The Plastic Failure of Frames During Earthquakes

by
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Introduction. - When a ductile frame structure is subjected to sufficiently strong ground motion, its oscillations will produce strains that exceed the elastic limit. The plastic straining will absorb some of the energy of vibration, and this may be sufficient to permit the structure to survive. If, however, the oscillations of the structure are very strong there may result a plastic collapse. The usual procedures of earthquake-resistant design are based on the supposition that very strong ground motion may cause overstressing but that the energy absorption capacity is sufficiently great to control the vibrations without being in danger of collapse. Our knowledge of the oscillatory motion and collapse of elasto-plastic structures during earthquake-type excitations is meager[1, 2, 3] and there is need for much additional study of this problem.

In this paper there will be considered a plastic analysis of collapse that takes into account the absorption of energy by the formation of yield hinges. This throws some light on the broad features of the problem, and indicates those areas where additional information is particularly needed.

As proposed in an earlier paper[4], it will be assumed that the vibrational energy to be safely contained by one component of motion of the structure is

$$E = \frac{1}{2} \frac{W}{g} S_v^2 \qquad \dots.(1)$$

where W is the total weight of the structure and S_v is the average velocity spectrum value corresponding to one horizontal component of ground motion[4, 5].

A Mass Supported by a Single Column. - Consideration will first be given to a cantilever column that supports a weight W as shown in Fig. 1. During strong ground shaking this system may fail in one of several ways. One possibility is that the vibrations will cause approximately equal plastic straining in alternate directions and that this will continue until the material breaks because of a fatigue failure. Another possibility is that all of the plastic straining will take place in one direction until the column collapses laterally because of excessive plastic drift. These two possibilities are extreme cases, and the probability of their occurrence is extremely small. The most probable failure is collapse due to plastic drift with a greater or lesser amount of energy having been absorbed in plastic straining in the opposite direction. In this case, collapse occurs when some fraction of the total energy pE is just equal to the energy required to produce collapse by plastic drift in one direction. In what follows, the factor p will be taken equal to unity as a matter of convenience, it being understood that a reduced value of S_v actually corresponds to failure, and that the safety factor must take this into account (see, for example, Eq. (42)).

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With sufficient overstress a yield hinge will be formed at the base of the column. It will be assumed that so far as plastic deformations are concerned the material is perfectly plastic and the yield moment has a constant value M_o. The value of M_o depends upon the yield stress, σ_y, of the column, the size and shape of the column, and the magnitude of the axial load W. The interaction relationship between M_o and W is easy to compute and it is approximately described by the following equation[6]:

$$M_o = a\left(1 - \frac{P}{P_y}\right) M_y \qquad \ldots.(2)$$

$$(\text{for } P > 0.05\, P_y)$$

where P is the axial load, P_y is equal to the yield stress, σ_y, times the area of the column, and M_y is the yield moment for P = 0. The coefficient a has the value 1.2 for bending about the strong axis of a steel H-section and has the value 1.7 for bending about the weak axis.

The energy absorbed by the yield hinge is $M_o \phi$ and this must be equal to pE plus the work done by W which is equal to $Wh(1 - \cos\phi)$ $\approx Wh\, 1/2\, \phi^2$, so that the condition for stability may be written:

$$\frac{1}{2}\frac{W}{g} S_v^2 + \frac{1}{2} Wh\phi^2 = M_o \phi \qquad \ldots.(3)$$

and the required value of M_o that the column must provide is:

$$M_o = \frac{1}{2}\frac{W}{g}\left(\frac{S_v^2}{\phi} + gh\phi\right)$$

Equation (3) is illustrated in Fig. 2 where it is seen that the smallest value of M_o for which the equation is satisfied is when the curves are tangent at point P.

