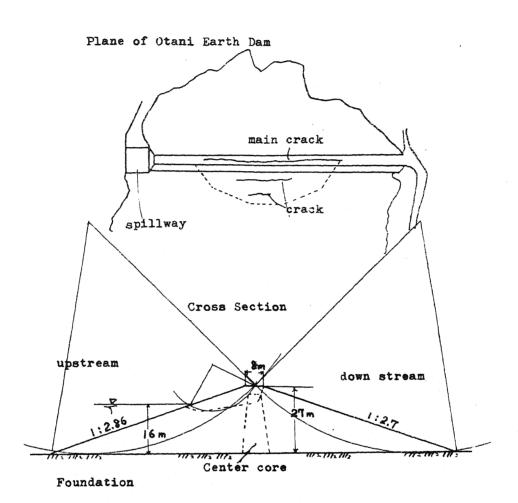


Fig. 5 : Otani Earth Dam.

I. Minami

Table 1 : About the scales of Otani Earth Dam,

				-
height	slope upstream	angle down stream	depth at the of rupture	
27 m	1: 2.86	1: 2.7	16 m	

Table 2 : Mechanical properties of Otani Earth Dam's soil,

unit	weight dow.	ton/m	up.	cohesion dow.ton	ı/m C	m/s
1.8	1.75		0.5	1.0	. 50	

Table 3 : Characteristic of the Nankaido Earthquake in Komatsu-

cho,

degree of vibrati	ion period of vibra	earthquake ation	amplitude
4 (	4) 2.	Sec.	

Table 4: Stability coefficients of Otani Earth Dam's slope against various degree of seismic coefficients.

(up.)

seismic coefficient	k = 0	0.1	0.2	0.3
stability factor	1.5	1.1	0.9	

(dow.)

seismic coeff.	k = 0	0.1	0.2	0.3	-
stability fact.	2.3	1.75	1.4	1.1	

# (sliding body)

seismic coeff.	k = 0.2	0.25		
stability fact.	1.1	1.0		