STUDY ON THE EARTHQUAKE RESISTANT DESIGN OF GRAVITY TYPE DAMS

by

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INTRODUCTION

For the earthquake resistant design of gravity dam, the method now adopted is based on the principle that no hair cracks should be allowed to occur. Accordingly, it may be approved that the behavior of dam subjected to the seismic motion is to be dealt with as an elastic vibration. This paper describes firstly that the fundamental natural period and mode of dam vibration, which are the basis of determining the seismic coefficient for design, are approximately expressible as those of bending-shear vibration by the beam theory, and also clarifies that in some high dams in Japan the values of predominant period of earthquake motion, the fundamental natural period of dam and resonant period of hydrodynamic pressure are very close to each other.

The vibration of dam in the case where the small number seismic waves having the period described above act suddenly, is also solved approximately, and the author's principle of earthquake resistant design is clarified in this report.

As the conclusion so far obtained, though it is not sufficiently perfect and further study is expected, a method of earthquake resistant design is proposed by the author.

SEISMIC COEFFICIENT OF DESIGN

In the present earthquake resistant design of gravity dam, the seismic coefficient of constant magnitude and of uniform distribution, which is based on the same static consideration as what has been taken in the designing of other common structures, is used. The rational seismic coefficient based on the so-called "method of seismic coefficient" (the method of lateral force coefficient) is obtained by dividing the lateral force of such magnitude by the weight of structure as to give the same amount of deflection as that of elastic structure caused by the seismic motion. (1),(2). And the seismic coefficient can be expressed by the sum of those of normal modes of vibration of each order.

In general, however, the normal mode having the nearly equal period to that of seismic motion is most influencial.

In a high dam, the predominant period of seismic motion of the dam foundation is nearly equal to the fundamental natural period of dam, and therefore, it seems quite sufficient that only the seismic coefficient for the fundamental mode is taken into account in an approximate solution. Accordingly, the seismic coefficient $K$ is expressed by,

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as one-dimensional problem

\[ K = \frac{1}{2} C Y A \]

\[ A = \frac{r^2}{\omega_0} \int_0^t e^{-\epsilon(t-\tau)} \alpha(\tau) \sin \omega(t-\tau) d\tau, \]

and as two-dimensional problem

\[ K' = \frac{1}{6} C' X Y A' \]

\[ A' = \frac{r^2}{\omega_0} \int_0^t e^{-\epsilon'(t-\tau)} \alpha(\tau) \sin \omega(t-\tau) d\tau. \]

The distribution of seismic coefficients are in direct proportion to \( Y \) and \( X, Y' \), and hence the absolute value of them are determined by \( A \) and \( A' \). To obtain the absolute values of \( K \) and \( K' \), it is necessary to know about \( \alpha(t) \). As to \( \alpha(t) \), many cases such as obtained by investigating the seismograms of strong earthquakes in the past, (3); by assuming the mean velocity of ground motion as a constant and adopting one of the resonant waves having the largest velocity (4); and by considering one sine (5) or cosine (6) wave, etc. have been discussed. At any rate, the most important data for determining the absolute value of seismic coefficient are the actual behaviors of dam and the ground motion which were caused by strong earthquakes in the past. At present it seems that there is no alternative but to determine, the above-mentioned coefficient by our technological judgement based on past experiences. Hence, in the following chapter, the fundamental natural periods and the modes of dam are discussed.

THE FUNDAMENTAL NATURAL PERIODS AND THE MODES

As to the vibration of gravity dam, Dr. Hatano (7) discussed each of the bending and shear vibrations as separate statitional vibration, for the fundamental triangular section of asymmetrical form. However, as already pointed out by the author (2), the influence of shearing force is considerably large in the dam having the ratio of base width \( b \) to height \( h \) of 0.8~0.9, such as in the gravity dam, and therefore the bending-shear vibration considering not only the bending moment but also shear, must be analysed. The fundamental natural period \( T_{bs} \) of bending-shear vibration is, in general, given by

\[ T_{bs} = C_s h \sqrt{\frac{h}{E}}. \]

