purpose.

In practice, quite obviously it is necessary to reproduce not just one particular spectrum but the envelope curve of the spectra corresponding to a given geographical zone.

The controversial question among geophisicists as to whether there really exists a "standard" spectrum for each individual zone is of no interest to the structural experimenter. The peak value of the spectrum and the corresponding value of T_o vary in all practical cases within moderate limits so that it is possible for the experimenter to reproduce also the "envelope of the envelopes of the spectra".

2) The second requirement, concerning the dynamic aspect of the problem, arises from the necessity to study the behavior of the model by introducing into it the possibilities the prototype possesses to free itself, both the "rigid" ones (as for instance, those provided by the discontinuities due to the presence of joints in dams) and the "plastic" ones. These possibilities may come into play already during earthquakes considered to be normal, and are certainly of decisive influence in limiting the strength reserves possessed by the structure in the case of exceptional earthquake.

The similitude conditions to be satisfied by the model in this case are rather complex.

a) First of all, since the principle of superimposition of effects is no longer valid in the presence of discontinuities and beyond the elastic range, it is necessary to reproduce simultaneously on the model all the main forces the structure is subjected to, viz.:

- dead load
- hydrostatic pressure (for dams)
- seismic action (which, in the case of dams, acts both directly and indirectly through the liquid mass).

b) The modelling has to reproduce also the characteristics of the prototype materials intervening in the dynamic phenomena and those which define their behavior outside the elastic range.

In what follows we shall set forth the quantitative relations brought about by the above-outlined considerations between the partaking quantities.

We shall assume the following fundamental ratios('):

a) (ratio between the lengths l), setting the "geometrical" similitude, i.e.:

$$\lambda = \frac{l}{l'}$$

(') The model quantities are denoted by an apex.

b) (time scale factor), which, together with λ , determines the "kinematic" similitude, or

$$\tau = \frac{t}{t}$$

c) (ratio between the surface forces), which, together with λ and τ , furnishes the "mechanical" similitude, i.e.:

$$\zeta = \frac{f}{f'}$$

Once the values of λ , τ and ζ are set, the ratios between all the other mechanical quantities coming into play are also defined, specially that between the accelerations

$$\alpha = \lambda \tau^{-2}$$

the one between the specific volume forces

$$\gamma = \zeta \lambda^{-1}$$

and that between the specific masses

$$\rho = \zeta \lambda^{-2}$$

When consideration is given only to the seismic action, all the three fundamental ratios can be selected at will, be thay the traditional λ , τ and ζ of theoretical mechanics or, as a more immediate reference to experimental practices, the three ratios

$$\lambda = \frac{l}{l'} \qquad \zeta = \frac{E}{E'} \qquad \rho = \frac{d}{d'}$$

between the lengths, the elastic moduli and the densities respectively, all the three being defined by the dimensions of the model and by the basic characteristics of the materials employed therein.

The model will then reproduce the seismic stresses σ (which, in the ultimate analysis, constitute the unknown of the problem) if the movement at the base of the model will satisfy, as to amplitudes, the scale factor λ , and as to the time, the time scale factor

$$\tau = \lambda \rho^{\frac{1}{2}} \zeta^{-\frac{1}{2}}$$

But, because of the reasons set forth at the beginning of this paragraph (and with special reference to dams), it is desirable to have the dead load and the hydrostaic pressure to act simultaneously with the seismic action. These are either volume forces or consequences of such forces. The scale of the volume forces is therefore defined by

$$\lambda \gamma = \zeta$$

Now, since the model has to be subjected to dynamic actions, it is practically impossible to satisfy the value of   given by expression (5) through the use of artifices (such as those employed in ordinary static model tests). Expression (5) can be met only "spontaneously" by applying the acceleration of gravity to the mass of the model, as given by ρ . Quite obviously, the acceleration of gravity is the same on both the model and the prototype, and hence $\tau = \sqrt{\lambda}$ and $\gamma = \rho$. Consequently, expression (5) becomes

$$\lambda\rho = \zeta$$

In conclusion, the requirement of applying various simultaneous actions leads in practice to imposing upon the model the condition (9), which arises from its connection with the surroundings and hence eliminates one of the three degree of dimensional freedom of the problem, rendering its solution much more difficult.