The minimum value of M_o is given by that ϕ for which $\partial M_o/\partial\phi = 0$, which is

$$\phi_c = \frac{S_v}{\sqrt{gh}}$$

and, therefore, the smallest M_o is

$$M_o = S_v \sqrt{\frac{h}{g}}\; W \qquad \ldots.(4)$$

The value of ϕ_c is the limiting value beyond which the column will collapse under the weight W. The above value of M_o is the least value that will keep the column from collapsing. The corresponding base shear is

$$V_b = \frac{S_v}{\sqrt{gh}}\; W \qquad \ldots.(5)$$

and the seismic coefficient is

$$c = \frac{S_v}{\sqrt{gh}} \qquad \ldots.(6)$$

It is informative to put Eq. (4) into a form in which it can be compared directly with the usual procedures of earthquake-resistant design. Substituting Eq. (2) into Eq. (4) gives:

$$M_y = \frac{\sigma_y}{\sigma_e} \sqrt{\frac{h}{g}} \frac{S_v}{a} W \qquad \ldots.(7)$$

where $\sigma_e = \sigma_y - W/A$

A = area of column.

Defining a section modulus by $Z = M_y / \sigma_y$ and substituting in Eq. (7) there results:

$$Z = \frac{S_v}{a\sqrt{hg}} \frac{Wh}{\sigma_e} \qquad \ldots.(8)$$

In this equation Z is the section modulus for which the maximum statically computed stress is equal to the yield stress; that is, for which

$$\frac{W}{A} + \frac{M}{Z} = \sigma_y$$

where M = cWh. Equation (8) may be written:

$$Z = c_p \frac{Wh}{\sigma_e} \qquad \ldots.(9)$$

Therefore the equivalent elastic-limit seismic coefficient is

$$c_p = \frac{S_v}{a\sqrt{hg}} \qquad \ldots.(10)$$

For strong ground motion such as the El Centro 1940 earthquake, a structure with small damping and with a period of vibration greater than about 0.4 seconds, a representative spectrum value might be $S_v = 2.5$ ft/sec. A one-story structure would have approximately $\sqrt{hg} = 20$ ft/sec. The equivalent static coefficient then would have the value

$$c_p = \frac{(2.5)}{(1.2)(20)} = \frac{1}{10}$$

As was stated above, it is extremely unlikely that all of the energy will go into producing uni-directional plastic drift and that only some fraction pE should be used. In this case, the value of the seismic coefficient is

$$c_p = \frac{S_v}{a\sqrt{hg}} (n\sqrt{p}) \qquad \ldots.(11)$$

It would seem conservative to take $p = 1/2$, that is, to assume that three quarters of the energy is absorbed in plastic drift in one direction and one quarter is absorbed in the other direction. In this case

$$c_p = 0.07$$

This would indicate that designing for 7% of gravity with yield point stresses would give a factor of safety against collapse of one.

The foregoing computations were made for bending about the strong axis of the column ($a = 1.2$). For bending about the weak axis ($a = 1.7$) there is obtained

$$c_p = 0.07 \frac{1.7}{1.2} = 0.10$$

It is thus seen that from the equivalent elastic limit design viewpoint the seismic coefficient depends upon the shape of the column or, in other words, it depends upon the ability of the column to absorb energy with a yield hinge.

It should also be noted that for the system under consideration the same amount of vibrational energy is associated with each component of motion and therefore the mass M has a total vibrational energy 2 pE and this fact might have an influence on the proper value of p to be used.

Factor of Safety. There are a number of different ways of defining the factor of safety for this problem. The most obvious way is to write Eq. (3) in the form

$$n\left(\frac{1}{2}\frac{W}{g}S_v^{\,2} + \frac{1}{2}Wh\phi^2\right) = M_o\phi \qquad \ldots.(3a)$$

This leads to a minimum M_o of

$$M_o = \frac{n\,S_v}{\sqrt{gh}} Wh \qquad \ldots.(4a)$$

where n is the factor of safety.

Another point of view is that in the case of an earthquake the uncertainty lies in the term pE and that the safety factor should be on that term:

$$\left(n\frac{1}{2}\frac{W}{g}S_v^{\,2} + \frac{1}{2}Wh\phi^2\right) = M_o\phi \qquad \ldots.(3b)$$

This leads to a minimum M_o of

$$M_o = \frac{\sqrt{n}\,S_v}{\sqrt{gh}} Wh \qquad \ldots.(4b)$$

A third possibility is to put the safety factor on S_v:

$$\left(\frac{1}{2}\frac{W}{g}(nS_v)^2 + \frac{1}{2}Wh\phi^2\right) = M_o\phi \qquad \ldots.(3c)$$

This gives the same result as did Eq. (3a), that is

$$M_o = \frac{n\, S_v}{\sqrt{gh}}\, Wh \qquad \ldots.(4c)$$

This method of defining the safety factor is used in the following portions of the paper since it has an easily understood meaning in terms of the spectrum, S_v.