Also as well known, the fundamental natural periods of bending vibration \( T_b \) and that of shear vibration \( T_s \) are expressed by the following equations respectively:

\[ T_b = \frac{4.098(bL/d) \sqrt{h/E}}{C_b h \sqrt{h/E}}, \]

\[ T_s = \frac{(2\pi h/2.4048) \sqrt{h/E}}{C_s h \sqrt{h/E}}. \]
The variations of $C_{BS}$ depending upon the slope of dam section $\alpha$ and Poisson's ratio $\sigma$ are shown in Fig. 1. a). In the figure, values of coefficients $C_S$, and $C_d$ in eqs. (4)(5) are also shown. $T_{BS}$ tends to $T_{B}$ when $\alpha$ becomes smaller, but on the contrary, to $T_{S}$ when $\alpha$ becomes large. As clearly seen in the figure, for such a slope of $\alpha = 0.9 \sim 1.3$ as found in the gravity or hollow gravity dam, the value of $T_{BS}/T_0$ is $1.3 \sim 1.6$ for $\sigma = 0.15$, it may be easily understood that the effect of shearing force is fairly large.

Therefore, the vibration of dams must be considered not as the bending vibration but as the bending-shear one.

Then, Fig. 1. b) illustrates an example of the fundamental mode $Y_{BS}$ of bending-shear vibration. For $\alpha = 0.9$ or so the normal mode is of bending-shear vibration, and is far apart from $Y_0$.

And, for $\alpha = 1.5 \sim 2.0$ the mode of bending-shear vibration $Y_{BS}$ is of nearly straight line. The value of $Y_{BS}/Y_0$ at the middle of dam height is $1.7 \sim 2.1$ ($\sigma = 0.15$) for $\alpha = 0.9 \sim 1.3$, and is considerably larger than that of $T_{BS}/T_0$. The variation of modes $Y_{BS}$ with that of Poisson's ratio $\sigma$ is very small and for $\sigma = 0.15 \sim 0.45$ at most 7 percent or so. Fig. 1 is shown for the dam with symmetrical triangular section. However, for such values of $\alpha$ as seen in the gravity and hollow gravity dams, the error that might be introduced into the above analysis due to the asymmetry of dam section is probably less than several percent.

Accordingly, the above figures may be used for the practical purposes with technologically sufficient accuracy.

According to some of the papers reported on the measurements of vibration of actual dams, it may be pointed out that measured periods are remarkably larger than those calculated by assuming as the bending or bending-shear vibration. (9). One cause of this fact is said to be the effect of the deformation of dam foundation. (9)(10). For simplicity, the vibration of dam will be discussed in the case where the dam and the foundation can be expressed by the one-mass system as shown in Fig. 2.

In this case, the natural period of dam with rocking motion $T_r$ becomes as follows (8):

$$T_r = \frac{T}{\sqrt{1 + \alpha/\sigma}}.$$  \hspace{1cm} (6)

As clearly shown by eq. (6), in the structure which is attended by not only the elastic deformation but also the displacement due to rocking motion, the natural period becomes as long as the value $\sqrt{1 + \alpha/\sigma}$ multiplied by the natural period $T$ obtained in the case of considering the elastic deformation only, neglecting the deviation by rocking motion. Then, the seismic coefficient for design $K_r$ in the above case is expressed by

$$K_r = \frac{2\pi^2}{\sigma} \int_0^t \alpha(t) \sin \alpha_r(t - \tau) d\tau.$$  \hspace{1cm} (7)

* Measured period of Miura and Taifu Dam in Table 2.

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On the other hand, the seismic coefficient $K$ of the structure of one-mass system, when only the elastic deformation is taken into account, is obtained by putting $c = \gamma = 1$ and $\varepsilon = 0$ in eq. (1),

$$K = \frac{n}{2} \int_{t_{0}}^{t} d(\tau) \sin n(t-\tau) d\tau.$$  \hspace{1cm} (8)

Comparison of eq. (7) with eq. (8) shows that eq. (7) is equal to eq. (8), when putting $n = n_r$. Therefore, as will be understood by the concept of acceleration spectrum (or seismic coefficient spectrum), it may be allowed to use $n_r$ (or $\tau_r$) instead of $n$ (or $\tau$). That is to say, even the case where the displacement due to rocking motion is accompanied, the acceleration spectrum (or seismic coefficient spectrum), is of similar form to that obtained in the case of the elastic deformation only, and does not show any particular change in the form due to rocking motion.

EFFECT OF THE RESERVOIR

(1) Hydrodynamic pressure working on vertical wall plane

The fundamental theory of hydrodynamic pressure was established by Dr. H.M. Westergaard (11), and was improved by T. Hatao (12) and by S. Sato (13). Lately S. Kotsubo discussed mathematically the effects of the topography, the dam shape and the shape of contact surface (exposed surface) of dam and water (14) on hydrodynamic pressure, as the problem of unsteady state.