In fact, it should be noted that λ is to be considered as fixed (at least in its order of magnitude) by practical requirements, and that ρ too can vary only within rather restricted limits ('). There follows that the condition $\lambda\rho = \zeta$ is very restrictive, especially for high values of λ .

To the conditions of similitude thus far examined and concerning the application of the external forces (including the inertia forces) to the model structure, the conditions governing the true "response" of the structures itself must be added.

These conditions regard the modelling of the rheological properties of the materials of the prototype, and can be determined on the basis of one of the theories by now well developed and which try to interpret analytically the elasto-plasto-viscous phenomena of solids. This will enable to define easily all the dimensionless ratios (since all the dimensions have already been fixed) which, in passing from the prototype to the model, should remain unaltered. These are, for instance: h/E, i.e., the ratio between the constant of plasticity (Von Mises, constant) and Young's modulus; μ/ET, or the ratio between the viscosity constant and the product of Young's modulus by the free vibration period; M/μ i.e., the functions of M (depending on the stresses and the rate of deformation) and the viscosity constant ('')

(') It is to be noted that in the case of dams the ratio ρ enters twice, namely for the liquid in the reservoir and the concrete.

('') For a more detailed treatment of the conditions of similitude derived from the theories of elasto-plastic-viscous bodies see "G. Oberti - Il comportamento statico delle strutture oltre il campo elastico studiato a mezzo di modelli - Varenna 1955".

It seems however that such a conception of the conditions of similitude is based on a too simplified interpretation of the complex phenomenon. Less elegantly, but with greater adherence to the well-known experimental results it seems to us that it is more appropriate to define the conditions of similitude as follows:

a) As far as the energy dispersion phenomena are concerned, reference should be made to the results of free vibration tests. These, as is known, are fairly well characterized by the parameter r (designated as coefficient of damping in the theory of vibration). Even though its dependence on the rheological properties is not well defined, it has advantage that it can be easily determined experimentally.

The parameter r has the dimension of a frequency. Hence, when the dimensionless ratios are directly taken, it can be stated that the similitude of the dispersion phenomena will be satisfactorily secured if the ratio

$$\nu = \frac{r}{f}$$

(where f is the free vibration frequency of the structure), remains unaltered in passing from the prototype to the model.

b) As to the stresses which may be considered as dangerous it would seem to be sufficient to refer, as is done analytically, to the sole conditions of ultimate compressive and plain tensile strength, which obviously are to be reproduced in the ratio ζ . A more complete reproduction on the model will be obtained by referring to the ultimate strength conditions defined by the complete intrinsic curves $\sigma(\tau)$ of the two materials (Mohr). In passing from the prototype to the model, these curves will have to correspond homothetically to one another in the ratio ζ.

c) Regarding the other known and unknown rheological parameters coming into play it seems to be adequate to rely on the "relationship" between the physical structure of the two materials. Thus, in the case so far examined of modelling concrete structures it is desirable that the model materials also be concrete. It is certainly not too far from being true to assert that when the reproduction of the above-discussed fundamental parameters, like modulus of elasticity, ultimate loads, etc., is assured (preparing, for instance, a concrete which is harmonically "weaker" than the concrete of the prototype but yet composed of a binder and an aggregate), the other parameters too, like viscosity, plasticity and so on, are reproduced with a good approximation.

PART II

Following is a brief description of the dynamic test equipment installed at I.S.M.E.S. and of the extent to which its performance meets the conditions of similitude outlined above.

The steel platform for the placement of the complete foundation has 3 m x 4 m in plan.

