THE EFFECT OF LATERAL EARTHQUAKE ON A HIGH BUTTRESS DAM

by

Jerome M. Raphael*

INTRODUCTION

In concrete gravity dams, constructed of monolithic units of masonry making contact at the transverse construction joints, no concern has been felt for the effect of earthquake motion parallel to the dam axis on the stability and stresses in the structure. In recent years, economy of construction has forced a gradual shift from solid concrete gravity dams to buttress dams, in which the water-bearing face is supported by a series of triangular walls or buttresses. As first developed, these walls were quite thin, closely spaced, and supported laterally at intervals by struts. With increased emphasis on high dams, buttresses have become substantial free-standing plate-like structures in which the water-bearing face is merely an upstream enlargement of the buttress itself and for which designs have been projected for heights up to 750 ft. Some concern has been felt for the stability of structures of this magnitude against earthquake in a direction normal to the plane of the buttress. This paper is a preliminary analysis of the stability of high buttresses against the effects of lateral earthquake.

THE STRUCTURE AND THE PROBLEM

An idealized view of the typical structure being considered is shown in Fig. 1. The entire structure is seen to consist of a series of independent roughly triangular walls, with maximum height of 750 ft, 60 ft thick, and spaced at 130 ft centers. The enlarged upstream portion of the buttress serves as the water barrier. This type of structure is termed a massive head buttress dam, and sometimes given more descriptive terms such as a diamond head or round head buttress dam, depending on the shape of the massive head. Each buttress is constructed independently, but makes contact with the neighboring buttress by the necessity for providing some sealing detail against the passage of water after the contraction joints open when the dam has cooled to final stable temperature. Although cement grout, asphaltic, or metallic seals are equally effective, for the design under consideration, it was assumed that the joints would be sealed with a continuous rubber cushion as shown in the drawing.

* Associate Professor of Civil Engineering, University of California, Berkeley, California.

MODE OF VIBRATION

Two types of structural action can be visualized in response to earthquake vibration normal to the plane of the buttress. In the first type, it might be presumed that each buttress will act as a free-standing structural element deflecting as a triangular cantilever about its base. This supposition was examined and discarded. Each buttress is in contact with its neighboring buttresses along the entire upstream face through the water seals or cushions. It would be impossible for all the buttresses of the dam to vibrate in unison. In the first place all the buttresses are slightly different in dimension, and hence differ in natural frequency. Thus, even if the ground everywhere had the same mode of vibration, the individual buttresses would tend to vibrate independently. Secondly, a dam of this size is of a much greater length than the wave length of the earthquake, and the ground in different parts of the dam would be moving in opposite directions. Hence any consideration of single mass vibration about the base must be set aside.

A second type of vibration may be considered which more closely approximates the physical conditions of the buttress. This would be to consider that the buttress is a triangular plate supported along two sides corresponding to the upstream face and the base of the dam, and completely free along the third side which corresponds to the downstream face of the buttress. The supported faces are neither completely free nor completely fixed. It was considered that along the upstream face, although each massive head is in contact with the neighboring massive head, the details of the water seals would be such that small shearing movements could take place, and hence small rotations of the massive head about an inclined axis parallel to the upstream face of the dam would be possible. In other words, the upstream face was considered to fulfill the conditions for simple support.

Since the base of the dam is constructed directly onto a rock foundation, it might be concluded that the base of the dam is fixed. However, even here more careful consideration of the situation shows that some slight rotation of the base can take place. When large masses of rock, such as form the foundation of a dam, are examined, it can be seen that the continuity of the rock mass is not perfect, but is interrupted generally by systems of seams and fissures. The larger the mass of rock considered in any analysis or test, the lower the resistance to deformation. Comparison of the results of tests of rock cores, bearing tests, pressure chamber tests in rock, and observation of the deflections of large dams confirm that the larger the mass of rock involved, the lower its apparent modulus of elasticity, and resistance to deformation. Hence it must be concluded that considering the large mass of the buttress dam, some foundation movement can be accommodated by opening and closing the fissures in the foundation. Secondly, even if the foundation was perfectly homogeneous, isotropic, and elastic, the support of the buttress at the base could not be considered to be completely fixed, since elastic movements of a perfectly elastic foundation would allow some rotation of the base. Considering these two factors together, it was concluded that the mathematical concept of perfect fixation could not be applied to the base of the buttress dam, and it was considered simply supported at the base.

VIBRATION OF A TRIANGULAR PLATE SIMPLY SUPPORTED ON TWO EDGES

The solution to the general problem of the vibration of a triangular plate simply supported on two edges was not found in the available literature on the mechanics of deformable bodies, and some improvisation was made from the solutions of those problems that had been attacked successfully.

In studying the vibration of a square plate with free edges, W. Ritz[1,2] found that in the second mode of vibration of the square plate, the diagonals were nodal lines, and the fundamental frequency of the plate could be determined by the equation

$$w = \frac{\alpha}{a^2}\sqrt{\frac{gD}{\gamma t}} \qquad (1)$$

where

$$D = \frac{Et^3}{12(1-\mu^2)} \qquad (2)$$

For the three lowest modes the values of the constant α are: $\alpha_1 = 14.10$, $\alpha_2 = 20.56$, $\alpha_3 = 23.91$. The modes of vibration corresponding to each of these fundamental frequencies are shown in Fig. 2.

Study of Fig. 2, for the second mode of vibration, shows that each quarter of the plate is analogous to the problem at hand. The outside edge of the plate is free, and the two diagonals represent lines of zero displacement but finite bending. Thus, as a first approximation, the vibration of the triangular buttress supported on two edges might be represented by the vibration of a 45° right triangle with the hypotenuse free and the two equal legs simply supported.

Circular frequency and natural period are related through

$$T = \frac{2\pi}{w} \qquad (3)$$

Substituting equations (1) and (2) in (3), we obtain

$$T = \frac{2\pi a^2}{\alpha}\sqrt{\frac{12\gamma(1-\mu^2)}{gEt^2}} \qquad (4)$$

or

$$T = \frac{2\pi a^2}{\alpha}\sqrt{\frac{12S(1-\mu^2)}{Et^2}} \qquad (5)$$

The modulus of elasticity enters into the computation for natural period, which varies with the $\sqrt{\frac{1}{E}}$. It is difficult to predict exactly the value for E, since it varies not only with the materials and method of manufacture of the concrete, but also with age, exposure, and the manner of loading. However, after studying a number of mass concrete mixes, the value $E = 4 \times 10^6$ psi was assumed to use in this dynamic application. In the first approximation, the 800 ft high buttress, 60 ft thick, and nearly equilateral, was approximated by a 45° right triangular plate of the same thickness and having the same area as the lateral area of the buttress shown in Fig. 3a. For this case, where a = 850 ft, the natural period was computed to be 1.92 seconds.

For a second approximation, the length of the free side of the triangle was made equal to the length of the free edge of the buttress, using equation (4) as before, but substituting a = 750 ft. Fig. 3b shows the specimen right triangle fitted into the buttress section. The free edge for this case was shorter than for the imaginary figure of the first approximation, and hence the plate was stiffer, and had a lower natural period. For an 800 ft high buttress, 60 ft thick, the natural period was computed to be 1.51 seconds.

As a third approximation, a method was sought by which the value of the characteristic parameter α could be varied according to the variation of the vertex angle. Variation of the stiffness parameter has been worked out by Cox and Klein[4] for the case of an isosceles triangular plate clamped on the two equal edges and simply supported on the third edge, varying the angle between the two clamped edges. The variation of the parameter β with the ratio h/a is shown in Fig. 4a. Although the relative rigidity of Cox and Klein's c-c-s plate is much greater than Ritz's s-s-f case, the assumption was made that the Cox and Klein parameter β and the Ritz parameter α varied similarly with h/a. Correspondence between the two cases is achieved when $\frac{h}{a} = 0.5$, the 45° right triangle. Increasing all values of α by the ratio β/β_o, where β_o is the Cox and Klein coefficient for $\frac{h}{a} = 0.5$, the second relationship for the variation of α with $\frac{h}{a}$ shown in Fig. 4 is established.

Despite the approximation involved in applying the variation in the parameter α with the variation in vertex angle when using equation (4), the published results of experiments tend to confirm its general validity. In studying the seismic behavior of hollow gravity and buttress dams, Niwa, Hatanaka, and Samukawa[5] performed a number of experiments on rubber and plastic models. Fig. 5 shows the measured deflection curve of the largest element of the rubber model of a buttress dam under lateral loading. From the inversion of the deflection curves near the supports, it can be seen that the upstream base and the foundation tend to fix the web of the buttress rather more than has been assumed in the above analysis. Thus it might be expected that the period for the model might be shorter than one computed using equation (4).

The rubber of which the model was constructed had the following properties:

Dynamic modulus of elasticity	42.9 kg/cm^2
Poisson's ratio	0.43
Specific gravity	1.27

As shown on the drawings, h and a were very nearly equal. From Fig. 4, for $\frac{h}{a} = 1$, α equals 48. For a length of free edge of 30 cm, and a thickness of 2.76 cm, the natural period computed using equation (4) is 0.21 seconds. The resonant period measured by Niwa et al was 0.180 seconds, which indicates that the relative fixation of the supported edges decreases the period by about 14 percent.

For the isosceles triangle fitted over the buttress shape shown in Fig. 3c as a third approximation, $\frac{h}{a} = \frac{500}{750} = 0.67$ and $\alpha = 28$. By making the appropriate substitutions in equation (4), the natural period for the 60 ft thick, 800 ft high buttress, was computed as 1.89 seconds.

The range of natural period found from all these approximations is not great, leading to identical response to seismic acceleration. Experiment and experience have shown that the maximum response of structures to seismic force is found for those structures having a natural period around 0.25 seconds.[6,7] For periods in excess of this critical value, the response of structures decreases in a relationship which varies inversely with the natural period. In general, analytical and experimental studies for zero damping have indicated structural response for certain periods which greatly exceed the maximum excitation. However, it has been repeatedly shown that when an allowance is made for the effects of damping, which is present in all real structures to some degree, the response is greatly reduced. Thus in studying the results of a number of such experiments, the Joint Committee[8] recommended alternative curves of response for California conditions, one to be used for design of buildings and a second for the design of other structures. Curve A of Fig. 6 represents the response of framed structures with varying amounts of damping due to partitions and curtain walls to ground accelerations of 0.10g and ranges from 0.06g to 0.02g as the natural period increases from 0.25 seconds to 0.75 seconds, according to the function $C = 0.015/T$. For structures other than buildings, which are generally simpler and with less damping, generally higher values are recommended, as shown in Curve B, in which C ranges from 0.10g to 0.03g, according to the equation $C = 0.025/T$. Accordingly, an average rounded value of 1.80 seconds was assumed as the fundamental period for an 800 ft high buttress, 60 ft thick. For buttresses of lesser height, the fundamental period was computed by letting it vary directly with the square of the height, as shown in column 2 of Table I. Applying these fundamental periods to the response spectrum of Fig. 6, the seismic factors shown in column 4 can be derived.

BUTTRESS STRESSES

The stresses in the free edge of the buttress due to lateral earthquake were found by taking a one-foot wide, 60 ft deep, beam element freely supported at the crest and at the toe of the dam. Uniform lateral load was applied to this beam element, equal to the product of the seismic factor and the weight of the beam. From this, the maximum bending stress was computed, as shown in column (6), neglecting any shear transfer from the edge element to the adjacent element. This is conservative, since any shear transfer tends to partially support the element, and reduces the computed tensile stresses. As the element vibrates under the action of seismic force, stress varies from maximum tension to maximum compression. In addition to these varying stresses, compressive stress due to dead load is always present, with possibly some additional compression due to the live load of the reservoir. The normal state of stress due to dead load and to reservoir load varies not only with the height of the reservoir, but with the relative size of the massive head of the buttress. Accordingly, to be perfectly general, the combination of dead and water load stress was taken as equal numerically in psi to the height of concrete above the point in question in feet. This assumption forms the basis for the values in column (7).

The combined stress in column (8) is the algebraic sum of the earthquake tensile stresses in column (6) and the usual compressive structural stresses in column (7). Study of the table shows that the 60 ft thick buttress has no objectionable net tensile stresses for any height of dam. Hence it must be concluded that the free-standing buttresses should be amply stable against lateral earthquake. It can also be concluded that nothing would be gained by constructing intermediate massive lateral bracing, say at mid-height, to reduce the unsupported length of the buttresses. This would only decrease the natural period in a manner yet to be determined, and probably increase the tensile stresses

Analytical research is continuing on the fundamental response of triangular plate structures with variable conditions of support and fixation at the edges. The results of experiments published by Niwa, Hatanaka, and Samukawa represent a step toward confirmation of vibration theory for buttress dams. The next logical step will be to conduct vibration tests on full-scale dams. It is hoped that vibration machines now under construction will be used to secure these needed data.

BIBLIOGRAPHY

1. W. Ritz, "Theorie der transversalschwingungen einer quadratischen Platten mit freien Rändern," Annalen der Physik, Vol. 28, 1909, p 737.

2. S. Timoshenko, "Vibration Problems in Engineering," Van Nostrand, 1955, p 446.

3. S. Iguchi, "Natural Frequency of a Free Rectangular Plate," Ingenieur Archiv, Vol. 21, 1953, p 303.

4. H. L. Cox and B. Klein, "Buckling and Vibration of Isosceles Triangular Plates Having the Two Equal Edges Clamped and the Outer Edge Simply-Supported," Journal Royal Aeronautical Society, Vol. 59, February 1955, p 151.

5. Y. Niwa, M. Hatanaka, and S. Samukawa, "Researches on Vibration Characteristics of Hollow Gravity Dams," Technical Report No. 46, Engineering Research Institute, Kyoto University, Kyoto, Japan, Vol. VIII, No. 6, 1958.

6. M. A. Biot, "Analytical and Experimental Methods in Engineering Seismology," Trans. ASCE, Vol. 108, 1943.

7. E. C. Robison, "Computations on the Response of One-Mass Undamped Structures to the El Centro Accelerograph Record," Watson Scientific Computing Laboratory, Columbia University.

8. Joint Committee of San Francisco Section, ASCE, and Structural Engineers Association of Northern California, "Lateral Forces of Earthquake and Wind," Trans. ASCE, Vol. 117, 1952.