The approximate formulas by Westergaard are very conveniently used technologically, but have some unreasonable aspects.

And furthermore the studies pursued by the latter researchers are reasonable but the calculations based on the studies are some what complicated for the practical use. Therefore, the author attempted to derive approximate formulas which are convenient for practical use, based on the studies made by the above-stated researchers in the following way. As it is well known, when the seismic motion is expressed by $a \sin (2\pi t/T)$, and $T \neq 1.3$ sec; the hydrodynamic pressure working on vertical wall plane is expressed as follows from Westergaard's formula for steady state:

$$p = \frac{7}{8} k w \sqrt{h y}.$$  \hspace{1cm} (9)

Eq. (9) is applicable when a rigid wall oscillates uniformly along the direction of stream flow. When the rigid wall makes rocking motion at the middle of dam bottom, the hydrodynamic pressure $p'$ is given by

$$p' = \frac{3}{8} k w \sqrt{h y}.$$  \hspace{1cm} (10)

When the seismic motion occurs suddenly, eq. (9) is rewritten as follows:

$$p = c \frac{7}{8} k w \sqrt{h y}.$$  \hspace{1cm} (11)
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For example, the coefficient \( C_0 \) in eq. (11) is about 1.5, when the ground motion of a half-wave of sine form having the same period as the natural period of dam occurs suddenly.

(2) The hydrodynamic pressure on the slope

Eqs. (9)(10) can be used approximately, even safely, for the gravity dam having the steep upstream surface, but they give overestimated values for the hollow gravity dam having the gentle slope surface unless the effects of slope are considered.

C.N. Zanger published the paper concerning experimental investigations in the steady state on this problem. (15). When the hydrodynamic pressure on the vertical plane obtained by Zanger's equation equals to that calculated by Westergaard's approximate eq. (9), a new multiplier coefficient 0.9 must be used for eq. (9).

Therefore, the hydrodynamic pressure \( \mathcal{F}_x \) on the slope, when the seismic motion occurs suddenly, can be expressed as follows:

\[
\mathcal{F}_x = C_0 C_m \frac{e^{3}}{8} a \mathcal{W}^2 \sqrt{\gamma \gamma} \]

(12)

(3) The effects of topography

The hydrodynamic pressure obtained above is solved in one-dimensional analysis, but it should be analysed as two-dimensional problem for actual dams. The problem has been discussed by Kotsubo who shows that the resonant period \( T'_w \) of hydrodynamic pressure can be approximately expressed by

\[
T'_w = T_w \sqrt{\gamma'/\gamma} \]

(13)

Furthermore, according to his investigation, the distribution shape of hydrodynamic pressure is not so distinctly different even when the topography changes greatly, but the absolute value depends upon the form of exposed water area and decreases in the following order: rectangle, semicircle, quadrant and triangle. When the period of ground motion is 1.0 sec, the hydrodynamic pressure for a triangle of exposed water area is about 0.7 times as large as that for a rectangular exposed water area.

COMPARISON OF THE MEASURED VALUES ON THE MODEL AND ACTUAL DAMS WITH THE CALCULATED ONES

In the preceding chapter, the results of calculation are described when the vibration of gravity type dam is assumed as the bending-shear vibration of a symmetrical triangular-section-dam.

In the following, in order to examine the propriety of the author's theory, they are compared with the experimental results obtained by himself, and with the actual measurements performed by Dr. Kishinoue, Dr. Hatano and author, etc., on the actual gravity dams.

(1) Experiment by agar-agar models (16)

The resonant periods and the deflection curves at resonance caused by the shaking table are to be assumed as the fundamental natural periods and the modes of models. In Fig. 3. a), the
relations between the slope of dam model and the fundamental natural periods are represented. The measured value of period shows some deviation from the calculated $\gamma_s$ curve, but the said deviation is to be found rather far smaller as compared with the difference between the above-stated measured value and the calculated $\gamma_s$ curve. Fig. 3. b) shows the fundamental modes.

In the case of $\alpha=2$, the measured values show good agreement with the calculated ones; in the case of $\alpha=1$, the measured values lie between the calculated $\gamma_{as}$ and $\gamma_s$ curves, and in the case of $\alpha=0.75$ the measured values coincide with $\gamma_s$ curve.

(2) Experiment by hollow gravity rubber models (17)

Table 1 shows a certain hollow gravity rubber model dam the scale of which is 1/200. The model dam is composed of ten elements and of four elements with different heights as shown in Table 1.