The entire unit can be considered as rigid (so as not to introduce appreciable parassitic vibrations) and is suspended from a steel frame by means of springs. It is thus possible to apply to the model both horizontal and vertical vibrations.

The generation of vibrations is obtained through three distinct devices.

a) The first one consists of a pendulum-spring set of the type devised for Stanford University by L. Jacobsen, well known to students of the dynamics of structures;

b) The second one is a system of electromagnetic vibrators whose electronic control permits to vary independently the frequency and the amplitude of the vibrations and to limit moreover the seismic action to a very small number of cycles.

These two devices enable to obtain "spectra" of the type shown in Fig. 1 and which are, as may be seen, essentially similar to the spectra calculated by Housner and Biot for various earthquakes.

Through a number of repeated shocks having pulsations between 10 and 25 Hz (on the model) it is thus possible to cover the entire range of "spectra" considered probable by the geophysicists.

c) The devices described under a) and b) meet well the theoretical requirements of modelling earthquakes and their effects (kinematic similitude). But due to the influence of damping, the analysis of the dynamic characteristics of the structures under test becomes rather difficult. Hence, together with these devices a vibrodyne with a system of eccentric masses is employed (the force developed is up to 10 tons). This enable the generation of strictly sinusoidal and undirectional vibrations, both vertical and horizontal, at a constant amplitude (which is very useful for the above-mentioned analysis) and a frequency ranging between 5 and 25 Hz.
This frequency, on the other hand, can be varied only slowly because of the inertia of the rotating mass, so that, in view of the great reduction brought about in time due to similitude, each test represents an earthquake of very great duration.

Therefore, the "spectrum" (relating to a certain interval of frequancies) obtained through this equipment (Fig. 2) is, as can be seen, very similar to the classical resonance curve of forced vibrations.

The peak value a/a_o is thus defined by the scale value of the coefficient of internal damping of the structure under test.

The materials thus far employed (January 1960) for reproducing concrete consist of mixes of litharge (aggregate) and plaster of Paris (binder). The mechanical characteristics of these materials can be considerably altered by varying the proportions and by adding appropriate admixtures. The extreme values so far realized and those of common use are given in table 1.

As far as the basic conditions of similitude are concerned it should be noted that, since the relation $\lambda = \rho\zeta$ is generally to be satisfied, the above-said three mixes are capable of reproducing the elastic properties of the concrete when the values 150, 75 and 45 are assumed for the length scale (for E = 300,000 Kg/sq. cm there will in fact be $\zeta_a = 100$, $\zeta_b = 50$ and $\zeta_c = 30$).

As to the strength conditions it ought to be pointed out that the materials listed under b) and c) retain the ratio ζ set for the elastic moduli also in the case of ultimate strength, whereas the materials of type a) reproduce a concrete which has a noticeably lower strength but is endowed with greater plastic characteristics (the ultimate strains, which in the case of strict similitude should remain unaltered when passing from the prototype to the model) are much larger for the material of the types (a).

The values given for the coefficient of damping have been obtained through some free and forced vibration tests on small cantilever beams.

The dependence of the damping variation law on the frequency of vibration and on the stresses induced in the structures is still under test both on the model materials and on some type of concrete.

Reproduction of the reservoir water (in the case of dam models) is done by using a saturated saline solution having a density of 1.5 tons per cu.m., which is in very good agreement with the density of 3.7 tons/cu.m. of the matarials reproducing concrete.

From what has been said in Part I it is possible to see that both the testing equipment and the model materials do not rigorously meet all the conditions of similitude set forth in the first part of this paper.

Let us now examine the limits and the methods by which it is all the same possible to make use of the test results.

For this purpose we shall introduce the concept of "distorted model".

Let be a certain quantity of the problem under consideration (in our case the stress, the main unknown). This will have to satisfy the well-known expression

$$\sigma_m = \sigma_p \cdot \varphi(\pi_1, \pi_2, \cdots\cdots \pi_m)$$

which, following Bukingham's classical notations, summarizes the conditions of similitude.

Obviously, the correspondence between σ_m and σ_p is assured in an absolutely general way when each and all of the dimensionless ratios π_1 remain unaltered in passing from the prototype to the model: then a perfect model is obtained.

But it is equally possible to secure a correspondence between σ_m and σ_p if the effects of possible alteration of a ratio π_h are offset by the alteration of another ratio π_k; it could then be spoken of a "distorted model". This offsetting should obviously be defined theoretically or a least checked experimentally ('), and the use of such a model will be subject to limitations contrarily to what occurs for the perfect model, as it will be legitimate

(') Essentially, the distorted model represents a midway between the modelling technique and the direct study of the phenomenon as it transfers to this second plane the difficulties which were not overcome on the first one, with all the uncertainties brought about by this transfer, at least in the case of very complex phenomena such as ours.

only with respect to the quantity (or quantities) for which the offsetting itself has been assured.

Bearing in mind what has now been said concerning the "distorted model" as well as the conditions required for obtaining a "perfect model" set forth in Part I of this paper, we shall in what follows examine briefly the testing possibilities afforded by the above-described equipment and the materials available at I.S.M.E.S.

a) Analysis of structures for which all the mass forces, excepting the inertia ones due to the seismic action, are negligible.

Once λ has been set in its order of magnitude by obvious practical requirements, the material available makes it possible to choose, at equal values of ρ (which thus get fixed), the value ζ for which the conditions of similitude with respect to strength are also well met (mixes outlined under b) and c)).

In this case therefore the time scale is fixed only after the rheological requirements have been satisfied, and it is $\tau = \tau(\lambda, \rho, \zeta)$. On the basis of this and of λ, the seismic movement is definitely set, and its reproduction, for the available values of, λ, ρ and ζ, presents no difficulty. To be correct, the time scale is influenced also by the dispersion phenomena and the pulsations ω of the structure prove to be altered in accordance with expression $\omega_r^2 = \omega^2 + r^2$ (if the usual theoretical schemes are accepted), but, since r is much smaller than ω, this influence may be disregarded.

Not to be overlooked, on the other hand, is the effect exerted by the damping on the vibration amplitude of the structure, i.e., on its "response" to the earthquake. Since this "response" forms the object of the investigation, it is absolutely necessary to assure its reproduction in any case (that is, also for $r : r' \neq \tau^{-1}$).

For such purpose, an artifice suggested by the above-mentioned theory of "distorted model" may be adopted.

If the vibration amplitude $A \simeq a\ \omega_r^{-2}$ proves to be altered at a ratio π_h with respect to what there ought to be, the acceleration of the seismic action applied to the model can, on the other hand, be altered in the ratio $\pi_k = 1 : \pi_h$. This can obviously be very easily done by changing its amplitude (the duration too can be acted upon if electromagnetic vibrators are used, but such operation is much more delicate). No variation should, instead, be impressed on the frequency, inasmuch as such a variation would lead not only to a change of the peak value but also of the shape of the "spectrum" $a : a_0$; whereas the shape has to remain unaltered (')

(') It is to be noted that corrections of this kind have been rarely used by us due to the uncertainty caused by the scarcity of data available about the damping in the prototype structures. To remedy this deficiency in data researches are now being conducted.

b) Analysis of structures for which all the mass forces are to be taken into consideration.

From what has been said earlier it is clear that all the three basic ratios λ ρ, and ζ, and hence , are practically fixed "a priori".

For $\lambda \leq 75$ the materials available (obtained through long and painstaking research, as nothing in this case is left to the free will of the experimenter) make possible that also the conditions governing the failure be automatically conformed to.

The development up to collapse occurs as for the models described under a), including the possible corrections due to dissipating effects. It is only to be pointed out that in case the model is that of a dam with a full reservoir, it is more appropriate that the correction of the peak values of the structure's acceleration be made by varying the duration rather than the amplitude of the seismic action. This is because amplitude variations, even though correcting the inertia forces, of the structure, would also produce an undesirable alteration of the hydrodynamic overpressures for which the damping problem has a very different setup.

For $\lambda \leq 75$, the materials now available permit a faithful reproduction of the seismic effects in case the material remain within the elastic range and also when phenomena such as opening of joints, overturning, etc., occur in thé structure. The failure governing conditions are, however, not conformed to simultaneously.

The model may equally be made use of when condidering separately the elastic vibration stage and the cracking or collapse stage.

In the former stage, the model is essentially a complete model, and the considerations outlined for the previous models (with $\lambda \leq 75$) are also valid. In the latter stage the model is to be regarded as a "distorted model". As time scale factor. it is still permissible to assure, as for first stage, $\tau = \sqrt{\lambda}$, since the internal rigidity of the structure (which is a function of the ratio $\zeta_E = E/E'$ between the elastic moduli) stays practically unaltered; unchanged are the free vibration frequencies and, hence, those of the seismic action are to remain unvaried.

As to the effects of the seismic action, they depend, in the cracking stage, on the ultimate stress σ_R, for which the new scale ratio $\zeta_R = \tau_R \zeta_E$ is valid. Now, the similitude with respect to the stresses depends on two conditions, i.e.; that the stresses coincide with the ultimate ones and that $\sigma_R \equiv a = AT^{-2}$. As regards the first condition, each case is to studied separately. As to the second condition, since in passing from the elastic stage to failure the time scale remains approximately unaltered, it can be confirmed to when the amplitude scale of the seismic action is varied in the ratio $1 : \pi_R$ This can easily be done with the available equipment.

The above-outlined scheme is, however, obviously one of first approximation. In fact, it neglects the influence of the variation of the other applied loads, such as dead load and hydrostatic pressure (varying with π_R), attributing a paramount importance to the seismic action; it, moreover, disregards the undoutedly very rapid variation of the damping coefficient, for which a new factor, corrective of the seismic movement amplitudes (and whose elavuations is, at the present stage of knowledge, unknown) ought to be introduced. Furthermore, it requires that the behavoir of the structure be governed by higher stresses, since at a stage far from failure other scales are valid for the stresses. As if all this were not enough, there appears to be omitted also the influence exerted by the high ultimate strains of the materials under consideration.

However, it seems, possible to state that if one limits oneself, as is the present practice at I.S.M.E.S., to scan on the model the initial stage of failure the moment and the load at which it occurs, the shape and position of the incipient cracks, then the model may, even at that stage, provide the designer with very useful indications.

PRACTICAL EXAMPLES

The above-outlined criteria have been first applied in 1955 when, through the initiative of Engineer Carlo Semenza, the seismic behavior of the Ambiesta dam has been examined by the aid of a model; other dam models have been tested subsequently. Table II shows the basic characteristics of the structures investigated and the most significant data regarding the modelling and the results obtained.

As an illustration and complement of what has been said above and still remaining within the limits of the modelling problem we wish to point out as follows:

- The phenomenon of greatest interest to the designer is the possible dinamic increase of the amplitude when the frequency of vibration coincides with that of the free vibration of the model. The example given in Fig. 3 shows how this phenomenon may be very clearly seen: the frequency of free vibration is correctly reproduced and so also, with the above-mentioned due precautions, the peak value of the vibration amplitude A.

- Of the phenomena taking place beyond the elastic range, the following are of special interest:

a) the "crinkle effect", i.e., that only a part of the structure enters in resonance. This phenomenon is obviously of a transitory nature, and its limits are often defined by the correct reproduction of all the forces in action, which reproduction is thus decisive.

Thus for instance, the central part of a dam overhanging heavily on the downstream side is sometimes brought into vibration in the manner shown diagrammatically in Fig. 4. The water pressure, stabilizing the continuity along the arches, limits this effect, reducing the danger.

b) the variation of the basic frequency of free vibration, because of the incipient plasticization (Fig. 5). This phenomenon is reproduced by the materials that we are now employing with great quantitative uncertainties. It is interesting however that the model should bring it to light at least qualitatively as one of the resources used by the structure in order to avoid as far as possible the dangerous effects of resonance.

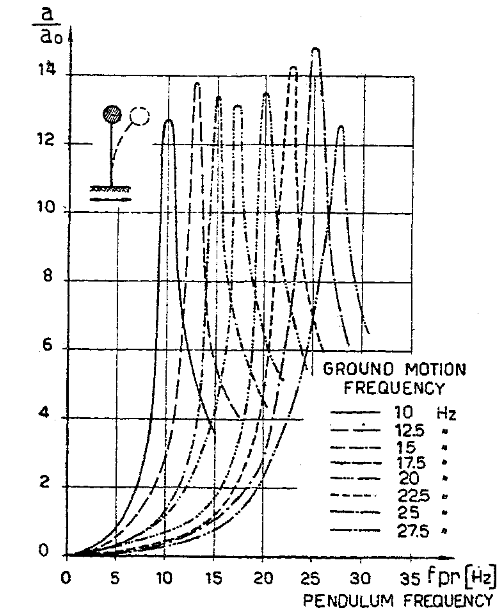

Fig. 1 "Seismic spectrum" for damped earthquake models and structu res with small damping coefficient.

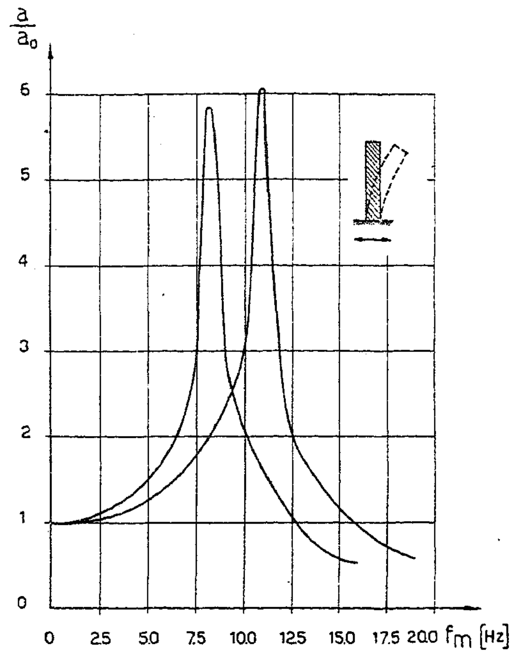

Fig. 2 "Resonance curves" for damped earthquake models and structu res with high damping coefficient.

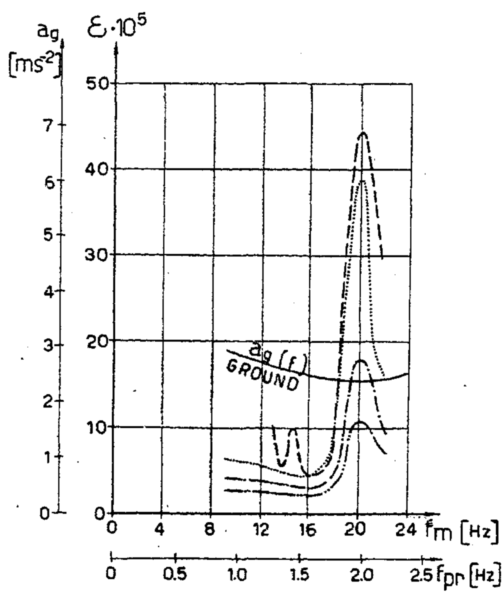

Fig. 3 Example of "dynamic amplification": strains in structure in function of the frequency of earthquake with very nearly constant acceleration.