NOMENCLATURE

a Length of side of square or base of triangle, ft

c Clamped or fixed supports

C Seismic coefficient, fraction of acceleration of gravity

D Flexural rigidity of plate

E Modulus of elasticity, psi

f Free edge condition

g Acceleration of gravity, ft per sec^2

h Altitude of isosceles triangle, ft

s Simply supported edge condition

S Specific gravity

t Plate thickness, ft

T Natural period, sec

w Circular frequency, radians per sec

α A constant, depending on edge conditions and mode of vibration

β A constant, depending on proportion h/a

γ Density of material, pcf

μ Poisson's ratio

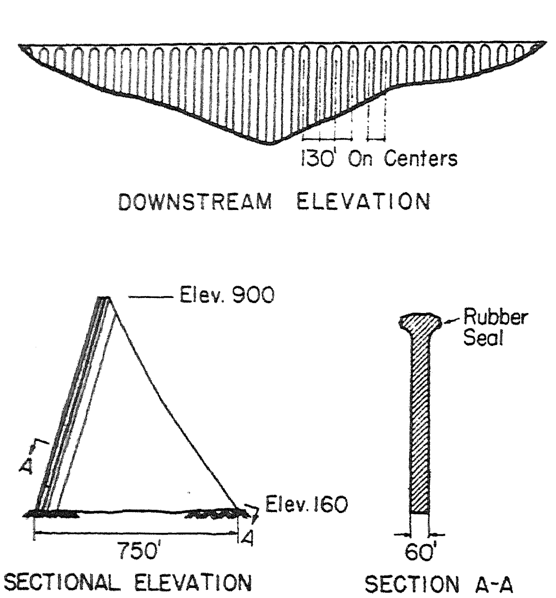

FIG. 1 - TYPICAL HIGH MASSIVE-HEAD BUTTRESS DAM

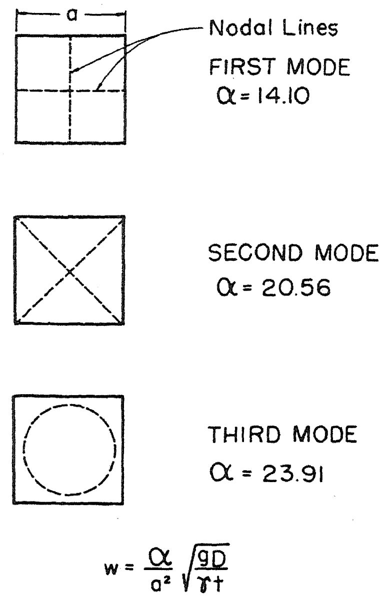

FIG. 2 - FREE VIBRATION OF SQUARE PLATES

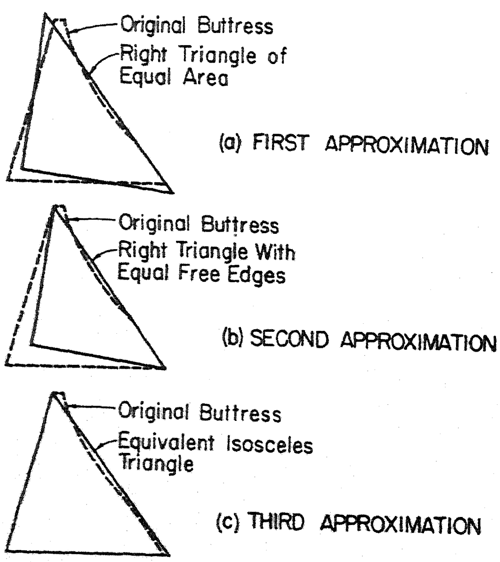

FIG. 3 - APPROXIMATIONS OF BUTTRESS SHAPE

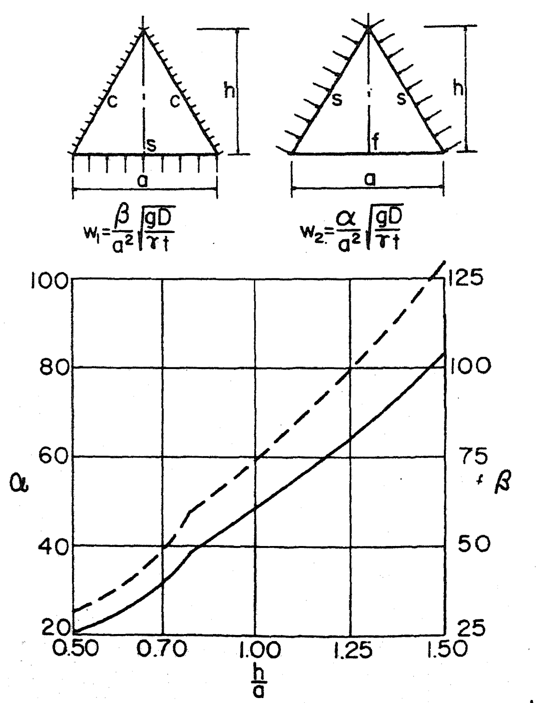

FIG. 4 - VARIATION OF α AND β WITH $\frac{h}{a}$

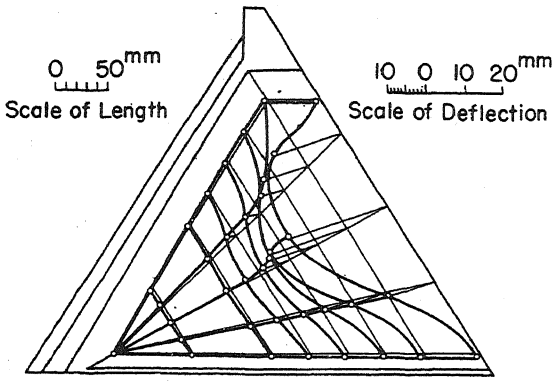

FIG. 5 - MEASURED DEFLECTION OF WEB OF RUBBER MODEL BUTTRESS

(After Niwa, Hatanaka, and Samukawa)

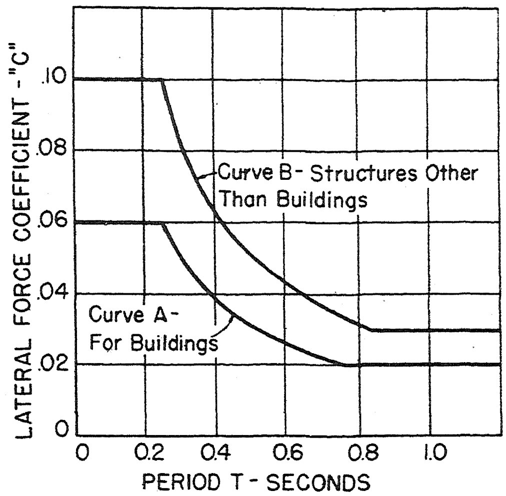

FIG. 6 - LATERAL FORCE COEFFICIENTS

TABLE I

EARTHQUAKE STRESSES IN TRIANGULAR BUTTRESS

Max Height ft	Period T sec	Seismic Factor C	Unit Load kif	Free Length ft	Bending Stress psi	Load Stress psi	Combined Stress psi
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
800	1.80	.030	.270	985	+380	-400	-20
700	1.38	.030	.270	860	+290	-350	-60
600	1.01	.030	.270	740	+214	-300	-84
500	0.70	.035	.315	615	+171	-250	-79
400	0.45	.056	.503	492	+175	-200	-25
300	0.25	.100	.900	370	+178	-150	+28
200	0.112	.100	.900	245	+ 78	-100	-22
100	0.028	.100	.900	103	+ 20	- 50	-30

DISCUSSION

M. Hatanaka, Kobe University, Japan:

Is there any record available in the United States on behaviour of dam structure and ground of dam site?

J. M. Raphael:

The U.S. Bureau of Reclamation has some comparative records of accelerations in the ground and at the crest of dams. I believe records were obtained at Hoover Dam and at Ross Dam.