The natural period and the mode of the elements are obtained from the resonant period and the deflection by the forced vibration caused by the shaking table. In this case the coefficient of the period of bending-shear vibration (the coefficient $C_{as}$ in eq. (3)) is 6.296 from Fig. 1. a). Calculations are made by using the above value and they are shown in Table 1 in company with the measured data.

As clearly seen from Table 1, the calculated values show good agreement with the measured ones. The mode of the model is represented in Fig. 4 in which the marks ‡ show the measured ones and the full-lines show the calculated values. The measured values lie between the bending-shear vibration curve $\gamma_{as}$ and shear vibration curve $\gamma_s$.

(3) Measured periods of actual dams

Table 2 shows the measured periods of a few actual gravity dams. In Taifu and Miura Dams, as it may probably be due to the difference in the vibration methods, the periods are cosiderably longer than those expected by the calculation. The deformation of foundation rock was considered as discribed before, as a cause of this discrepancy, but as seen from eq. (5), $T=1,41T$ and $T_0=1,73T$ even for $\gamma=\delta$ and $\gamma=\delta$, in other words, the period becomes longer by about 40 ~ 70 percent. Therefore, this way of thinking cannot explain the remarkable discrepancy between the calculated results and the measured ones. On the contrary, in Tsukabaru Dam the measured data agreed well with calculated value $\gamma_{as}$. The measured period of Tsukabaru Dam was obtained by the oscillator attached in the middle of the span especially on the top of the dam, and so it may be considered that it expresses accurately the natural period. So, some doubts arise whether the measured values on these dams represent their natural period or not.

(4) Two-dimensional vibration of dam

Although the dam must be substantially treated as a two-dimensional structure, the analogy from the author's studies (2) made on the earth dam suggests that one-dimensional treatment Fig. 1. a), b) may be used correctly enough for the dams having the ratio of the length to the height as given in Table 2.

Fig. 4. a), b) show the two-dimensional modes of Tsukabaru
Dam and of the hollow gravity dam of rubber model, but in the latter model dam, the above-mentioned elements were pasted up to each another so as to form one body. As clearly seen from Fig. 4, the measured values show good coincidence with the calculated ones.

(5) Vibration characteristics of dam in the case of full reservoir

Only a few data are available which provide the results of measurements taken for both full and empty cases of the reservoir formed by one of the actual dams. Therefore, the effects of stored water are considered by using the measurements made on Tsukabaru Dam and the models of hollow gravity dam. The natural periods measured on the rubber model dam for the full reservoir are shown in column 5 of Table 1. Coupled oscillation of dam and hydrodynamic pressure is considerably difficult to be treated with.

Accordingly, by aiming at the fundamental natural period of dam only, and also by considering that the hydrodynamic pressure is a virtual mass added to the dam, the vibration of dam is analysed as that of one-mass system. The vibration amplitude of the top of the model dam in resonance becomes about 10 times as large as that of the ground motion, and the deflection curves in resonance are not so different as those obtained for empty reservoir.

Therefore, as the simplest case by using eq. (10) which gives the hydrodynamic pressure when a rigid wall takes the rocking oscillation centered on the foundation, the virtual mass $\Delta M$ which is to be added to the one-mass system having the thickness $d$ can be calculated as follows:

$$\Delta M = 0.218 \text{Cm} \omega \nu d^2 / g.$$  \hspace{1cm} \text{(14)}

Column 6 of Table 1 points out the natural period for full reservoir, calculated by using $\Delta M$ in eq. (14), the mass and the natural period of the model dam for empty reservoir. Comparison shows that the simple calculation described above gives fairly good approximation to the measured values. So, by using the hydrodynamic pressure where the deformation of dam is taken into account, as a density function and by computing the natural period by the energy method, better approximation is obtainable.

The natural period and mode of Tsukabaru Dam shown in Table 2 and Fig. 4 are obtained for full reservoir. The natural period for empty reservoir, computed from the period 0.146 sec in Table 2 by using the above method, is 0.134 sec. This shows that stored water elongates the period by 9 percent. These values are obtained when the rigid wall takes rocking oscillation. If the rigid wall takes parallel oscillation, the elongation of the period as discussed above is 22 percent which is obtained from calculation under the same assumption. Calculation performed by Kotsubo by assuming as the coupled oscillation of dam and stored water, gives that stored water elongates the period by 15 percent, and it coincide with the mean value of the above two calculated ones by the author.