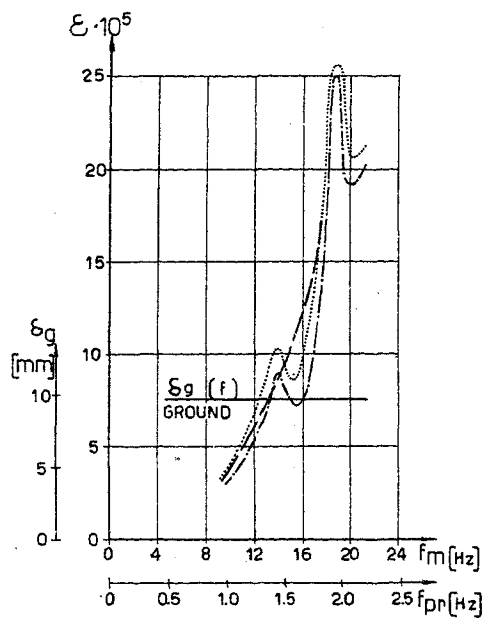

Fig. 4 Example of "crinkle effect".

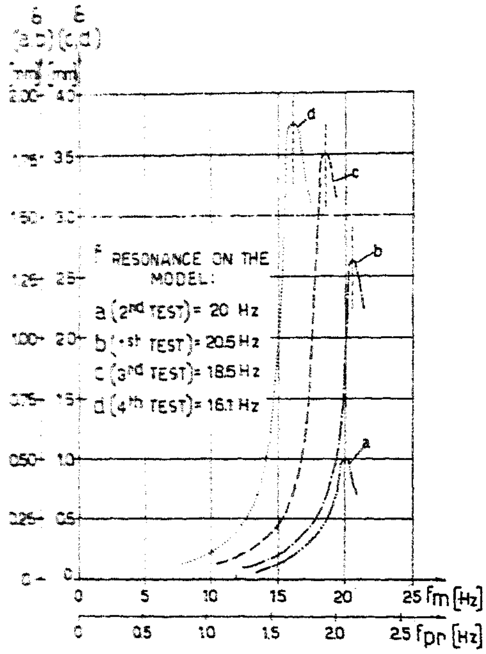

Fig. 5 Example of decrease in natural frequency of structure during successive testing.

Table I

CHARACTERISTICS OF MIXES:

density d, Young's modulus E, ultimate strengths in compression σ_C and flexure σ_F, dimensionless coefficient of damping r/f_o

MIX	d ($t\ m^{-3}$)	E ($Kg\ cm^{-2}$)	σ_C ($Kg\ cm^{-2}$)	σ_F ($Kg\ cm^{-2}$)	r/f_o
Type "A"	3,6	3.000	2,0	0,6	0,23
Type "B"	3,7	6.000	6,0	1,5	0,21
Type "C"	3,7	10.000	11,0	2,8	0,20

Table II

MODELS TESTED

D a m	Company	Chord (m)	Height (m)	Years of testing	Models tested	Length scale	Type of vibrations (')
Ambiesta	S.A.D.E. (Italy)	118	56	1957	3	75	a - b - d -
Kurobe IV	Kansai Electric Power Co. (Japan)	385	184	1958	4	180	a - b - c - d -
D e z	Development and Resources Corp. (U.S.A.)	200	184	1959	4	160	a - b - c - d -
Soledad	Comision Federal de Electricidad (Mexico)	130	91,5	1959	3	100	a - c - b -
Santa Rosa	Comision Federal de Electricidad (Mexico)	118	107	1959	3	100	a - c - d -
Grancarevo	Energoinvest (Jugoslavia)	340	123	1960	3	180	a - c - d -

(') a vertical undamped vibrations
b horizontal damped vibrations parallel to the chord
c horizontal undamped vibrations normal to the chord
d horizontal undamped vibrations parallel to the chord