" ACCURACY AND SIMPLIFICATION OF THE SEISMIC DESIGN "

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1.- From an examination of the seismic problem, it becomes imperative to express the seismic coefficient not as a constant for all structures of the same geographical region, but as a function of:

a) The rigidity of the structure and the mass distribution, expressed in terms of the natural period T_S of the fundamental motion of the structure.

b) The physical property of the material of the structure and the ground.

On the other hand, since during the oscillations, the seismic coefficient of the structure changes as a function of time, the most unfavourable moment of the fundamental motion, which produces the maximum base shear, must be determined, taking into account the increase due to resonance and the decrease due to damping.

For these it is proposed to use the following terms:

a) <u>External seismic coefficient C_o or seismic coefficient of the ground:</u>

We define thus, the ratio of the observed maximum horizontal component of the seismic acceleration of the ground to the acceleration of gravity.

b) <u>Mean or particular seismic coefficient C of the structure</u>:

We define thus, the ratio of the maximum resultant of the lateral seismic forces (base shear) to the weight of the whole structure.
Hence, by the definition :

$$R = cW \cdots\cdots\cdots (1)$$

where: R= the resultant of the horizontal (lateral) seismic forces.

W= the weight of the structure.

Obviously, because all the points of the structure have not the same seismic acceleration, the centre of the seismic forces does not coincide with the centre of gravity of the structure.
The particular seismic coefficient C of every structure depends on its rigidity.
The measure of rigidity is given by the natural period T_S of the structure.
The maximum particular seismic coefficient C of the building, after an impulse of the ground motion, is produced in the case of stereostatic (perfectly rigid) structure and it is equal to C_o
The particular seismic coefficient of a structure can be written:

$$c = \lambda \mu c_o \cdots\cdots\cdots (2)$$

where: λ = a function of the rigidity

μ = a factor depending on the resonance and the damping.

2.- <u>Calculation of the mean or particular seismic coefficient of a structure.</u>

A.- The capacity of absorption of seismic kinetic energy by the structure is limited and depends on its rigidity.

By equating the work done by the ideal stereostatic structure and the real structure (elastic) during an earthquake impulse (half a ground oscillation), we have the relation :

$$C = C_o \frac{\sum_{k=1}^{n} a_k W_k}{a_o W} \qquad \text{..........} (3)$$

where: W_k = the weight of the k^{th} floor

$W = \sum_{k=1}^{n} W_k$

n = the number of floors

a_o = the amplitude of the horizontal component of the motion of the ground

a_k = the translation of the k^{th} floor during the time of displacement a_o on the ground, <u>Before the structure starts to give out the absorbed seismic energy</u> (before the beginning of the vibration of the structure) (Fig.1).

Clearly the ratio :

$$\frac{\sum_{k=1}^{n} a_k W_k}{a_o W} = \lambda \qquad \text{.........} (4)$$

is less than unity and is called <u>rigidity factor</u>.

When the velocity v of transmission of the motion of the ground in the overlying structure is given by the equation :

$$v = \frac{2H}{T_s} \qquad \text{...........} (5)$$

where: H = is the height of the structure, the magnitude of the translation a_k is given by the equation :

$$a_k = \frac{1}{2} a_o \left[1 + \sin \frac{\pi}{2}\left(1 - \frac{2h_k}{H_o}\right)\right] \qquad \text{........} (6)$$

where: h_k = the distance from the foundation to the point A_k under consideration (Fig.1)

H_o = the distance from the foundation to the point M (Fig.1) over which the motion has not yet been transmitted when the foundation has moved through the amplitude $2a_o$

Hence:

$$H_o = \upsilon \frac{T_e}{2} = H \frac{T_e}{T_s} \quad \dots\dots\dots (7)$$

where: T_e = the period of the ground forcing motion (earthquake)

From equations (6) and (7) we have:

$$a_k = \frac{1}{2} a_o \left[1 + \sin \frac{\pi}{2} \left(1 - \frac{2h_k}{H} \frac{T_s}{T_e} \right) \right] \quad \dots\dots\dots (8)$$