For the model test the law of similarity is very important. In this test using a rubber model having a small value of modulus of elasticity, the variation in the mode due to stored water is
not so large. In addition, the deflection curve for Tsukabaru Dam
in resonance, as shown in Fig. 4, is in good accordance with the
mode $\gamma =\gamma_5$ of bending-shear. Hence, it may be considered that the
distribution of seismic coefficient for design is expressed by $\gamma =\gamma_5$
curve for both full and empty cases of the reservoir.

DEFORMATIONS ON THE TRANSIENT VIBRATION OF DAM

According to the data ever measured in Japan, the predominant
period of seismic motion at dam site is $0.1~0.3$ sec, and, on the
other hand, the fundamental natural period of high dam lies in the
range as shown by Fig. 5. Therefore, it can be imagined that the
dam resonates to a small number of waves—one to a few waves—of
the seismic wave, and that it is under the maximum stress condition.

According to the author's investigation by the mechanical
analyser with multiple mass (23), if the seismic motion of sine
or cosine type the period of which is the same as the fundamental
natural period of dam occurs suddenly, the deformation of dam
after the ceasing of seismic motion, may be considered approximately
as the fundamental mode $\gamma =\gamma_5$ and the deformation caused by that time
is close to $\gamma =\gamma_5$ rather than to $\gamma =\gamma_5$. The deformation of gravity
dam can be approximated by the fundamental mode $\gamma =\gamma_5$ for both cases
of full and empty reservoir. Therefore, the distribution of
seismic coefficient of design is also expressed by $\gamma =\gamma_5$.

However, when taking into account the present method of assum-
ing the uniform distribution of seismic coefficient, the tri-
angular distribution along the height and the sine form distribution
along the length can be assumed. All described above is discussed
for a case where the acceleration of ground motion which is uni-
formly distributed along the length of dam, acts along the stream
flow direction. But, if the structure is long like the dam, the
phase difference between the ground motions at both shores becomes
a problem. Therefore, when considering such a seismic motion as
an object, it is necessary to take into consideration the seismic
coefficient for design the distribution of which is of sine type
as shown in Fig. 6. b) curve, (24). To determine the absolute
value of seismic coefficient, it is necessary to know the damping
characteristics of dam besides the natures of seismic motion itself.

As to the seismic motion, our country has not had such perfect
standards of seismogram as those obtained by San Francisco Com-
mitee. As previously described, it may be imagined that the high
dams in Japan are expected to be subjected to the seismic motions
frequently the periods of which are very close to the fundamental
natural periods of dams themselves, and in these cases the
duration time of the seismic motion, that is, the number of reso-
nant waves is a problem. Further investigations have to be made
on the seismic motion, but the problem is ascribable to the point
how much value obtained by multiplying the seismic coefficient of
ground motion is to be used for the seismic coefficient for
design. For example, if the seismic motion having a half sine-
wave with the same period as that of dam acts suddenly, the
multiplier is $1.77$ for no damping force $\varepsilon =0$ (natural circular
frequency of dam $\omega _1$ / circular frequency of ground $\omega_g =1.620$).
Then, as to the damping coefficient, only a few measured data are available now except the values as given in Table 3.

Furthermore, all described above such as the predominant period of seismic motion, the damping coefficient, etc., were obtained by the experiments of micro-seismic motion or micro-vibration, and hence the values in the case of strong seismic motion should be clarified by further studies. The vibration of dam in the case of full reservoir is very complicated, and the natural period of dam in the case is about 15 percent longer than that in the case of empty reservoir. As illustrated in Fig. 5, the resonant period of hydrodynamic pressure is generally longer than the natural period of dam. In some cases, as pointed out by Kotsubo, the period of hydrodynamic pressure is $1 \sim 0.7$ times as short as that of one-dimensional hydrodynamic pressure dependent on the topography, and, on the other hand, the natural period of dam becomes $1 \sim 1.7$ times as long as the said value, due to the deformation of rock foundation. Therefore, the case can be assumed as the critical (most dangerous) condition, where the following three values: the period of seismic motion, the natural period of dam and the resonant period of hydrodynamic pressure coincide with each other.

If the seismic motion of a half sine-wave having the same period as the resonant period of hydrodynamic pressure, acts suddenly, the hydrodynamic pressure working on the vertical rigid wall is about 1.5 times as large as that obtained by Westergaard's approximate eq. (9), and, in an example of natural seismic motion, it is found to be about 1.6 times as large as the said one (10).