Elastic Design. - If the design is to be such that the structure remains elastic, instead of Eq. (3) the energy balance requires

$$\frac{1}{2}\frac{W}{g} S_v^{\,2} = \frac{1}{2}\,\frac{M^2 h}{3\,EI} \qquad \ldots.(12)$$

and

$$M = \left(\frac{3EIW}{hg}\right)^{1/2} S_v$$

and in place of Eq. (8) there is obtained

$$Z = \left(\frac{2\pi}{Tg}\, n_e S_v\right)\frac{Wh}{\sigma_e} = c_e\,\frac{Wh}{\sigma_e} \qquad \ldots.(13)$$

where T is the natural period of vibration of the structure. Instead of Eq. (7) the following equation gives the coefficient for elastic design:

$$c_e = \frac{2\pi}{Tg}\, n_e\, S_v \qquad \ldots.(14)$$

where n_e is the factor of safety against yield point stress. It is seen that the period of vibration enters differently in Eq. (12) than it does in Eq.(7), remembering that S_v is a function of the period. The condition under which $c_e = c_p$ is when the plastic safety factor is:

$$n_p = n_e\,\frac{2\pi}{T}\, a\sqrt{\frac{h}{g}} \qquad \ldots.(15)$$

From Eq. (15) it is seen that structures designed to remain elastic on the basis of Eq. (12) will have quite different factors of safety against collapse depending upon the values of T. The implication of Eq. (15) is that the design computations should be made twice, once for a c_e that will insure elastic stresses for ground motion of a specified intensity, and a second time for a c_p that gives the required factor of safety against collapse for stronger ground motion.

A Mass Supported on a Two-Column Frame. - If the weights W_1 and W_2 are supported by a two-column frame as shown in Fig. 3, a plastic collapse will require the formation of four plastic hinges, one at each end of each column. An analysis similar to that made for the single-column case gives

$$M_{01} + M_{02} = n\sqrt{\frac{h}{g}}\;\frac{S_v}{2}(W_1 + W_2) \qquad \ldots.(16)$$

The corresponding shear force is

$$V = \frac{n\,S_v}{2\sqrt{gh}}\;W \qquad \ldots.(17)$$

The static seismic coefficient c for which the columns will just reach yield moments M_{01} and M_{02} with a factor of safety n against collapse is

$$c = \frac{n\,S_v}{2\sqrt{gh}} \qquad \ldots.(18)$$

It is seen that when the columns can develop yield hinges at both ends the same factor of safety against collapse is given for a value of c that is only one-half as large as for a cantilever column.

A Mass Supported on Four Columns. - Consider a four-column frame, as shown in Fig. 4, where each column carries one-quarter of the weight and can develop yield hinges at each end. For lateral collapse

$$n\left(\frac{1}{2}\,\frac{W}{g}\,S_v^{\,2} + \frac{1}{2}\,Wh\phi^2\right) = 8M_o\phi \qquad \ldots.(19)$$

and from this the minimum required value of yield moment for this type of collapse is found to be

$$M_o = \frac{n\,S_v}{2}\sqrt{\frac{h}{g}}\;\frac{W}{4} \qquad \ldots.(20)$$

There is another mode of collapse, however, which requires a larger M_o. If collapse occurs by twisting about one column as shown in Fig. 4b, the condition to be satisfied is

$$n\left(\frac{W}{g}\,S_v^{\,2} + \sum_i \frac{W_i h \phi_i^{\,2}}{2}\right) = \sum_i 2\,M_o\phi_i \qquad \ldots.(21)$$

Note that with this type of collapse the vibrational energies of both horizontal components of motion are involved so that the total vibrational energy to be absorbed is 2 pE. The angles associated with the columns are

$$\phi_1 = \frac{b}{h}\theta$$

$$\phi_2 = \sqrt{2}\,\frac{b}{h}\,\theta$$

$$\phi_3 = \frac{b}{h}\theta$$

$$\phi_4 = \theta$$

These give for the required yield moment

$$M_o = 1.7\left(\frac{n}{2}\,S_v\sqrt{\frac{h}{g}}\;\frac{W}{4}\right) \qquad \ldots.(22)$$

This is 1.7 times as large as required for lateral collapse and even if only pE is to be absorbed in collapse, M_o is still 1.2 times as large as required for lateral collapse. It can therefore be expected that collapses of such frameworks will almost certainly involve twisting*.

Multi-Story Buildings. - The collapse of multi-story buildings will also almost certainly involve twisting; however, for convenience, the following analysis will treat lateral collapse and the resulting yield moment will be somewhat smaller than would be obtained for a twisting collapse. It is assumed that the building is uniform with equal story heights h, and with each floor (and roof) of equal weight. Each column receives a weight w from each floor (and roof) so that for an N-story building the column carries a total weight Nw. Each column is assumed to form two yield hinges in each story.

Suppose the collapse occurs in the kth story from the top, and that the total vibrational energy is absorbed in this story. The energy balance for one column is then

$$\frac{1}{2}\,\frac{Nw}{g}(n\,S_v)^2 + \frac{1}{2}\,k\,wh\phi^2 = 2\,M_o\phi \qquad \ldots . \;(23)$$

From this the required M_o is found to be

$$M_o = \frac{1}{2}\sqrt{\frac{N}{k}}\sqrt{\frac{h}{g}}\;nS_v(kw) \qquad \ldots . \;(24)$$

The equivalent static lateral shear in the story is thus

$$V = \frac{1}{\sqrt{k}}\,n\,S_v\sqrt{\frac{N}{gh}}\;(kw) \qquad \ldots . \;(25)$$

The equivalent static seismic coefficient for the kth story from the top is

$$C_k = (\sqrt{k} - \sqrt{k-1}\,)\,nS_v\sqrt{\frac{N}{gh}} \qquad \ldots . \;(26)$$

Equations (25) and (26) give shear and seismic coefficient distribution similar to the old Uniform Building Code formula**which in the notation of this paper is

$$V = \frac{0.6}{k+4.5}\;(kw) \qquad \ldots . \;(27)$$

and this gives

$$C_k = \left(\frac{k}{k+4.5} - \frac{k-1}{k+3.5}\right)0.6 \qquad \ldots . \;(28)$$

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* The plastic collapse of x-braced elevated water tanks can also be analyzed in the same way with the same conclusions. These have been observed to collapse with twisting motion during earthquakes.

**This specified that the seismic shear was given by ($\frac{0.6}{N+4.5}$) times the weight above with N being the number of stories above the story under consideration.

It is seen that Eq. (25) gives a base shear

$$V_b = \frac{n S_v}{\sqrt{gh}} W \qquad \ldots. (29)$$

Another possibility is to assume that the total vibrational energy is absorbed equally by all floors. In this case Eq. (23) takes the form

$$n\left(\frac{1}{2}\frac{w}{g} S_v^2 + \frac{1}{2} kwh\phi^2\right) = 2 M_o\phi \qquad \ldots. (30)$$

From this there is obtained:

$$M_o = \frac{1}{2}\frac{1}{\sqrt{k}}\sqrt{\frac{h}{g}}\; nS_v (kw) \qquad \ldots. (31)$$

and the story shear is

$$V = \frac{1}{\sqrt{k}}\frac{nS_v}{\sqrt{gh}} (kw) \qquad \ldots. (32)$$

and the corresponding seismic coefficient is

$$C_k = (\sqrt{k} - \sqrt{k-1})\frac{nS_v}{\sqrt{gh}} \qquad \ldots. (33)$$

For this case the base shear is

$$V_b = \frac{nS_v}{\sqrt{gh}}\frac{1}{\sqrt{N}} W \qquad \ldots. (34)$$

Since, for a uniform shear building, the period of vibration, T, is proportional to the number of stories N, the formula for the base shear may be written (setting T = 0.1N):

$$V_b = \frac{nS_v}{\sqrt{gh}}\frac{\sqrt{0.1}}{\sqrt{T}} W \qquad \ldots. (35)$$

This may be compared with the newly proposed California code which takes V_b to vary as $1/\sqrt[3]{T}$.

A third possibility, that is most attractive, is to assume that the total energy is absorbed approximately according to the sizes of the columns. In this case, if it is assumed that the size of the columns varies linearly from the top story down, the energy balance for the kth story may be written:

$$n\left(\frac{1}{2}\frac{k}{N}\frac{w}{g} S_v^2 + \frac{1}{2} kwh\phi^2\right) = 2 M_o\phi \qquad \ldots. (36)$$

From this there is obtained:

$$M_o = \frac{1}{\sqrt{N}} \sqrt{\frac{h}{g}} \; n S_v (kw) \qquad \ldots. \; (37)$$

$$V = \frac{1}{\sqrt{N}} \frac{n S_v}{\sqrt{gh}} (kw) \qquad \ldots. \; (38)$$

$$c_k = \frac{1}{\sqrt{N}} \frac{n S_v}{\sqrt{gh}} \qquad \ldots. \; (39)$$

Equation (39) gives a constant seismic coefficient for each floor. If the period is again set equal to 0.1N, the equation for the base shear may be written

$$V_b = \frac{n S_v}{\sqrt{gh}} \frac{\sqrt{0.1}}{\sqrt{T}} W \qquad \ldots. \; (40)$$

To obtain an idea of numerical values, Eq. (40) takes the following form for $S_v = 2.5$ ft/sec, $\sqrt{gh} = 20$ ft/sec:

$$V_b = n\left(\frac{0.04}{\sqrt{T}}\right) W \qquad \ldots. \; (41)$$

The only indeterminate quantity in Eq. (41) is the factor n. The correct value of n must reflect the following considerations:

a) n must contain a proper factor of safety against collapse, perhaps 2 or 3.

b) It must reflect the fact that collapse would actually involve twisting and hence a factor of perhaps 1.7 should be introduced.

c) It must reflect the fact that the energy will not be uniformly absorbed by all of the stories and this might introduce a factor of 2 or so.

d) The spectrum value $S_v = 2.5$ corresponds to the El Centro, 1940 ground motion with very low damping. As will be discussed later, a structure may have appreciable damping and hence $S_v < 2.5$ would be appropriate and n should reflect this.

e) Equation (41) is based on the supposition that the absorption of energy occurs with plastic drift always in the same direction. Actually the plastic drift takes place in both directions with a cancelling effect, and the net drift in one direction may be very much smaller. It is conceivable that the net drift might correspond to only 1/10 of the energy. It appears that the determination of this quantity is the key to the problem.

f) Equation (41) does not allow for the fact that some of the vibrational energy would be left in the structure at the time of collapse (the yield-point strain energy), and for long-period structures this might be significant.

Plastic Drift. - The exact determination of plastic drift of realistic structures is a difficult problem. A crude approximation is given by the following reasoning. Suppose E is the energy to be absorbed plastically, and t is the duration of the ground motion up to the time of collapse, and T is the period of vibration of the structure. There will thus be approximately t/T times when the structure has excursions in one direction, and t/T times when the excursions are in the other direction. The situation might then be idealized as a random walk problem in which the length of a step is E ÷ 2t/T and the number of steps is 2t/T. On the average the net amount of energy absorbed into the resultant plastic drift is then

$$E_{pd} = \frac{E}{2\,t/T}\sqrt{\frac{2t}{T}} = \frac{E}{\sqrt{2t}}\sqrt{T} \qquad \ldots\ldots (42)$$

If this is incorporated in Eq. (41) there is obtained

$$V_b = n\left(\frac{0.04}{\sqrt{2t}}\right) W \qquad \ldots\ldots (43)$$

This corresponds to a constant seismic coefficient. To obtain an idea of magnitudes, let a factor of 1.7 be taken for twisting collapse, and 3 for non-uniform absorption among the stories. Include 3/4 to allow for normal damping, and take the duration of ground motion to be 30 sec. Equation (43) then has the form

$$V_b = n' \frac{(1.7)(3)(3/4)}{\sqrt{60}}\ 0.04\ W = n'\ (0.02)\ W \qquad \ldots\ldots (44)$$

This would indicate that a design which would reach yield moments M_o at all columns for a 2%g design would have a safety factor of one against collapse. It is, of course, not very precise in view of all the assumptions involved.

Multi-story Elastic Design. - It is clear from the foregoing that the design of a structure should satisfy two different criteria. First, it should be designed so that no damage will be incurred when it is subjected to moderate ground motion of relatively high probability of occurrence. This would require an appropriate "elastic design." Second, the structure should not collapse if it is subjected to less probable but very strong ground motion. This requires a design based on a plastic analysis, and the general requirements will be different in this case from what they would be for the "elastic design." The final design should, of course, satisfy both sets of conditions.

When making an "elastic design" there is some question as to how much damping to assign to the structure. A welded, unclad, steel frame would have very low damping if all the stresses were within the elastic limit. In an actual building, the design will not be such that all columns in a story reach the elastic limit at the same time. The diagram of shear force vs. relative story displacement will then have rounded knees as shown in Fig. 5. Fully plastic action will therefore not be reached until the displacement reaches the point P. The area (OP δ_o) represents energy that must be absorbed before plastic drift sets in, and hence twice the energy (OP δ_o) must be included in the equivalent viscous damping used in the "elastic design." Hence, by definition, any structure that does not develop fully plastic action under the design conditions is said to have an elastic design. The equivalent viscous damping may be quite large, depending upon the design of the structure.

There would be a very real advantage in making the design such that the equivalent viscous damping is large. For example, if, after making a regular elastic design (taking into account relative rigidities, etc.), several columns were arbitrarily increased in stiffness so that they would exceed the yield point while the other of the columns remained elastic, the equivalent viscous damping would be much increased. It follows that in the case of very strong ground motion the energy that must be absorbed in fully plastic action is then much decreased. In such a design, certain of the columns are forced to be the damping mechanism for otherwise elastic vibrations. The base shear coefficient for elastic design varies approximately as S_v/T. For structures with damping S_v is given approximately by

$$S_v = \frac{c_1 T}{c_2 + T}$$

and hence the base shear coefficient varies as

$$\frac{1}{c_2 + T}$$

Conclusions. - The analysis of this paper examines the broad features of the problem of plastic collapse and hence does not arrive at very reliable values of design coefficients. The following broad conclusions can be drawn, however:

1) The phenomenology of plastic collapse is as follows: A certain amount of plastic straining occurs before fully plastic action is established, and the energy dissipated by this is logically lumped into the equivalent viscous damping of the structural oscillations; after fully plastic action has developed, plastic drift sets in and this will tend to progress further in one direction until collapse occurs.

2) The axial load carried by a column is a very significant factor in the ability of the column to absorb vibrational energy without collapsing.

3) The equivalent static seismic coefficients for plastic design (factor of safety against collapse) are quite different from those proper to elastic design (factor of safety against yield-point stress). The old 10%g method of design is a reasonable approach for plastic design, but it is not known what factor of safety against collapse the 10%g design would give.

4) Plastic collapse is almost certain to involve a twisting motion of the structure.

5) For plastic design of multi-story structures, it appears that the base shear coefficient has a different variation with period than for elastic design.

6) It is advantageous to design the structure so that the force-displacement diagram for relative motion between floors has a well-rounded knee so that the development of fully plastic action is delayed for as long a time as possible.

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Nomenclature

A = cross-sectional area of column

a = effectiveness factor

c_e = seismic coefficient, elastic

c_p = seismic coefficient, plastic

h = story height

k = number of floors from the top

M_o = yield moment for axially loaded column

M_y = yield moment for zero axial load

N = number of stories in structure

n = factor of safety

S_v = velocity spectrum value

T = natural period of vibration

W = weight of structure

w = column load per floor

ϕ = angle of rotation of column

σ_y = yield point stress

σ_e = σ_y - W/A

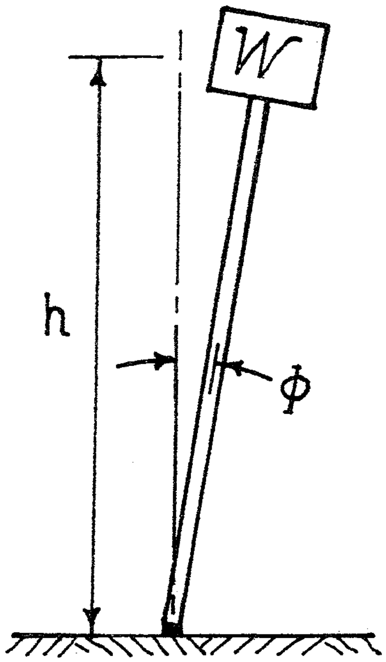

FIG. 1

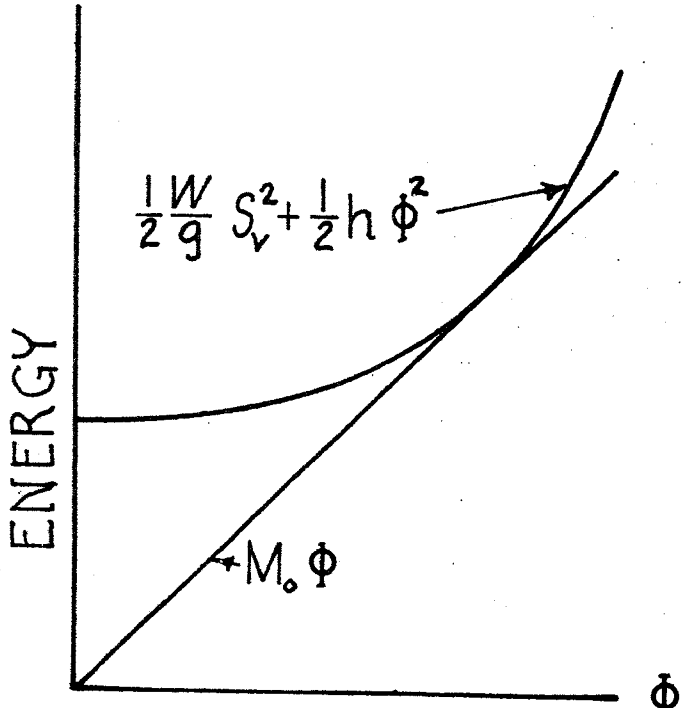

FIG. 2

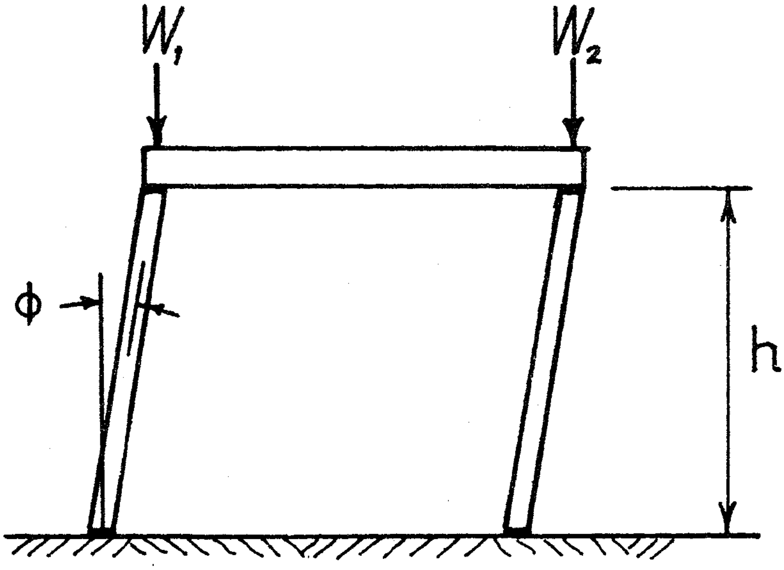

FIG. 3

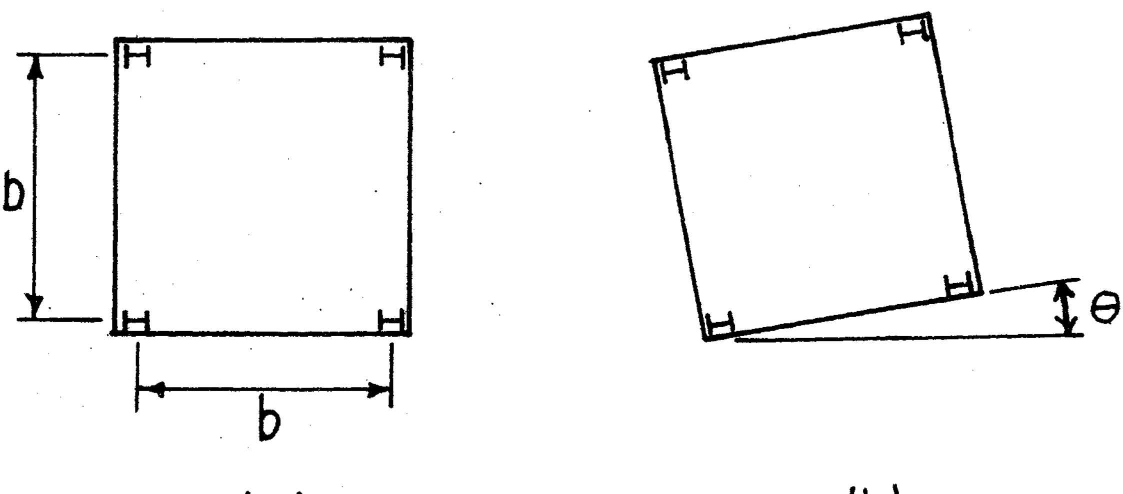

FIG. 4

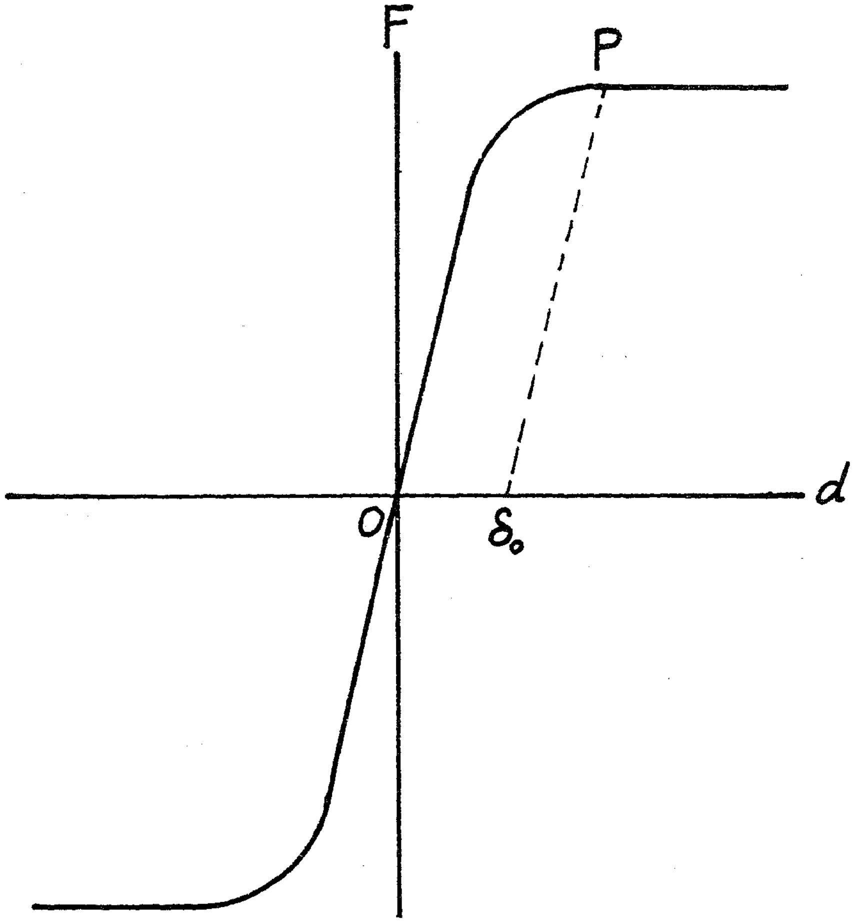

FIG. 5