DESIGN SEISMIC FORCES FOR REINFORCED CONCRETE BUILDINGS

by YUKIO OTSUKI

INTRODUCTION

The Japanese Code requirements for the design seismic force specifies the use of a seismic coefficient, K, calculated as a percentage of the gravitational acceleration, g. The Code requirements were contained in an article by the writer presented to the First World Conference on Earthquake Engineering held at Berkley, California in 1956 (1).

This form of specification is simple to apply to actual design calculations and gives reasonable results for average building structures as long as the buildings are squatty and fairly rigid. But the writer feels somewhat sceptical about the validity of applying the same requirements to non-squatty or slender buildings, the vibrational behaviors of which are obviously so different.

The design seismic force for a particular building structure should be determined from the vibrational characteristics of that building. A remarkable achievement in utilizing such building characteristics is the recent San Francisco Building Code requirements in which the design base shear coefficient for a building structure is given as a function of the structure's natural period of vibration. The form of representing this force coefficient is as percentages of gravity acceleration similar to other existing Code requirements.

Up-to-this time, therefore, whether or not the natural period of vibration of the structure is taken into account, all Code requirements specify the statical equivalent coefficient which is expected to give appropriate values for the design storey shears. Sometimes this procedure results in excessive overturning moments for the lower portions of the building, particularly as the height of the structure increases. Some special procedures become necessary to handle such large moments.

A fact that earthquake ground motion is not steady indicates that a building structure will never vibrate accordingly during an earthquake. Therefore, the most adverse conditions will never occur simultaneously throughout the building structure. If the coefficients are so specified as to insure sufficient strength for the upper portions of a building, then the lower portions of the same building certainly will be overdesigned, and vice versa. Such contradicting tendencies can be eliminated by the introduction of some localized design conditions in addition to overall requirements.

Further, current design methods in which a building structure is assumed to deform in the shearing mode during an earthquake have never been proved inadequate. Supposing therefore, that a certain storey of a building structure is deformed in the shearing mode, it is obvious that the force

B.Eng., M.S., Chief Engineer of Atomic Power Division, The Shimizu Construction Company, Ltd.

induced in this storey will be equal to the product of the amount of storey deformation and the storey rigidity.

Therefore, when the vibrational pattern of a building structure can be determined with reasonable accuracy, it is possible to calculate all storey deformations, and hence the storey shears if all storey rigidities are known.

This becomes the basic philosophy of this proposal. The important quantities in this proposal are:

Natural period of the structure
Natural period of the site ground, and
Design velocity of the structural vibration.

GROUND MOTION AND STRUCTURAL RESPONSE

In order to cope with the problem dynamically, it is necessary to define the ground motion during an earthquake. As described previously, the ground motion during an earthquake is not stationary. The best way to define the effect of the ground motion is to define the structural response instead of the ground motion itself. Then, considering the ground, or subsoil, to be an elastic medium, a stratum or strata has its own natural period of vibration. Accordingly, a structure which is built on this soil will be excited in its natural modes of vibration by the earthquake ground motion which has been modulated by the vibrational nature of the site strata.

This is equivalent to what Messers. Martel, Housner and Alford maintained in the discussion of a relatively small slender tower on top of a building (2). Mr. H. Tajimi has made an extensive study on this problem (3). He has concluded that tendencies coinciding with Housner's results (4) occur only if the abscissa of Fig. 6 of Reference (4) is changed into the period ratio of the structure to the ground, Ts/Tg, in place of the simple structural period, Ts.

The response velocity certainly will vary with the structural damping. However, for practising engineers, it is a difficult problem to assume an appropriate value of damping for any structure under immediate consideration. However, if the scope of application is limited to cover only reinforced concrete structures, with or without encased fabricated structural steel frames, the range of damping will be limited to within a comparatively narrow range that can be deducted by formula.

It is well known that shear walls placed in a building structure are very effective in minimizing the structural deformation under earthquake loading. The shear walls will be the first elements to become overstressed when the building vibrations become violent. They then dissipate energy and prevent the rest of the structure from becoming greatly overstressed. Further, it has been observed that when shear walls have been provided in sufficient numbers in a building structure, the amount of shear deformation is reduced to a remarkable extent as compared to the integral-unit movements such as rocking and swaying displacements. Both of these shear wall factors mean an appreciable increase of damping.

Soil conditions must be carefully considered also; past experience shows that the intensity of the earthquake shock is always greater on soft aluvium than on harder soils.

Mr. H. Kobayashi's studies indicate that the taller the structure, the greater will be the velocity of vibration. The empirical relationship between the height of the structure and the vibrational velocity can be expressed approximately as the two thirds power of the height.

Considering all the contributing factors expressed herein, it is proposed that the design vibrational velocity be calculated by use of Eq. (1).

NATURAL PERIOD OF A BUILDING STRUCTURE

In order to define the vibrational velocity of a building structure, the correct assumption of the natural period of the structure is of paramount importance. The requirements of the San Francisco Code gives an underestimation of the period giving a margin of safety. In this proposal, underestimation of the period will not result in such a margin of safety. Therefore, a more accurate formula for the period estimate is a necessity.

Accumulated vibration test data (6) clearly indicate that the majority of the existing buildings tested rock and sway, and behave differently from a fixed base condition. In the case of a shear beam, the natural period of the beam is theoretically proportional to the product of the square root of the tip deflection under the force of horizontal gravity and the length of the beam. It is to be emphasized that the deflection of the beam tip consists of two components: the deformation of the beam proper and the displacement due to the deformation of the support. Therefore, it is a suggestion of this proposal that the total displacement of the building top be divided into two parts: the elastic deformation displacement of the superstructure and the displacement due to the deformation of the substructure.

Another factor to consider is the fact that a very flat structure such as a machine foundation where the height, H, is practically zero, still has its own natural period, which is finite.

Considering all the pertinent factors, a formula something like Eq.(2) is developed.

In the determination of the magnitude of factors which represent structural deformation and subsoil deformation, it is best to avoid becoming too theoretical such as in the assumption of the rigidities of structural members or the spring constant of the soil. Eq. (2) is absolutely empirical, therefore, but never the less it can be expected to predict the natural period of a building within about twenty percent; which considering the possibilities of error is quite close.

The natural period so predicted is the natural period of small amplitude, or the initial period. It has often been noted that when the amplitude becomes large as in a violent earthquake, the period of a structure will be elongated appreciably. This is probably due to localized damage of both superstructure and substructure. However, until such local damage does occur in a structural system, the structure will certainly be subjected to a force system consistent with the initial state.

In case where the period becomes elongated because of such local damages, such elongation will be inversely proportional to the square root of the rigidity. As long as the period ratio of the structure to the ground remains less than unity, the design velocity is proportional to the natural period, again inversely proportional to the square root of the rigidity. This means, the overall deflection, hence the storey deformation is inversely proportional to the rigidity. Now, since the storey shear is given as product of the rigidity and the deformation, the storey shear will remain unchanged as long as the natural period of the ground remains constant. For the range where the structural period is longer than the ground period, the design velocity remains constant, and therefore the elongation of the structural period reduces the design shear.

For reasons stated previously, it is proposed to use the initial period of a structure as the basis of the design analysis.

It may be pointed out that the natural period of the ground also will become longer in a violent earthquake. However, the elongation of the ground period always tends to the favor of the situation.

In this proposal, it becomes necessary to estimate the period of the second mode vibration. It must be born in mind that although the end conditions have appreciable influence on the fundamental period, they have but little influence on the second mode. Therefore, the second mode period can be computed using Eq. (2.6).

NATURAL PERIOD OF THE GROUND

Since the design velocity is given as a function of the period ratio of the structure to the ground, it is equally important to know about the predominant period of the ground. However, as mentioned previously, the accuracy of the ground period will not matter as seriously as in the case of the structural period. The elongation of the ground period tends to decrease the design shear, and therefore, it is proposed to take the initial period as the basis of the design analysis. Such predominant period of the ground can be determined from the frequency distribution of small ground tremors (7). Where such data is unavailable, provisions are made so that one can choose an appropriate standard value according to the formation of the subsoil strata. It is needless to say that the more accurately the ground period is estimated, the more reasonable the design shear that can be obtained. In any cases, the ground period should not be assumed as excessively long.

DEFORMATION PATTERN OF A BUILDING STRUCTURE

Since the natural periods of a building structure and the subsoil are known, it is possible to define the design velocity by assuming the quantity of wall elements to be provided in the building refering to the structural plans. The next step then is to find out what deflection pattern of the building structure is both in the fundamental and the second modes.

This can be done only if the rigidity distribution along the height becomes known. This process may be quickly done by use of a computor. Even if such facilities are not available, the modified Stodola method (8) and Holzer method (9) will give both a quick and good approximation for design purposes for the fundamental and the second modes, respectively.

The next step then is to calculate each storey deformation. For the second mode, the storey deformation can be obtained readily from the deflection curve by subtracting the deflection of the certain storey in question from that of the upper and adjacent floor level. But in the case of the fundamental mode, since it contains also the bodily displacement due to the soil deformation, it is nacessary to calculate the pure structural deformation by subtracting that amount from the apparent deformation. When all of these procedures are complete, a table can be set up in which all of these storey deformations are listed.

DESIGN SHEARING FORCE

When all storey deformations are calculated, the last step is then to calculate the design shear for each storey. It is done simply by multiplying each storey rigidity by the corresponding storey deformation.

Now, there are two components for each storey deformation, one for the fundamental mode and the other for the second mode. The former is to be used to calculate a force system with the equilibrium of the complete building structure while the last is for local adjustment disregarding the equilibrium as a whole. This adjustment is applied in such a manner that the storey shear calculated from the fundamental mode is multiplied by the ratio of the sum of the absolute values of the fundamental and second mode storey deformations to the fundamental storey deformation. The adjustment is to be applied storey by storey, and the adjusted shear should not be considered as exerted simultaneously for all storeys.

A question may arise then as to what the appropriate value is for the rigidities of storeys to be used in this step. The answer is the rigidity which is consistent with the design natural period which is given by Eq. (2) of this paper.

In the course of calculating the vibrational mode, it is necessary to assume storey rigidities, or at least rigidity ratios for all storeys. But since this assumption is made quite independent of Eq. (2), the computed value based on those rigidities will not coinside with the predicted period. Therefore, correction as shown by Eq. (6) should be applied.

PROPOSED METHOD FOR DETERMINATION OF EARTHQUAKE FORCES

Scope of Application: The following provisions are applicable to framed reinforced concrete building structures with or without encased fabricated structural steel which are taller than 16 meters above the ground; or to such a building the structural nature of which is comparatively different from the conventional structure and considered inappropriate for application of the current Code provisions.

Design Velocity: The design velocity is to be determined from the following formula:

$$V = C_1 \times C_2 \times C_3 \times V_0 \times \left(\frac{H}{30}\right)^{\frac{2}{3}}, \tag{1}$$

where C_1 is a coefficient due to locality of the site, and same as specified in the present Code (5).

C_2 is a coefficient which is determined as follows;

Let T_s be the natural period of a building and T_g be the natural period of the ground, both in seconds,

a) For $\frac{T_s}{T_g} \leqq 1.0$, $C_2 = \frac{T_s}{T_g}$, (1.1)

b) For $\frac{T_s}{T_g} > 1.0$, $C_2 = 1.0$. (1.2)

c) For soft ground, the natural period of which is longer than 0.6 seconds, or if it is difficult to define its natural period, then

for $T_s \geqq 0.5$ seconds, $C_2 = 1.0$, and

for $T_s < 0.5$ seconds, $C_2 = 2\,T_s$. (1.3)

d) When the soil is definitely of hard formation, but is difficult to define its natural period, Tg can be assumed equal to 0.25 seconds.

C_3 is a coefficient which is a function of the wall quantity and is given as follows:

$$C_3 = 1.3 - \frac{w}{25}, \tag{1.4}$$

where w is the wall quantity per unit floor area in terms of cm / m², in respective directions. The value for w shall be the minimum for all storeys in the respective directions.

V_0 is the basic velocity which is given as follows; according to the nature of the subsoil formation:

a) For tertiary or older 12 cm/s

b) For deluvial formation or alluvial formation, the thickness of which is not deeper than 5 meters 15 cm/s

c) For alluvial formation the thickness of which is equal to, or deeper than 5 meters 20 cm/s

Natural Period of Building Structure: The natural period of a building structure shall be computed by the following formula:

$$T_{S1} = 0.007 (H + 15) \sqrt{f_1 + f_2} \tag{2}$$

where

$$H = H_o + \frac{1}{3} H_u \quad (m) \tag{2.1}$$

H_o = Building height above the ground in meters.
H_u = Depth of basement in meters.

$$f_1 = 0.07 (20 - w) \tag{2.2}$$

$$f_2 = K_1 \times K_2 \times \frac{1 + \lambda^2}{\lambda}, \tag{2.3}$$

$$K_1 = (15 + w) H / 2000 \tag{2.4}$$
for direct foundation on base rock, and

$$K_1 = (25 + w) H / 2000 \tag{2.5}$$
for foundations other than above.

K_2 = Coefficient due to type of substructures given as follows:

Direct foundation on base rock 1.0
Bearing piles or piers

	Length in meters			
	0 - 5	5 - 10	10 - 15	15 and up
Piers	1.0	1.1	1.2	1.5
Piles	1.5	1.5	1.8	2.5

Friction piles 2.5
Direct foundation on other than rock 3.0 - F/25,
where F = Allowable bearing power in t/m^2.

λ = H/D, where D is the depth of building measured along the direction under consideration, in meters. Where H/D is less than unity, λ is assumed to be equal to unity.

$$T_{S2} = 0.4 T'_{S1}. \tag{2.6}$$

where $$T'_{S1} = 0.007 (H + 15) \sqrt{f_1}. \tag{2.7}$$

Observed periods for various buildings are shown in Table 2 in comparizon with the computed periods by Eq. (2).

Natural Period of Ground: The natural period of the site ground is to be determined by the frequency distribution data obtained from the microtremor observations at the site. Where such measurement is not feasible, the following figures can be used;

Hard rock	0.15 seconds
Soft rock	0.25 seconds
Deluvial formation	0.40 seconds
Alluvial formation	0.60 seconds

Determination of Design Shear:

a. Design velocities V_1 and V_2 and natural periods T_{s1} and T_{s2} are to be calculated for the fundamental and second modes of vibration by Eqs. (1) and (2).

b. Deflection patterns for the above mentioned modes, X_1 and X_2, are to be determined. The following constants then can be determined.

$$\beta_1 = \frac{\sum W_i X_{1i}}{\sum W_i X_{1i}^2}, \tag{3.1}$$

$$\beta_2 = 1 - \beta_1 . \tag{3.2}$$

c. Top deflections for the respective modes are determined as follows:

$$y_{1T} = \frac{V_1 T_{s1}}{2\pi} \cdot \beta_1 \cdot \frac{f_1}{f_1 + f_2}, \tag{4.1}$$

$$y_{2T} = \frac{V_2 T_{s2}}{2\pi} \cdot \beta_2 . \tag{4.2}$$

The complete deflections are then calculated for the respective modes as follows:

$$y_1 = y_{1T} X_1, \quad \text{and} \tag{5.1}$$

$$y_2 = y_{2T} X_2 \tag{5.2}$$

d. All storey deformations are to be calculated for respective modes.

e. Storey shears are to be determined by multiplying each storey rigidity by the storey deformation obtained. The rigidity figures as herein used must be consistent with the natural period used in (c).

Let R be an assumed rigidity and R' be the corrected rigidity which is consistent with the natural period used in (c). Denoting the computed period basing on R by T_m, and the period as given by Eq. (2.7) by T'_{s1}, then it follows:

$$R' = \left(\frac{T_m}{T'_{s1}}\right)^2 \cdot R \tag{6}$$

f. The fundamental mode shear is to be used for calculation of the equilibrium state of the building as a whole, while the loçal shear, which is the sum of the absolute values of the fundamental and second mode shears, is to be applied locally to each corresponding storey.

NUMERICAL EXAMPLE AND COMPARISONS

A sample calculation in accordance with this proposal has been carried out and the results are compared with current Code values, and additionally with triangular distribution in which the base shear coefficient is adjusted to be equal to 0.2.

It is observed, as shown in Fig. 1, that this proposal requires more shear for the upper part of the building, but less for the lower part than has been required by current Code. The general tendency of the design storey shear distribution is similar to one calculated from triangular distribution. However, a comparative difference can be seen between these two for the overturning moment distribution. Such tendency is expected to become remarkable as the building height increases. The shear distribution pattern is generally similar to the results in References (10) and (11).

Because of the factor C_2 , as the building height becomes taller and the natural period of the fundamental mode becomes longer than the period of the ground, the ratio C_{22}/C_{21} will increase, i.e., the contribution of the second mode is expected to become pronounced.

CONCLUDING REMARKS

This proposal was drafted initially by the author and has been discussed at the meetings of the Vibrational Committee of the Architectural Institute of Japan. Messers. K. Kanai, T. Hisada, K.Nakagawa, H. Kobayashi, N. Ando, T. Tajime, H. Tajimi, Y. Osawa, Y. Murata, T. Iwashita and T. Fukuchi participated in the discussions. To date, the committee has not been able to reach a final conclusion. However, the author believes that the following three points have been generally accepted by the committee members:

1) The design shear for a building structure is to be determined according to its vibrational behaviors, such as the natural periods of both the building and the ground.

2) The introduction of local treatment presents a good solution for the present problem.

3) Eq. (2) for the prediction of the natural period of a building gives a good approximation.

One of the main points of those discussed at the meetings was that the proposal seemed to be a little too complicated to be uniformly applied as a Code requirement. Another point was that the requirements in the proposal were not uniform; some parts are too theoretical as compared with some other more practical parts. Some members prefer to give a seismic coefficient form to the present deformation and rigidity proposal.

These discussions are very worthy in that they represent the thoughts of men fully qualified. The scope of application is therefore limited to tall or otherwise unconventional building which are best designed only by well qualified structural engineers.

The non-uniformity of the requirements can not be avoided because of the nature of the proposal, unless of course all emperical requirements can be replaced by those more theoretical and competently handled. It is to be stressed that this proposal tries to specify requirements in simplest form as long as is possible without such simplification resulting in the invalidation of the problem.

It would not be impossible to specify the design seismic factor in terms of seismic coefficients, with a certain adjustment for local shear due to the higher modes, if a simple formula can be established to give appropriate values. The author feels, however, that a representation such as seismic coefficients would not be effective for any building structure in which the rigidity distribution along the height is unconventional.

This proposal requires more time for design shear calculations than engineers are required to expend when following the simpler method delineated by most present Codes. The time required for the calculations following the method outlined herein will be somewhere between two to six hours according to experience. This is, of course, only a fraction of the time required for the complete structural analysis. It is believed that the added time spent compensates in that it gives the engineer a more complete grasp of the general structural behavior pattern of the building during an earthquake.

The author hopes that this proposal will help to provide a design method satisfactory enough to alleviate the present Japanese Code restrictions against buildings for the taller buildings of this type of construction, although the Code restriction is not for the structural reasons. It is hoped also that it may help those persons in other seismic countries who are forced with the same problem.

ACKNOWLEDGEMENT

The Author wishes to express his deep appreciation to the members of the Vibration Committee of the Architectural Institute of Japan mentioned in this paper for their cooperation and advice and especially for their generosity in allowing the use of their finding in this paper.

BIBLIOGRAPHY

1. "Development of Earthquake Resistant Building Construction in Japan" by Y. Otsuki, Proc. WCEE, 1956, pp. 16-1 - 16-17.

2. "Discussion of Lateral Forces of Earthquake and Wind" (Published in April, 1951) by G. W. Housner, R. R. Martel, and J. L. Alford, May, 1952.

3. "Basic Theories on Aseismic Design of Structures" by H. Tajimi, Report of The Institute of Industrial Science, University of Tokyo, No.4,Vol. 8, March, 1959.

4. "Limit Design of Structures to Resist Earthquakes" by G. W. Housner, Proc. WCEE, 1956, Fig. 6, p. 5-13.

5. Same as Ref. (1), p. 16-17.

6. By K. Nakagawa, Unpublished.

7. For example, "Seismic Characteristics of Ground" by K. Kanai, R. Takahashi, and H. Kawasumi, Proc. WCEE, 1956, pp. 31-1 - 31-16.

8. See Appendix to this paper.

9. For example, "Engineering Vibrations" by L. S. Jacobsen and R. S. Ayre, McGraw Hill Book Co., 1958, pp. 407-416.

10. "Numerical Analysis of Earthquake Response of a Tall Building" by T.P. Tung and N. M. Newmark, Bulletin SSA, No.4, Vol. 45, October, 1955, pp 269-278.

11. "On the Importance of Higher Modes of Vibration in the Earthquake Response of a Tall Building" by R. W. Clough, Bulletin SSA, No. 4, Vol. 45, October, 1955, pp. 289-301.

NOMENCLATURE

β_1	Coefficient of excitation, fundamental mode	
β_2	Coefficient of excitation, second mode	
C_1	Coefficient due to locality of the site	
C_2	Coefficient due to period ratio of building to the ground	
C_3	Coefficient due to wall quantity	
D	Depth of building	m
F	Allowable bearing capacity	t/m^2
f_1	Factor representing structural deformation	
f_2	Factor representing building displacement due to foundation yield	
H	Building height	m
H_o	Building height above ground	m
H_u	Depth of basement below ground level	m
K_1	Coefficient representing foundation yield	
K_2	Coefficient representing foundation yield	
λ	Inertia factor	
M	Overturning moment	t-m
Q	Design shear	t
R	Rigidity of building	t/cm
R'	Corrected rigidity of building	t/cm

T_g	Natural period of the ground	sec
T_s	Natural period of building	sec
T_m	Period corresponding to assumed rigidity	sec
T_{s1}	First mode period	sec
T^{*}_{s1}	First mode period for fixed base	sec
T_{s2}	Second mode period	sec
V	Design velocity	cm/s
V_o	Basic velocity	cm/s
V_1	Design velocity for fundamental mode	cm/s
V_2	Design velocity for second mode	cm/s
w	Wall quantity in one direction	cm/m^2
W_i	Load at i-th floor level	t
X_1	Normalized deflection, fundamental mode	
X_{1i}	Normalized deflection at i-th floor level, fundamental mode	
X_2	Normalized deflection, second mode	
X_{2i}	Normalized deflection at i-th floor level, second mode	
y_1	Design deflection, fundamental mode	cm
y_{1T}	Design deflection at the top of building, fundamental mode	cm
y_2	Design deflection, second mode	cm
y_{2T}	Design deflection at the top of building, second mode	cm

APPENDIX

The welknown Stodola method was primarily prepared for the bending vibration of a beam, but the same principle can apply to the vibration of a shear beam.

The following tabular form is self explanetory and convenient for the mode calculation for the fundamental mode.

(1) N	(2) y_n	(3) W_i	(4) (2)x(3)	(5) $\downarrow\sum$(4)	(6) D_i/h_i^2	(7) (5)/(6)	(8) $\uparrow\sum$(7)	(9) (2)/(8)
R	-----	-----	-----				-----	-----
				-----	-----	-----		
N	-----	-----	-----				-----	-----
				-----	-----	-----		
N-1	-----	-----	-----				-----	-----
				-----	-----	-----		
---	---	---	---				---	---
				↓				
							↑	
---	---	---	---				---	---
2	-----	-----	-----				-----	-----
				-----	-----	-----		
1	0	-----	0				0	

In the above Table, column (8) gives the figure for y_{n+1} , and D_i denotes the rigidity figure as given by $D_i = R_i/(12EK_0)$.

The similar calculation is to be repeated until all of $\left(\frac{y_n}{y_{n+1}}\right)_i$ values become uniform. Then, it follows

$$\left(\frac{2\pi}{T}\right)^2 = \left(\frac{y_n}{y_{n+1}}\right) \times 12EK_0 g.$$

TABLE 2

	COMPRESSION	TENSION
STRUCTURAL STEEL	2400 kg/cm^2 (34200 psi)	2400 kg/cm^2
CONCRETE	2/3 OF COMPRESSIVE STRENGTH	1/10 OF ALLOWABLE COMP. STRESS