CONCLUSIONS

Based on the discussions described above, the author clarifies the principle of earthquake resistant design of gravity type dam and proposes, as the conclusions, the method of earthquake resistant design.

An author's proposal for seismic coefficient for design

**For empty reservoir**

1. Distribution of seismic coefficient: Assume a triangular distribution along the height, and a sine type distribution along the length (containing the corrected ones) (See Fig. 6).

2. Absolute value of seismic coefficient: Assume the area of distribution of seismic coefficient as the value "seismic coefficient of ground times height of dam" multiplied by $\beta$. And determine the seismic coefficient of ground by considering the regional degree of danger of earthquake (25), the effects of ground foundation, and the effects of dam failure upon the downstream area.

**For full reservoir**

1. Distribution of seismic coefficient: Same as for empty reservoir.

2. Absolute value of seismic coefficient: Assume the seismic force working on dam body as the same as for empty reservoir, and use the corrected Westergaard's approximate eq. (12) for hydrodynamic pressure.
Allowable stresses

Determine the allowable stresses for compression, tension and shear by dividing by proper factors of safety the values obtained by the standard strength tests.

Namely, the author proposes to take the actually working load as the external load at the time of earthquake occurrence, and to adopt the smaller value of safety factors than those adopted now for statical loads in determining the allowable stresses.

The author's opinion is that the value of \( \beta \) is to be chosen as \( 1.2 \sim 1.8 \), the value of \( C_s \) in eq. (12) as \( 1.2 \sim 1.6 \), and the factor of safety as \( 3 \sim 4 \). Even if the above method is used, there remains the most important problem namely how the value of seismic coefficient of ground motion is to be assumed. Further studies on this problem are expected to be developed, and now it seems that there are no data except the fact that the dams designed by the conventional seismic coefficient (26) have been safe for the past strong earthquakes.

In ending this paper, author wish to add the following information for reference. When the stresses in a dam are calculated for cases of the vertical uniform ( \( K = 0.12 \) ) and of the equivalent triangular distributions of seismic coefficient for design, small values of tensile stresses occur in upstream surface for empty reservoir and in downstream surface for full one, respectively. (16).

BIBLIOGRAPHY


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NOMENCLATURE

A : Acceleration spectrum, one-dimensional.
A' : Ditto, two-dimensional.
b : Base width of dam.
C : Constant depending upon the normal mode, one-dimensional.
C' : Ditto, two-dimensional.
C_{bs} : Coefficient of bending-shear vibration, (constant determined by the form of dam section \( \alpha = b/\rho \) and Poisson's ratio \( \sigma' \) of dam material).
C_{b} : Coefficient of bending vibration, (constant determined by the \( \alpha \)).
C_{s} : Coefficient of shear vibration, (constant determined by Poisson's ratio \( \sigma' \)).
C_{o} : Constant to be determined by the ratio of characteristics (period, wave form, duration time) of ground motion to the natural period of dam.
C_{m} : Zanger's coefficient of the inclined wall.
E : Young's modulus of dam material.
\( g \) : Acceleration of gravity.
G : Shear modulus of elasticity of dam material.
H : Dam height, water depth or maximum water depth.
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\( h' \): Mean water depth.

\( K \): Seismic coefficient, one-dimensional.

\( K' \): Ditto, two-dimensional.

\( K_r \): Ditto, with rocking motion.

\( K_e \): Ditto, for the hydrodynamic pressure.

\( K' \): Ditto, ditto, at the top of wall.

\( n \): Natural circular frequency of dam without damping, one-dimensional.

\( n' \): Ditto, two-dimensional.

\( n_r \): Ditto, with rocking oscillation.

\( p \): Westergaard's approximate hydrodynamic pressure.

\( p' \): Hydrodynamic pressure in the case of rigid wall caused by rocking oscillation.

\( p_3 \): Transient hydrodynamic pressure, caused by parallel and rocking oscillation.

\( T \): Natural period of dam, \( T = \frac{2\pi}{n} \).

\( T_r \): Ditto, \( T_r = \frac{2\pi}{n_r} \), with rocking motion.

\( T_B \): Ditto, in the case of bending vibration.

\( T_{B3} \): Ditto, in the case of bending-shear vibration.

\( T_s \): Ditto, in the case of shear vibration.

\( T_w \): Fundamental resonant period of hydrodynamic pressure of the rectangular form of exposed water area.

\( T_{w3} \): Ditto, of the any form of exposed water area.