For structures with storeys of constant height and constant concentrated static loads i.e. when:

$$H = n h_o \quad \dots\dots\dots (9)$$

$$W = n W_k \quad \dots\dots\dots (10)$$

where: n = number of storeys

h_o = the height of every storey,

the value of the rigidity factor λ is derived from equations (4), (8), (9) and (10) :

$$\lambda_p = \frac{\sum_{k=1}^{n} a_k}{n a_o} = \frac{1}{2n} \left[n + \sin \frac{\pi}{2} \left(1 - \frac{2T_s}{nT_e} \right) + \sin \frac{\pi}{2} \left(1 - \frac{4T_s}{nT_e} \right) + \right.$$

$$\left. + \dots\dots\dots + \sin \frac{\pi}{2} \left(1 - 2 \frac{T_s}{T_e} \right) \right] \quad \dots\dots\dots (11)$$

For continuous uniformly distributed static loads, equation (11) becomes:

$$\lambda_p = \frac{\int_0^H a_k \, dh}{H a_o} = \frac{1}{2H} \int_0^H \left[1 + \sin \frac{\pi}{2} \left(1 - \frac{2h_k}{H} \frac{T_s}{T_e} \right) \right] dh \quad \dots (12)$$

From equations (11) and (12) the tables I and II are set out, the first for concentrated masses on the floors (point loads) of the multistory buildings and the second for continuous mass distribution, along the height of the structure.

It is of special importance to note that the part A_oM and especially the region M (Fig.1) is subjected to the maximum danger of damage because of the small radius of curvature of the deflected structure.

The point M lies on the structure when it is slender and flexible.

Generally the total lateral seismic force R for one earthquake impulse is given by :

$$R=(\lambda_P W_P+\lambda_p W_p)c_0 \cdots\cdots\cdots (13)$$

where: λ_P = the rigidity factor for point loads,
λ_p = the rigidity factor for continuous loads,
W_P = the resultant of the point loads,
W_p = the resultant of the continuous loads.

B.- Resonance and damping

Definitions:

T_0 = the duration of maximum seismic intensity. This is a function of the magnitude of the earthquake and can be taken equal to 6 seconds.
ρ = damping coefficient during half a period $\frac{T_s}{2}$ of the structure fundamental motion.

$$\rho=e^{-\frac{K}{T_s}} \cdots\cdots\cdots (14)$$

where: K = Constant depending on the physical properties of the material. For reinforced concrete structures K can be taken equal to 0,20.

μ = Resonance factor i.e. factor of increase of the particular seismic coefficient of the structure because of resonance, taking into account the damping.
a_k = the amplitude of undamped oscillation of the point A_k (Fig.2) without the influence of resonance.
a'_k = the amplitude of oscillation of the point A_k after m ground impulses taking into account the effects of resonance and damping

Hence, by definition :

$$a'_k=\mu a_k \cdots\cdots\cdots (15)$$

The value of the resonance factor μ can be calculated from the equation:

$$\mu=1+\rho\frac{\sin\left(\frac{\pi}{2}\frac{T_s}{T_e}\right)}{\sin\frac{\pi}{2}}+\rho^2\frac{\sin\left(3\frac{\pi}{2}\frac{T_s}{T_e}\right)}{\sin\frac{\pi}{2}}+\cdots\cdots+\rho^{m-1}\frac{\sin\left[(2m-3)\frac{\pi}{2}\frac{T_s}{T_e}\right]}{\sin\left[(2m-3)\frac{\pi}{2}\right]}+\rho^{m}\frac{\sin\left[(2m-1)\frac{\pi}{2}\frac{T_s}{T_e}\right]}{\sin\left[(2m-1)\frac{\pi}{2}\right]}\cdots\cdots\cdots (16)$$

where: m = number of impulses of ground motion given by the relation:

$$m=2\frac{T_0}{T_e} \cdots\cdots\cdots (17)$$

From equation (16) the table III is set out, for $T_e = 1{,}0^{sec}$ (soft ground).
If in equation (16) we introduce:

$T_e = 0{,}3^{sec}$ (for hard ground)
or $T_e = 0{,}6^{sec}$ (for medium stiff ground),

we can set out two tables which with table III cover all needs of the aseismic design.

3.- Distribution of the total lateral seismic force along the height of the structure.

By shear deflection (Fig.3) of multistory structures the distribution of the total lateral shear R, given by the relation :

$$R = \sum_{1}^{n} P_k = cW = \lambda \mu c_o W \ldots\ldots \quad (18)$$

is obtained by the equation :

$$P_k = \frac{cW}{\sum\limits_{k=1}^{n} (W_k \sqrt{h_k})} W_k \sqrt{h_k} \ldots\ldots \quad (19)$$

which assumes a parabolicdeflection of the structure.

When the deflection is due to shear and to bending (stacks) the distribution of the total lateral force along the height of the structure, can be obtained by the equation :

$$P_k = \frac{cW}{\sum\limits_{k=1}^{n} (W_k h_k)} W_k h_k \ldots\ldots \quad (20)$$

which assumes a linear deflection of the structure. (Fig.4)

The lateral shear Q on the Kth floor (Fig. 3 and 4) is given by the relation:

$$Q_k = \sum_{k}^{n} P \ldots\ldots \quad (21)$$

4.- Distribution of the shear Q on the columns and walls.

For the distribution of the shear Q on the seismic load bearing vertical members, it is required to determine its point of application. Because the coordinates of the point of application of the shear Q are functions of the coordinates of the pole of the horizontal rotational oscillations of the slab-discs of the floor under consideration, whose position depends on the position of the centre of the seismic forces, the coordinates of the pole is determined by the method of successive approximations as a function of the coordinates of the centre of seismic forces by the following equations which are valid in the case that Q_x is parallel to the axis of Ox (Fig.5) :

$$Y_{cx} = \frac{\Phi_x \sum Y_k D_{kx} - Q_x}{\Phi_x \sum D_{kx}} \quad \ldots\ldots\ldots (22)$$

$$\Phi_x = \frac{Q_x (Y_E - Y_I)}{(Y_E - Y_I)(\sum Y_k D_{kx} - Y_{cx} \sum D_{kx}) + \sum x_k^2 D_{ky} - x_I \sum x_k D_{ky} + \sum D_{\omega k}} \quad \ldots (23)$$

$$Y_E = \frac{J_x - Y_{cx} S_x}{S_x - Y_{cx} W_x} \quad \ldots\ldots\ldots\ldots (24)$$

$$Y_I = \frac{\sum Y_k D_{kx}}{\sum D_{kx}} \quad \ldots\ldots\ldots (25)$$

$$x_I = x_{cx} = \frac{\sum x_k D_{ky}}{\sum D_{ky}} \quad \ldots\ldots (26)$$

where: D_{kx}, D_{ky} = the indices of resistance in bending and shear of the columns and walls correspondingly in the direction of axes Ox and Oy

x_k, Y_k = the coordinates of the centre of section of the column or wall k

Φ_x = the angle of maximum horizontal rotational oscillation

x_E, Y_E = the coordinates of the centre E of shear Q (seismic forces)

x_I, Y_I = the coordinates of the elastic centre i.e., the centre of gravity of the indices D_{ky}, D_{kx}

$D_{\omega k}$ = the indices of resistance in torsion.

By the first step of calculation we assume that the centre E (Fig.5) of seismic forces, coincide with the centre of gravity G

The distribution of the resultant Q in the columns and walls is carried out by the following relation:

$$P_{kx} = \Phi_x (Y_k - Y_{cx}) D_{kx} \quad \ldots\ldots\ldots (27)$$

$$P_{ky} = -\Phi_x (x_k - x_{cx}) D_{ky} \quad \ldots\ldots\ldots (28)$$

where: P_{kx}, P_{ky} = the components of the seismicforce per column or wall corespondingly parallel to the axes

J_x, J_y = the moment of inertia of the static loads parallel to the axes

S_x, S_y = the static moment of static loads parallel to the axes.

W_x = the static loads.

Obviously, if the pole of rotational oscillations is taken as origin, the equation (24) becomes

$$Y_E = \frac{J_x}{S_x} \quad \ldots\ldots\ldots (29)$$

Omitting the influence of the index of torsional resistance, the ordinate

Y_{Cx} of the pole of rotation is given by :

$$Y_{Cx} = \frac{\sum Y_k^2 D_{kx} - Y_E \sum Y_k D_{kx}}{\sum Y_k D_{kx} - Y_E \sum D_{kx}} \quad \ldots\ldots\ldots (30)$$

5.- Indices of resistance and static calculation.

It is aimed to obtain the indices of resistance and the static calculation of a multistory frame by breaking it up into equivalent one-column multistory frames.

The indices of resistance D are calculated from the equations :

$$D = \frac{1}{\delta} \quad \ldots\ldots\ldots\ldots (31)$$

$$\delta = \left(\frac{\gamma h^3}{3J} + \frac{3h}{F}\right)\frac{1}{E} + \beta h \quad \ldots\ldots\ldots (32)$$

where: $\gamma =$ coefficient of relative rotation of the tangents at the upper and lower ends of the columns or walls

$h =$ the height of the column or wall

$J =$ the moment of inertia of the section

$F =$ the area of the section

$\beta =$ the angle of elastic rotation of the tangent at the lower end (Fig.6).

The value of γ varies between 0,25, when the tangents remain parallel, after the displacements, and 1,0 in the case of free upper end.

The cooefficient γ of the column AB is calculated by the relations :

$$\gamma = \frac{3\alpha - 1}{2} \quad \ldots\ldots\ldots\ldots (33)$$

$$\alpha = \frac{\Phi_{AB}^{B}}{\Phi_{AB}^{A} + \Phi_{AB}^{B}} \quad \ldots\ldots\ldots (34)$$

$$\Phi_{AB}^{B} = \frac{k_B^r + k_B^l + k_B^u}{k_B^r + k_B^l + k_B^u + k_{AB}} \quad \ldots\ldots (35)$$

$$\Phi_{AB}^{A} = \frac{k_A^r + k_A^l + k_A^o}{k_A^r + k_A^l + k_A^o + k_{AB}} \quad \ldots\ldots (36)$$

where: k_B, k_A = the stiffness value of the right (r), left (l), lower (u) and upper (o) members of the nodes B and A (Fig.6)

Φ_{AB}^{B}, Φ_{AB}^{A} = the degrees of fixity of the column or wall AB, correspondingly of the lower and upper ends B and A.-

The values of the degree of fixity vary from unity (complete fixity) to zero (free end).

For the calculation of degree of fixity Φ the lengths l' of the equivalent spans are introduced.

In the case of beams with undisplaced supports the equivalent span is given by the relation :

$$l' = l\left(1 - \frac{\Phi}{4}\right) \quad \ldots\ldots\ldots (37)$$

and the term stiffness value :

$$k = \frac{J}{l'} \quad \ldots\ldots\ldots (38)$$

In the case of <u>free horizontal displacements</u> of the ends of the columns the stiffness values become :

a) In the case of loading of the storey AB of the column by a horizontal force applied at the upper end A, the stiffness value of AB is given by the relation : (Fig 6)

$$k_{AB} = \frac{J}{1{,}5\,h_{AB}}, \qquad \text{i.e.} \quad l' = 1{,}5\,h_{AB} \quad \ldots\ldots\ldots (39)$$

b) In the case of loading of the storey AB of the column by a moment M constant along its height AB (Fig.7), the stiffness value AB of column AB is given by the relation :

$$k_{AB} = \frac{J}{3h_{AB}} \qquad \text{i.e.} \quad l' = 3\,h_{AB} \quad \ldots\ldots\ldots (40)$$

c) In the case of loading of the floor AB of the column, by a moment M constant along the height AB (Fig.7), the stiffness value of BC of the underlying floor BC of the column is given by the relation:

$$k_{BC} = \frac{\Phi_C^{o} J_{BC}}{3\,h_{BC}}, \qquad \text{i.e.} \quad l' = \frac{3\,h_{BS}}{\Phi_C^{o}} \quad \ldots\ldots (41)$$

where: Φ_C^{o} = the degree of fixity of the column, (floor BC), at the lower end C.

For the complete separation of a multistory frame-column from the multistory frame building and to take into account the transmitted actions from the remaining building frame, the spans of the beams are replaced by the following equivalent freely supported spans L (Fig.8) :

$$L_A = \frac{\varphi_A^{u}\,l_A' + \varphi_B^{u}\,l_B''}{\varphi_A^{u} + \varphi_B^{u}} \quad \ldots\ldots\ldots (42)$$

where: φ^u_A, φ^u_B = distribution factors of the moments.

The above determination of indices of resistance D and correspondingly the static calculation on the assumption of free horizontal displacement i.e., without developpment of horizontal reactions at the columns by the slab disc, is valid when the sections of the columns or walls remain the same or change proportionally from floor to floor, for-all the colums or walls. In a different case the calculation is corrected by the introduction of the term "degree of prevention" ζ. (Fig.9)

$$\zeta = \frac{\Delta_{BC}}{\Delta_{AB}} = \frac{D_{BC}/\Sigma D_{BC}}{D_{AB}/\Sigma D_{AB}} \qquad \ldots\ldots\ldots\ldots (43)$$

by means of the relation :

$$M^o_C = -M^u_B \frac{\Phi^o_c}{2} + \left(1 + \frac{\Phi^o_c}{2}\right) M^u_B \zeta$$

For the calculation of the degree of prevention we may accept with sufficient accuracy the indices of resistance as calculated by any approximate formula.-

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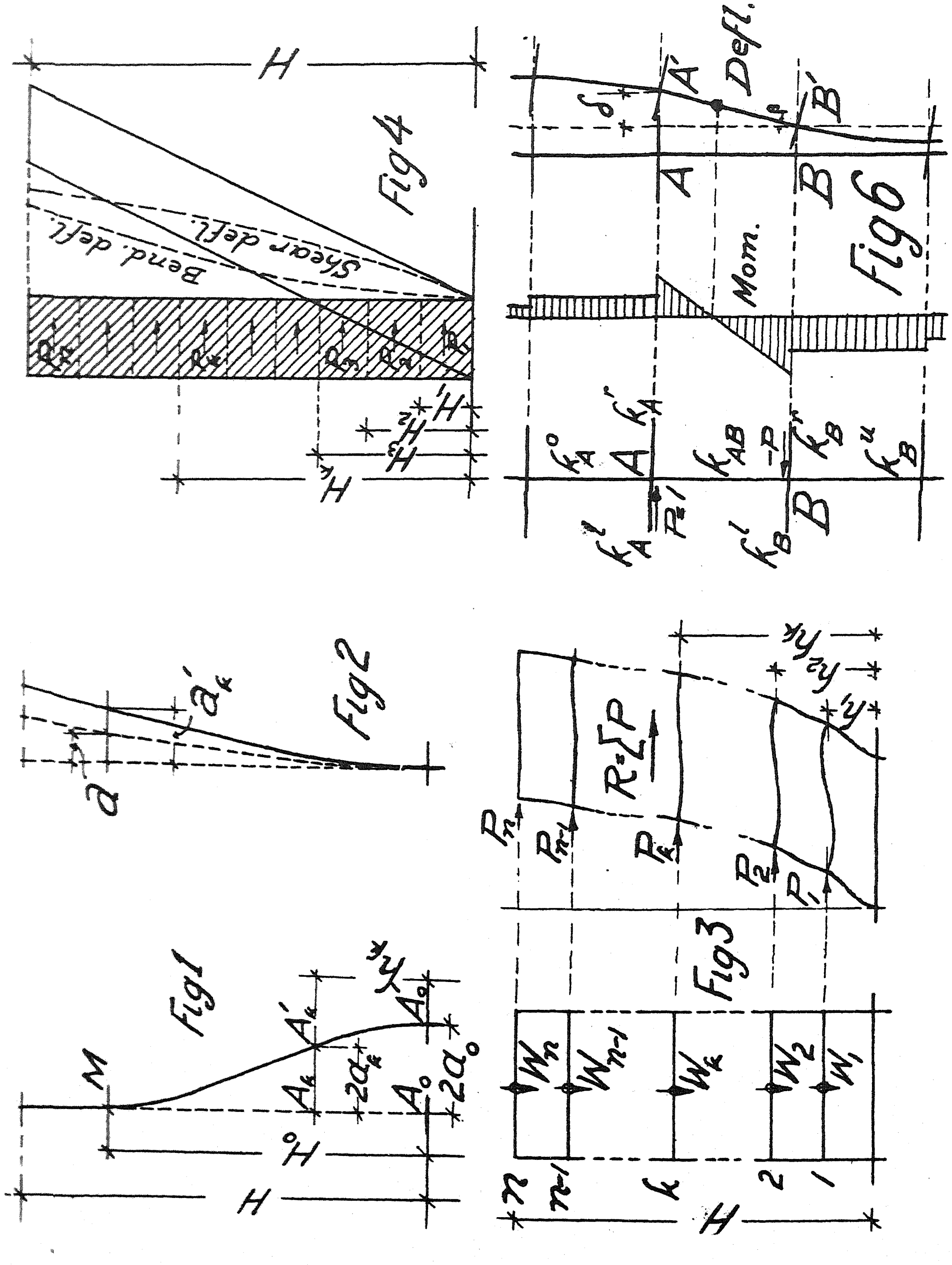
Fig4
Bend. defl.
Shear defl.
Fig6
Mom.
Defl.
Fig1
Fig2
Fig3
R=ΣP

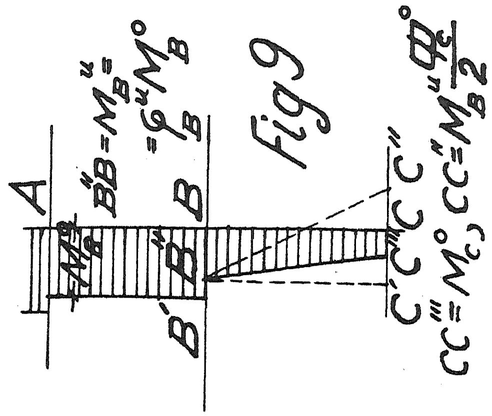
Fig 9

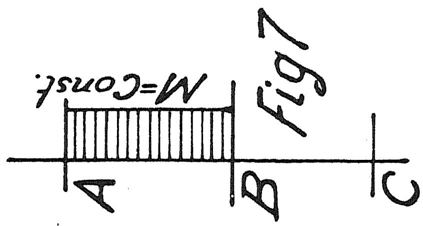

Fig 7

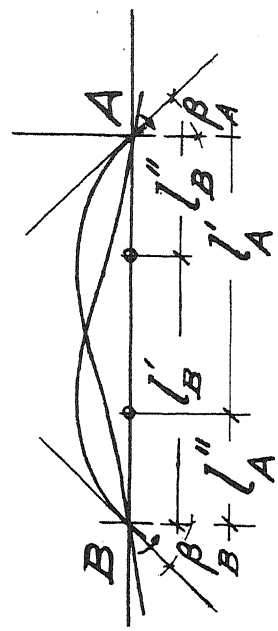

Fig 8

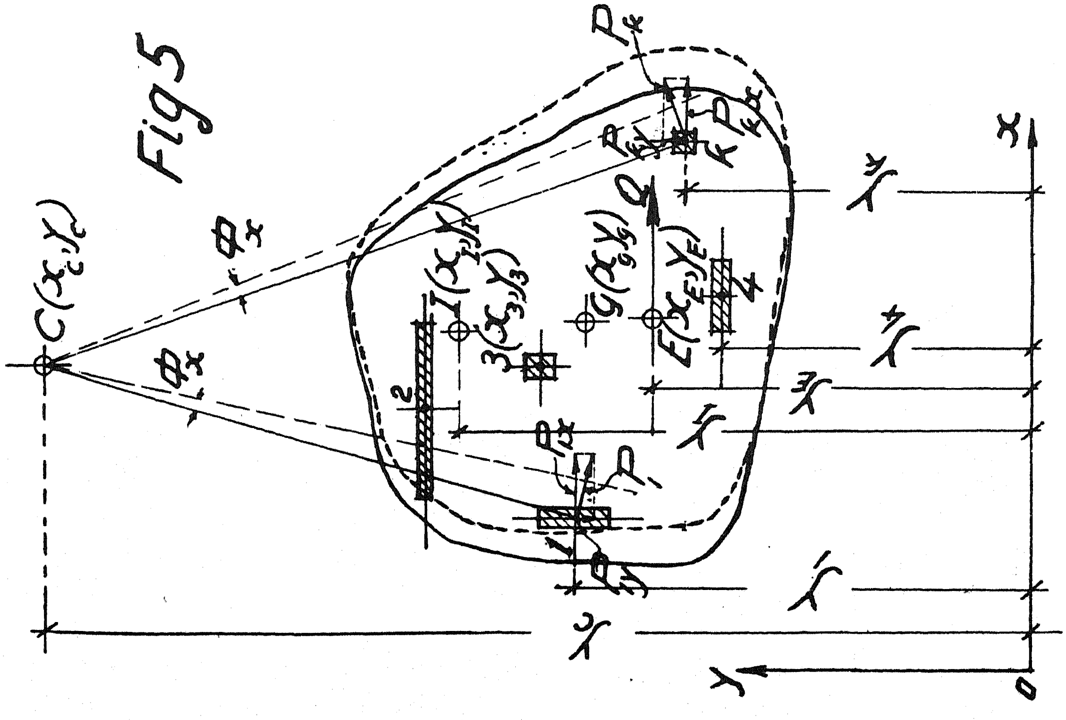
Fig 5

Table I

Values of rigidity factor λ_p for equal concentrated static loads and storeys of constant height.

$\frac{T_s}{T_e}$	λ_p										
	1	2	3	4	5	6	7	8	9	10	= n
0,1	0,95	0,96	0,97	0,98							
0,2	0,82	0,89	0,91	0,92	0,92	0,93					
0,3	0,63	0,77	0,79	0,82	0,83	0,84	0,84	0,85	0,85		
0,4	0,43	0,63	0,67	0,71	0,73	0,74	0,75	0,75	0,76	0,76	
0,5	0,25	0,49	0,56	0,60	0,62	0,63	0,64	0,65	0,65	0,66	
0,6	0,12	0,38	0,46	0,50	0,52	0,53	0,54	0,55	0,56	0,57	
0,7		0,29	0,38	0,42	0,44	0,45	0,46	0,47	0,48	0,49	
0,8		0,22	0,31	0,35	0,37	0,39	0,40	0,41	0,42	0,43	
0,9		0,17	0,25	0,29	0,32	0,34	0,36	0,37	0,38	0,39	
1,0		0,13	0,21	0,25	0,28	0,30	0,31	0,32	0,33	0,34	
1,2			0,15	0,20	0,25	0,26	0,26	0,27	0,27	0,27	
1,4				0,15	0,21	0,22	0,22	0,23	0,23	0,23	
1,6					0,17	0,18	0,18	0,19	0,19	0,19	
1,8					0,11	0,12	0,13	0,14	0,15	0,16	
2,0						0,11	0,12	0,13	0,14	0,14	

T_s = natural period of the structure
T_e = period of ground motion
n = number of storeys

Table II

Values of rigidity factor λ_p for continuous loads
T_s = natural period of the structure
T_e = period of the ground motion

T_s/T_e	λ_p	T_s/T_e	λ_p	T_s/T_e	λ_p	T_s/T_e	λ_p	T_s/T_e	λ_p
0,1	0,95	0,6	0,60	1,1	0,34	1,6	0,23	2,5	0,15
0,2	0,92	0,7	0,52	1,2	0,31	1,7	0,22	3,0	0,12
0,3	0,85	0,8	0,46	1,3	0,29	1,8	0,21	3,5	0,11
0,4	0,77	0,9	0,41	1,4	0,27	1,9	0,20	4,0	0,10
0,5	0,68	1,0	0,37	1,5	0,25	2,0	0,19		

Table III

Values of resonance factor μ
Period of the ground motion $T_e = 1,0$ sec.
Duration of maximum seismic intensity $T_0 = 6$ sec.
Number of impulses $m = 2\frac{T_0}{T_e} = 12$

T_s	μ	T_s	μ	T_s	μ	T_s	μ	T_s	μ
0,1	1,02	0,5	1,47	0,9	2,72	1,3	1,88	1,7	1,40
0,2	1,11	0,6	1,58	1,0	5,05	1,4	1,70	1,8	1,28
0,3	1,23	0,7	1,76	1,1	2,97	1,5	1,62	1,9	1,14
0,4	1,35	0,8	2,10	1,2	2,23	1,6	1,52	2,0	1,00

T_s = natural period of the structure
T_e = period of the ground motion
$[T_s]$ = sec.

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Supplement to the paper under the title

" accuracy and simplification of the seismic design

Equation (3) and (4) of the above mentioned paper,must be substituted by the following relation (3a) which is obtained from the ratio of the equations which express respectively the total lateral seismic force of the elastic structure and that of the stereostatic (rigid) structure

$$\lambda = \frac{C}{Co} = \frac{\sum_{K=1}^{n} a_K W_K}{a_o W} \dots\dots\dots (3a)$$

Relations (11) and (12) are respectively substituted by the following equations (11a) and (12a).

$$\lambda_p = \frac{\sum_{K=1}^{n} a_K}{n a_o} = \frac{1}{2n}\Big[n + \sin\frac{\pi}{2}\Big(1 - \frac{2T_s}{nT_e}\Big) + $$

$$+ \sin\frac{\pi}{2}\Big(1 - \frac{4T_s}{nT_e}\Big) + \dots\dots + \sin\frac{\pi}{2}\Big(1 - 2\frac{T_s}{T_e}\Big)\Big] \dots\dots (11a)$$

$$\lambda_p = \frac{\int_0^H a_K\, dh}{H a_o} = \frac{1}{2H}\int_0^H \Big[1 + \sin\frac{\pi}{2}\Big(1 - \frac{2h_K}{H}\,\frac{T_s}{T_e}\Big)\Big] dh \dots\dots (12a)$$

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