\( w \): Weight of unit volume of water.

\( \lambda \): Fundamental normal mode of dam, in the direction of length.

\( \gamma \): Ditto, in the direction of the height.

\( \alpha \): Ratio of the base width of dam \( b \) to height \( h \), \( \alpha = \frac{b}{h} \).

\( d(t) \): Acceleration of earthquake motion.
\( \sigma \): Deflection of the point where unit load acts on mass, in the case where the foundation is fixed.

\( \gamma \): Ditto, in the case where the foundation can only rotate.

\( \varepsilon \): Damping coefficient of dam (ratio to critical damping), one-dimensional.

\( \varepsilon' \): Ditto, two-dimensional.

\( \rho \): Density of dam material.

\( \sigma_0 \): Poisson's ratio.

\( \omega \): Natural circular frequency of dam with damping, one-dimensional.

\( \omega' \): Ditto, two-dimensional.
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Fig. 1 a) Coefficients $C_a$, $C_b$, and $C_d$.

Fig. 1 b) An example of fundamental modes $Y_a$, $Y_b$, and $Y_c$.

Fig. 2 a) The model of dam with rocking motion.

Fig. 2 b) The model of dam with rocking motion.

Fig. 3 a) Relation between the calculated periods $T_{aa}$, $T_{ab}$, $T_{ac}$, and the measured one, (graph-only model).

Fig. 3 b) Relation between the calculated modes $Y_a$, $Y_b$, $Y_c$, and the measured, (graph-only model).

Fig. 4 a) Relation between the calculated modes and the measured ones. Fixed base dam (actual gravity dam), and the rubber model dam (rubber gravity dam).

Fig. 4 b) Relation between the calculated modes and the measured ones. Fixed base dam (actual gravity dam), and the rubber model dam (rubber gravity dam).
### Table 1. Natural periods of the hollow gravity dam model.

<table>
<thead>
<tr>
<th>Height of dam element (cm)</th>
<th>Reservoir empty</th>
<th>Reservoir is full</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured period (sec)</td>
<td>Calculated period (sec)</td>
</tr>
<tr>
<td></td>
<td>Dam</td>
<td>Dam (one body)</td>
</tr>
<tr>
<td>I  36.0</td>
<td>0.039-0.4</td>
<td>0.035</td>
</tr>
<tr>
<td>II 33.0</td>
<td>0.036-0.45</td>
<td>0.026</td>
</tr>
<tr>
<td>III 23.5</td>
<td>0.026-0.27</td>
<td>0.026</td>
</tr>
<tr>
<td>IV 16.0</td>
<td>0.026-0.27</td>
<td>0.026</td>
</tr>
</tbody>
</table>

### Table 2. Natural period of the gravity dam, (plate type).

<table>
<thead>
<tr>
<th>Name of dam</th>
<th>Height (m)</th>
<th>φ = b/A</th>
<th>Calculated period (sec)</th>
<th>Measured period (sec)</th>
<th>Vibrating force</th>
<th>References</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td>E = 25×10^6 (N/m^2)</td>
<td>E = 30×10^6 (N/m^2)</td>
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<td></td>
</tr>
<tr>
<td>Taisu</td>
<td>5.9</td>
<td>0.90</td>
<td>0.046</td>
<td>0.085</td>
<td>0.6-0.7</td>
<td>Natural earthquake over flow</td>
</tr>
<tr>
<td>Mura</td>
<td>8.1</td>
<td>0.91-0.99</td>
<td>0.155</td>
<td>0.181</td>
<td>0.6, 1.2</td>
<td>Blasting (Reservoir empty) Blasting (Reservoir is full)</td>
</tr>
<tr>
<td>Tsukabara</td>
<td>8.8</td>
<td>0.88</td>
<td>0.147</td>
<td>0.134</td>
<td>0.146</td>
<td>Oscillator (Reservoir is full)</td>
</tr>
<tr>
<td>Hikiharana</td>
<td>6.6</td>
<td>0.86</td>
<td>0.122</td>
<td>0.111</td>
<td>0.13</td>
<td>Natural earthquake (Reservoir is full)</td>
</tr>
</tbody>
</table>

### Table 3. Damping coefficient of dam, (plate type).

<table>
<thead>
<tr>
<th>Name of dam</th>
<th>Type of dam</th>
<th>Height (m)</th>
<th>Damping Coefficient ratio to Critical Reservoir empty</th>
<th>Reservoir is full</th>
</tr>
</thead>
<tbody>
<tr>
<td>Namahata (cliff dam)</td>
<td>Arch</td>
<td>12.8</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Tsukabara</td>
<td>Gravity</td>
<td>8.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Togakusa</td>
<td>Arch</td>
<td>6.2</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
DISCUSSION

S. Okamoto, University of Tokyo, Japan:

In order to establish a rational aseismic design procedure for dams, I think that, it is a most important problem to obtain the more complete informations about the properties of earthquake motion in mountain district. How do you think about it?

M. Hatanaka:

From the view-point of achieving a more reasonable, economic, aseismic design of dams, I believe that it is very significant to obtain seismograph records of earthquake motion at the dam site as well as the motion of a dam itself.

An observation of earthquake ground motion and the response of gravity dams is being carried out by Dr. Hatano of the Central Research Institute of Electric Power Industry, Tokyo, and also I have been making a similar observation since 1957 at Hikihara Dam in Hyogo Prefecture, Japan. The Hikihara Dam is a gravity dam, 86 meters in height, and is now equipped with a total of 6 seismometers. Namely, each pair of accelerographs and displacement seismometers is located at the top and bottom of the central part of the dam, respectively, and two other displacement seismometers are set on the both sides of the dam.

However, only a few records of the small-size earthquakes were obtained so far in the observation, probably because the area has experienced very few earthquakes.

Prof. Raphael stated a couple of hours ago that such observations are not carried out in the United States, but I hope this kind of observations will herewith be made in every earthquake countries in the world.

R. Tanahashi, Kyoto University, Japan:

1. For large-scale structures such as dams, it is often noticed that a certain difference in the phase, or phase angle, of earthquake ground motions may be observed at the boundaries of the structures, namely that vibration of the structure at its one end will be different in phase from that at the other end. Is there any considerations on the fact of phase difference, which are reflected currently in the aseismic design of dams?

2. Although it is of great significance that people at Kobe University are making an observation of response of gravity dams due to small-scale earthquakes, I realize that such an observation or study is not enough if this is carried out by only a university or an institute, and that a more comprehensive study on the problem in a larger scale, by organizing and in coopera-
tion of a number of institutions, will be indispensable. It seems to me that the government should help for these important investigations so as to maintain them with an expectation of early success. On the occasion of the 2nd World Conference on Earthquake Engineering, I strongly would like to emphasize this point.

M. Hatanaka:

1. As to the first part of Question 1, I am in complete agreement with Prof. Tanabashi's opinion. It is true that in case of high dams the phase difference between the earthquake responses of the both sides of a dam is a problem.

In general, the foundation of concrete dams is usually on a solid layer of rock, for which it is said that the longitudinal wave of earthquake travels at a speed of about 3,000 to 5,000 m/sec. If the value of Poisson's ratio of the rock layer is assumed to be ⅓, the velocities of traveling transverse and Rayleigh waves will be in the range of about 1,700 to 2,900 m/sec and 1,600 to 2,700 m/sec, respectively. On the other hand, the predominant period of earthquake ground motions observed so far ranges about 0.1 to 0.3 sec. Therefore, the half wave length of the longitudinal, transverse and Rayleigh wave components of seismic waves will be about 150 to 750 meters, 85 to 435 meters, and 80 to 405 meters for each of them written in that order.

Consequently, it is easily seen that a dam with the top span length of such a scale may possibly have a relative displacement between the both sides of the dam, in the direction parallel to the stream.

Nevertheless, I do not think that this fact is taken into consideration in the prevalent design criteria for earthquake-resistant dams. But, this problem is of great importance especially for arch dams. And I have once made a proposal of a new distribution of the seismic factor or base shear coefficient, by laying an emphasis on the phase difference in the motions of both sides of dams subjected to earthquake excitation. (Ref. 1)

Also, in my paper presented at this Conference and now being discussed, I am making another proposal of a distribution of design coefficient for base shears, which varies in a pattern of one wave length of sinusoidal function. (see Fig. 6, b)

2. I am in favor with Dr. Tanabashi's comment. While investigations on earthquake response of building structures are carried out much in Japan, more attention should be paid to the aseismic design of dam structures. As Dr. Tanabashi mentioned, it is our hope that a comprehensive investigation on the earthquake problems of dam structures will be in progress in a nation-wide scale and further in an international scale under the support of governmental organizations.
Earthquake Resistant Design of Gravity Type Dams

I deeply appreciate Profs. Okamoto and Tanabashi for their valuable discussions and comments.

Reference: