LATERAL STABILITY OF A SUSPENSION BRIDGE SUBJECTED TO FOUNDATION-MOTION

by

A. Hirai [1], T. Okumura [2], M. Ito [3] and N. Narita [4]

ABSTRACT

The seismic coefficient method which has been used for design of structures in Japan is inadequate to very slender structures like a suspension bridge. A fundamental idea of analyzing the lateral stability of suspension bridge structures is presented here which may serve to make their aseismic design criterion.

The paper deals with the lateral vibration of a suspension bridge and its elastic stability under lateral forces, both theoretically and experimentally. In consequence, it is suggested that the aseismic design of a suspension bridge superstructure having a long span should be made under the condition of a given lateral displacement and the lateral buckling behavior of stiffening frame is to be checked.

INTRODUCTION AND GENERAL DESCRIPTION

Recently long-span suspension bridges tend to be proposed in Japan since across-the-strait bridges are progressively gaining their necessity. The plan makes it requisite to have resistance to strong earthquakes and storms. Especially, careful study should be made with regard to the lateral stability of such a long-span suspension bridge, because its rigidity in lateral direction is determined exclusively on the basis of these factors.

Someone says that the lateral rigidity of the stiffening frame of a suspension bridge will be governed not by the resistance to earthquakes but by that to wind forces. However, the numerical calculations for suspension bridges with a very long span over 800m show that it is not true, so far as the conventional seismic coefficient method is employed according to the Japanese bridge-design specification codes. Now an attention must be paid on the facts that a long-span suspension bridge is a very flexible structure and its fundamental natural period of lateral vibration will be much longer

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than the periods of earthquake motion. Therefore, it is doubtful whether the current seismic coefficients is usable or not.

An instructive information concerning the aseismic design criterion of a suspension bridge is found in the report of the Golden Gate Bridge 1). But the greatest acceleration assumed in this case was very small, especially for cables and suspended spans under earthquake waves acting transversely to the bridge, in contrast with the circumstances in Japan.

The authors and their research group have devoted themselves to the study of aerodynamic stability of suspension bridges over the past years and found that the lateral buckling behavior of stiffening frame might be a fatal condition of a suspension bridge subjected to lateral forces. Later in this paper, discussions will be made at this point in relation to the lateral stability of the bridge under seismic action.

It is believed that the aseismic analysis of such a flexible structure as suspension bridge should be carried out taking into consideration of its dynamic characteristics. Accordingly, the small lateral vibration of a suspension bridge will be discussed at first in this paper, followed by the stability problem and the results of the model experiments concerned. Since the present research aims at only obtaining some basic conceptions with regard to the lateral stability of a suspension bridge, the conclusive design criteria against earthquake waves are not related here.

LATERAL VIBRATION OF A SUSPENSION BRIDGE

In reality, the coupling between lateral, torsional and vertical motions should be taken into consideration when a suspension bridge is acted by wind forces or earthquakes transversely. However, as it leads to very intricate calculations, only the small lateral displacement is assumed here in order to obtain the basic characteristics of lateral vibrations of a suspension bridge structure.

Neglecting the additional cable tension caused by the displacement of bridge and considered very small in this case, the fundamental differential equations of vibratory lateral motion (see Fig. 1) of a suspension bridge are represented by

$$\frac{w_f}{g}\frac{\partial^2 v}{\partial t^2} + EI_v\frac{\partial^4 v}{\partial x^4} + \frac{w_f}{h(x)}(v-u) = P_f(x,t) \quad \text{- - - - - (1 a)}$$

$$\frac{w_c}{g}\frac{\partial^2 u}{\partial t^2} - H_w\frac{\partial^2 u}{\partial x^2} - \frac{w_f}{h(x)}(v-u) = P_c(x,t) \quad \text{- - - - - (1 b)}$$

where the last term of the left side of the equations

$$r(x) = \frac{w_f}{h(x)}(v-u)$$

shows the restoring force caused by the inclination of hangers the length of which $h(x)$ becomes for parabolic cables

$$h(x) = h_T - \frac{4f}{l^2} x(l-x) \quad \text{-------- (2)}$$

The above equations (1 a) and (1 b) express the coupled oscillations of main cables and suspended frame.

Natural Frequencies: Putting $p_c(x,t) = p_f(x,t) = 0$ and

$$v = \bar{v}(x) e^{i\omega_n t}, \quad u = \bar{u}(x) \cdot e^{i\omega_n t} \quad \text{-------- (3)}$$

in Eqs.(1 a,b) and considering the boundary conditions as transversely simply supported, the lateral vibration modes can be assumed

$$\bar{v}(x) = a_n \sin \frac{n\pi}{l} x, \quad \bar{u}(x) = b_n \sin \frac{n\pi}{l} x \quad \text{-------- (4)}$$

Then the following frequency equation is obtained according to the virtual work principle

$$\begin{vmatrix} EI_v\left(\frac{n\pi}{l}\right)^4 + \frac{w_f}{A_n} - \omega_n^2 \frac{w_f}{g} & -\frac{w_f}{A_n} \\ -\frac{w_f}{A_n} & H_w\left(\frac{n\pi}{l}\right)^2 + \frac{w_f}{A_n} - \omega_n^2 \frac{w_c}{g} \end{vmatrix} = 0 \quad \text{------ (5)}$$

where $A_n = h_T - 2f\left(\frac{1}{3} + \frac{1}{n^2\pi^2}\right)$ (see Fig. 1).

If $h(x)$ of Eq.(2) is assumed to be constant, h, the natural frequency corresponding to the vibrational mode of Eq.(4) becomes

$$\omega_n^2 = \frac{\alpha \pm \sqrt{\alpha^2 - 4\beta}}{2} \quad \text{------ (6)}$$

where

$$\alpha = \frac{(n\pi/l)^4}{w_f/g}\left\{EI_v + \left(\frac{l}{n\pi}\right)^2 H_w\left(\frac{w_f}{w_c}\right)\right\} + \frac{g}{h}\left(1 + \frac{w_f}{w_c}\right)$$

$$\beta = \left\{\frac{EI_v}{w_f/g}\left(\frac{n\pi}{l}\right)^4 + \frac{g}{h}\right\}\left\{\frac{H_w}{w_c/g}\left(\frac{n\pi}{l}\right)^2 + \frac{w_f}{w_c}\frac{g}{h}\right\} - \frac{w_f}{w_c}\left(\frac{g}{h}\right)^2$$

Equating the results of Eq.(5) and Eq.(6) the reduced or mean value of hanger length, h, is obtained, and making use of this constant value of h the forced lateral vibrations of a suspension bridge will be treated more easily, though the process is an approximate one.

Incidentally, Mr. Silverman [2)] proposed the formula to calculate the natural frequencies just related, but the fundamental equation of vibratory motion he derived has some doubts about the coupling motion of cables and suspended structure. The results obtained from his formula did not agree with the test results as shown later.

If the following expression is used as an approximate deflection form instead of Eq.(4), the accuracy of the solution will be improved but the calculation becomes very complicated.

$$\left.\begin{aligned}\bar{v}(x) &= \sum a_k \sin\frac{k\pi x}{\ell} \\ \bar{u}(x) &= \sum b_k \sin\frac{k\pi x}{\ell}\end{aligned}\right\} (k = 1,\ 2,\ 3,\ --) \quad --\ (7)$$

For example, the natural periods of lateral vibrations in a tentative design proposed for the George Washington Bridge were calculated by Eq.(5) and the following results were obtained.

n	Period of Vibration (sec)
1	21.0 or 4.2
2	8.6 or 4.1
3	4.2 or 3.1
4	2.5 or 2.4

As understood from the above results, the natural period of lateral vibration of a long-span suspension bridge is generally very long. Two values of the natural period obtained for each value of n are corresponding to the same phase and the opposite phase of the vibratory motions of cables and stiffening frame, respectively.

Response to Forced Vibrations: If the movement of ground perpendicular to bridge axis is expressed by a sinuous time function with the period of $2\pi/\omega$, the differential equations of the forced lateral vibration of a suspension bridge become

$$\left.\begin{aligned}\frac{w_f}{g}\frac{\partial^2 v}{\partial t^2} + EI_v\frac{\partial^4 v}{\partial x^4} + \frac{w_f}{h(x)}(v-u) &= \frac{w_f}{g}\bar{a}\omega^2 \sin\omega t \\ \frac{w_c}{g}\frac{\partial^2 u}{\partial t^2} - H_w\frac{\partial^2 u}{\partial x^2} - \frac{w_f}{h(x)}(v-u) &= \frac{w_c}{g}\bar{b}\omega^2 \sin\omega t\end{aligned}\right\} \quad ---\ (8)$$

where the constants $\bar{a}$ and $\bar{b}$ will be determined from the nature of earthquake motion and the dynamic characteristics of towers and the term of damping force is neglected.

Assuming that $h(x)$ is a constant and making use of the vibration mode as in Eq.(4), the solutions for transient- and steady-state oscillations are obtained. In this case, the dynamic magnifiers for the latter become

$$\left.\begin{aligned}\vartheta_f &= \frac{\frac{4\omega^2}{n\pi g}\left[w_f\left\{\frac{w_f}{h} + H_w\left(\frac{n\pi}{\ell}\right)^2 - \frac{w_c}{g}\omega^2\right\} - \frac{\bar{b}}{\bar{a}}w_c\frac{w_f}{h}\right]}{\left[EI_v\left(\frac{n\pi}{\ell}\right)^4 + \frac{w_f}{h} - \frac{w_f}{g}\omega^2\right]\left[H_w\left(\frac{n\pi}{\ell}\right)^2 + \frac{w_f}{h} - \frac{w_c}{g}\omega^2\right] - \left(\frac{w_f}{h}\right)^2}; \text{ for frame} \\ \vartheta_c &= \frac{\frac{4\omega^2}{n\pi g}\left[w_c\left\{\frac{w_f}{h} + EI_v\left(\frac{n\pi}{\ell}\right)^4 - \frac{w_f}{g}\omega^2\right\} + \frac{\bar{a}}{\bar{b}}w_f\frac{w_f}{h}\right]}{\left[EI_v\left(\frac{n\pi}{\ell}\right)^4 + \frac{w_f}{h} - \frac{w_f}{g}\omega^2\right]\left[H_w\left(\frac{n\pi}{\ell}\right)^2 + \frac{w_f}{h} - \frac{w_c}{g}\omega^2\right] - \left(\frac{w_f}{h}\right)^2}; \text{ for cable}\end{aligned}\right\} \quad --\ (9)$$

However, an actual seismic motion is different from the expression in Eq.(8). In order to gain the real dynamic response in the system the following procedures will have to be employed.

1) To obtain a better approximation, the duration of seismic motion, namely the period and the number of earthquake waves, should be assumed. Then after applying some calculation process like Duhamel's integral to Eq.(8), the dynamic response of the system can be gained.

At that time, the method proposed by Professors S. Okamoto and K. Kubo [3] will be useful, and the energy spectra should be considered together with damping effect.

2) Response spectra would be obtained in a satisfactory accuracy for optional ground-motion if proper analog computer is available.

The evaluation of damping force plays an important role in aseismic design of structures. But the reliable data of damping factor usable for the lateral vibration of a suspension bridge has not been found yet.

SUSPENSION BRIDGES SUBJECTED TO STATICAL LATERAL FORCES

Solutions according to Seismic Coefficient Method: As mentioned previously, the conventional seismic coefficients specified in the codes are not suitable for aseismic design of a suspension bridge having long span. Furthermore, it is necessary to take into consideration the fact that the rigidity of a structure governs its response to dynamic forces. Since the problem is in a category of dynamics, the fundamental period of a structure largely determines the magnitude of the induced seismic forces, and it is a reason why the natural period of the lateral vibration of a suspension bridge has evaluated in the previous section. Generally speaking, flexible structures having longer natural periods respond less than rigid structures. It is also believed that, if the distribution of seismic forces which will vary to a great extent in a case of suspension bridge structure is known, the seismic coefficient method may be usable in the present problem.

Accordingly, the displacement of a suspension bridge subjected to static forces proportional to the mass of structural elements will be calculated here. In connection with Eqs.(1 a, b), the following approximate relations to solve are obtained.

$$\left.\begin{aligned} EI_v \frac{d^4 v}{dx^4} + \frac{w_f}{h(x)}(v-u) &= k w_f \\ -H_w \frac{d^2 u}{dx^2} - \frac{w_f}{h(x)}(v-u) &= k w_c \end{aligned}\right\} \text{- - - - (10)}$$

Taking the displacements of stiffening frame and cable as

$$\left.\begin{aligned} v &= a_1 \sin\frac{\pi x}{l} + a_3 \sin\frac{3\pi x}{l} + a_5 \sin\frac{5\pi x}{l} \\ u &= b_1 \sin\frac{\pi x}{l} + b_3 \sin\frac{3\pi x}{l} + b_5 \sin\frac{5\pi x}{l} \end{aligned}\right\} \text{- - - (11)}$$

and making use of the variation method, the following equation (12) on the next page to determine the unknown constants a_i and b_i is obtained.

Similar problem was discussed by Mr. L.S. Moisseiff [4] in connection with the wind forces applied to a suspension bridge. His

$$\begin{bmatrix} \alpha^2 A_1+\delta & \frac{243}{2}\alpha\beta & \frac{3125}{18}\alpha\beta & -\delta & 0 & 0 \\ \frac{3}{2}\alpha\beta & 81\alpha^2 A_3+\delta & \frac{9375}{8}\alpha\beta & 0 & -\delta & 0 \\ \frac{5}{18}\alpha\beta & \frac{1215}{8}\alpha\beta & 625\alpha^2 A_5+\delta & 0 & 0 & -\delta \\ -\gamma_f & 0 & 0 & \alpha A_1+\gamma_f & \frac{27}{2}\beta & \frac{125}{18}\beta \\ 0 & -\gamma_f & 0 & \frac{3}{2}\beta & 9\alpha A_3+\gamma_f & \frac{375}{8}\beta \\ 0 & 0 & -\gamma_f & \frac{5}{18}\beta & \frac{135}{8}\beta & 25\alpha A_5+\gamma_f \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_3 \\ a_5 \\ b_1 \\ b_3 \\ b_5 \end{bmatrix} = K \cdot \begin{bmatrix} \delta F \\ \delta G \\ \delta H \\ \gamma_c F \\ \gamma_c G \\ \gamma_c H \end{bmatrix} \quad \text{-- (12)}$$

$$\alpha = \frac{\pi^2}{l^2}, \quad \beta = \frac{f}{l^2}, \quad A_i = h_T - 2f\left(\frac{1}{3} + \frac{1}{i^2\pi^2}\right)$$

$$F = \frac{4}{\pi}\left(h_T - \frac{8f}{\pi^2}\right), \quad G = \frac{4}{3\pi}\left(h_T - \frac{8f}{9\pi^2}\right), \quad H = \frac{4}{5\pi}\left(h_T - \frac{8f}{25\pi^2}\right)$$

$$\delta = \frac{w_f}{EI_v}, \quad \gamma_f = \frac{w_f}{H_w}, \quad \gamma_c = \frac{w_c}{H_w}$$

theory is known as the Uniform Distribution Method and the Elastic distribution Method, which are based on the different point of view from the authors' just mentioned but have applied to the aseismic design of suspension bridges so far.

Numerical calculations were made with the tentatively proposed design for the George Washington Bridge, and the results obtained from Eq.(12) and from the Moisseiff's Elastic Distribution Method were compared as shown in Fig. 2. In consequence, the problem is how to estimate the seismic coefficients for a suspension bridge structure, and its solution will be obtained from the standpoint of dynamics.

Lateral Elastic Stability of Suspension Bridges: It is fully conceivable that the dynamic behavior of a suspension bridge under seismic action is somehow connected with that under wind action. The aerodynamic stability of suspension bridges has been investigated over the past years by one of the authors, A. Hirai[5], and his colleagues. They have pointed out several conditions for the aerodynamic stability of suspension bridges, among which the stability condition against the lateral buckling behavior of stiffening frame is to be noted.

In reality, the coupling of vertical, torsional, and lateral motion should be considered when a suspension bridge is subjected to lateral forces. In this case, the lateral buckling - or "Kippung" - of stiffening frame with one-noded asymmetric buckling mode might be fatal, if the stiffening frame has small torsional rigidity and large flectional rigidity in lateral direction. This phenomena were observed experimentally [6] when the laterally distributed load was statically applied to suspension bridge models.

According to the theoretical analysis by A. Hirai and H. Chikuma, the critical lateral load which is uniformly distributed along bridge axis and causes the lateral buckling behavior is approximately given by [6]

$$q_{cr} \doteqdot \kappa \frac{62.68\sqrt{\mathit{EJ}\cdot \underline{GK}}}{l^3} \qquad ------ (13)$$

where, $\mathit{EJ} = EI + \frac{l^2}{4\pi^2} H_w \,, \quad \underline{GK} = GK + \frac{\pi^2 b^2}{l^2} \mathit{EJ}$

$$\kappa = \frac{c^2 \cosh \frac{c}{2}}{8\left(\cosh\frac{c}{2} - 1\right)} \,, \quad c = l\sqrt{\frac{H_w}{EI_v}}$$

Consequently, it is necessary to increase dead load and torsional rigidity of bridge in order to augment the value of q_{cr}. On the other hand, another critical load $(q_{cr})_m$ for bending moment in the stiffening frame is considered to exist and the calculation brings

$$(q_{cr})_m \doteqdot \kappa \frac{2\sqrt{128} \cdot I_v}{l^2 \cdot b} \sigma_a \qquad ------ (14)$$

Comparing Eq.(13) with Eq.(14), one can find that the rate of increase of q_{cr} due to the increase in dead load is higher than that of $(q_{cr})_m$. Accordingly, if $(q_{cr})_m < q_{cr}$ the stiffening frame will collapse through simple bending due to lateral force before it buckles. The condition for $(q_{cr})_m < q_{cr}$ is

$$c > 2.2\sqrt{\frac{\sigma_a}{E}}\,\frac{l}{b} \qquad ------ (15)$$

The installation of center diagonal stays which tie the cables to the stiffening frame at the lowest point of the cable is desirable to improve the stability of suspension bridges. In this case the lateral buckling mode of the frame becomes symmetrical one with two nodes in general, and the critical lateral load corresponding to Eq.(13) is

$$q_{cr} \doteqdot \kappa \frac{100.4\sqrt{\mathit{EJ}\cdot \underline{GK}}}{l^3} \qquad ------ (16)$$

where, $\mathit{EJ} = EI + \frac{l^2}{9\pi^2} H_w \,, \quad \underline{GK} = GK + \frac{9}{4}\frac{b^2\pi^2}{l^2} \mathit{EJ}$

But it should be reminded that some margins or plays in stays decrease the above critical load.

DYNAMIC RESPONSE OF TOWERS

As compared with the stiffness in the longitudinal direction, the lateral stiffness of towers is assumed to be so high that the acceleration due to earthquake is extended over the full length of this structure. The seismic coefficients specified in the current design codes will have to be applied to the design of towers. The horizontal force acting on each tower due to the acceleration of suspended spans and cables will be able to be estimated from the results previously investigated.

The natural frequency of towers in lateral direction is calculated as a rigid frame structure with two or three layers. The reaction from cables and stiffening frame should be taken into consideration.

MODEL TESTS AND THEIR RESULTS

Test Equipments: The vibration-table (see Fig.3) used in the small-scaled model tests has a very simple structure but is specially designed for the lateral vibration tests of bridge models. The table to place a model has the dimensions of 4.0m in length and 0.6m in width, and is supported by eight plate-springs. Lateral vibration of the table is given by the rotating unbalanced weights attached directly to the table and driven by a motor. The weight of the table is about 370kg and is much larger than that of bridge models in the present experiments.

The amount of unbalance of the counter-weights can vary into 8 stages which were numbered from 0 to 7 (see Fig.4). The characteristic diagram of this vibration-table is illustrated in Fig.4. Vibration tests of bridge models making use of this table can be conducted in the ranges of 0.3g - 1.2g in acceleration, 0.5 - 2.8mm in amplitude of displacement, and 5 - 18cps in frequency.

Suspension Bridge Models: The suspension bridge models used in the tests had a single span, straight backstays, and the following dimensions and details.

Span: l = 300cm , Cable Sag: f = 30cm , Sag Ratio: 1/10
Height of Towers above Stiffening Frame: h_T = 33cm,
Main Cables: ϕ2mm stranded wires, Hangers: Jute thread.

Towers and stiffening frames were made of steel or brass and the former was designed as a rigid frame.

Four types of stiffening frame were used in tests and the dead weight of cables and stiffening frame was variable. Consequently, eight cases of modifications were tested as summarized in the tables on the next page.

Dynamic stresses were measured by the use of electrical resistance strain-gages and recorded through electro-magnetic oscillograph. The strain-gages were mounted on main parts of the bridge model, and the stresses in stiffening frame, tower, and main cable were measured.

Free Lateral Oscillations: The natural periods of vibration and the damping factor were read from the recorded oscillopapers. But in most cases it was almost impossible to get the reliable data of damping

TYPE	I-A	I-B	II-A	II-B	III-A	III-B	IV-A	IV-B
SECTION								
EI ($\times 10^8$ gr-cm^2)	1.75	1.75	0.17	0.17	0.135	0.135	0.57	0.57
EI_v ($\times 10^8$ gr-cm^2)	1.75	1.75	25.8	25.8	18.3	18.3	36.2	88.7
w_g (gr/cm)	7.85	16.25	6.7	13.6	15.14	15.14	14.86	14.86
w_c (gr/cm)	0.12	0.12	0.24	0.24	0.24	3.17	0.24	0.24
H_w (kg)	2.99	6.14	2.61	5.19	5.77	6.89	5.66	5.66
q_{cr} (gr/cm)	318	325	341	382	263	275	138	138

SUSPENSION BRIDGE MODELS USED

factor. In the model tests, the natural frequencies were obtained up to the 3rd mode and it was proved that the theoretical values calculated from Eq.(5) were in good agreement with the test results. But in order to get the better results as to the fundamental period of lateral vibration, it will be necessary to take two terms in Eq.(7) instead of the assumption of Eq.(4).

Logarithmic decrement obtained from tests was around 0.02 - 0.05, and the damping factor in lateral vibrations was generally lower than in vertical vibrations.

Forced Vibrations: Some examples of the dynamic responses obtained from the model tests were displayed in Figs.5 - 8. Figs.5, 6, and 7 show the lateral bending moment induced in stiffening frame due to the periodic foundation-motion, and the theoretical values according to the seismic coefficient method were also shown by curves. Fig.8 is for the towers in the same case.

The resonant frequency for the stiffening frame is not always coincide with the natural frequency of suspended spans, if towers are not so rigid in lateral direction.

Except for the regions in the neighborhood of resonant frequency, the maximum bending moment is constant independently of the forced acceleration and is rather proportional to the amplitude of foundation motion. Moreover, in the above case the test results were much lower than the values estimated under the assumption that the statical inertia forces proportional to the given acceleration were applied to structures. However, the transient motion should be experimentally investigated too.

Unless the vertical and/or torsional oscillations were induced, the additional cable tension due to inertia forces was negligible as compared with the cable tension due to dead load. It must be noted that, if the mass of cables is large, the cables might oscillate independently of stiffening frame. This phenomenon was observed in the test with Model III-B.

When the torsional rigidity of stiffening frame is large like Models I-A, B, the motion is considered to be purely lateral vibration. On the other hand, in the case of the stiffening frame with small torsional rigidity such as Models II or III, the coupling of vertical and torsional motions with lateral motion was observed and the catastrophic stage of bridge was undoubtedly caused by the lateral buckling of stiffening frame.

Judging from the test results in Fig.8, the seismic coefficient method may be applicable to towers. Furthermore, it should be reminded that the dynamic characteristics of towers also affect the dynamic response of suspended spans.

CONCLUSIVE REMARKS

The outline of the lateral vibration and the lateral stability of suspension bridges was discussed, but the paper presents only the basic ideas or suggestions for the above subjects, and there remain a lot of problems to investigated. Anyway, as far as the aseismic design of a suspension bridge in its lateral direction is concerned, the dynamic behavior of the structure must be taken into consideration. Especially, it is necessary to know the energy spectra at the time of earthquakes and the dynamic behavior of towers.

BIBLIOGRAPHY

1) L. S. Moisseiff, "Provision for Seismic Forces in Design of the Golden Gate Bridge"; Civil Engineering, January 1940, Vol.10, No.1.

2) Silverman, "Lateral Stiffness of Suspension Bridges"; Proc. ASCE, EM Div., Vol.83, 1957.

3) K. Kubo, "The Aseismic Design of a Suspension Bridge in its Longitudinal Direction"; Proceedings of the 2nd World Conference on Earthquake Engineering, Tokyo, 1960.

4) L. S. Moisseiff, "Suspension Bridges under the Action of Lateral Forces"; Trans. ASCE, Vol.98, p.1080, 1933.

5) A. Hirai, "Aerodynamic Stability of Suspension Bridges"; Publ. 5th International Congress for Bridge and Structural Engineering, Lisbon, 1956.

6) A. Hirai and H. Chikuma, "The Aerodynamic Stability of Suspension

Bridges with reference to the Wakato Bridge" (Part II); The Bridge Engineering Laboratory, University of Tokyo, and The Wakato Bridge Construction Office, Japan Road Corporation; 1960.

NOMENCLATURE

b : Distance of main cables

f : Cable sag

l : Span length

x : Distance along bridge axis

u : Lateral displacement of main cables

v : Lateral displacement of stiffening frame

$h(x)$: Length of hangers

h_T : Height of towers above stiffening frame

t : Time

K : Horizontal seismic coefficient

g : Gravitational acceleration

E : Modulus of elasticity

EI : Flectional rigidity of stiffening frame in vertical direction

EJ : Reduced flectional rigidity of a suspension bridge in its vertical direction

GK : Torsional rigidity of stiffening frame

$\underline{GK}$: Reduced torsional rigidity of a suspension bridge

EI_v : Flectional rigidity of stiffening frame in lateral direction

H_w : Horizontal component of cable tension due to dead load

w_f : Dead weight of stiffening frame per unit length of bridge

w_c : Dead weight of cables per unit length of bridge

q_{cr} : Critical lateral buckling load per unit length of bridge

$(q_{cr})_m$: Critical lateral load against the bending of stiffening frame per unit length of bridge

$r(x)$: Restoring force due to the inclination of hangers per unit length of bridge

ϑ : Dynamic magnifier

ω : Circular frequency of periodic foundation-motion

ω_n : Natural circular frequency of a suspended span

σ_a : Allowable bending stress intensity of the material consisting stiffening frame

Other notations are given where they appear.

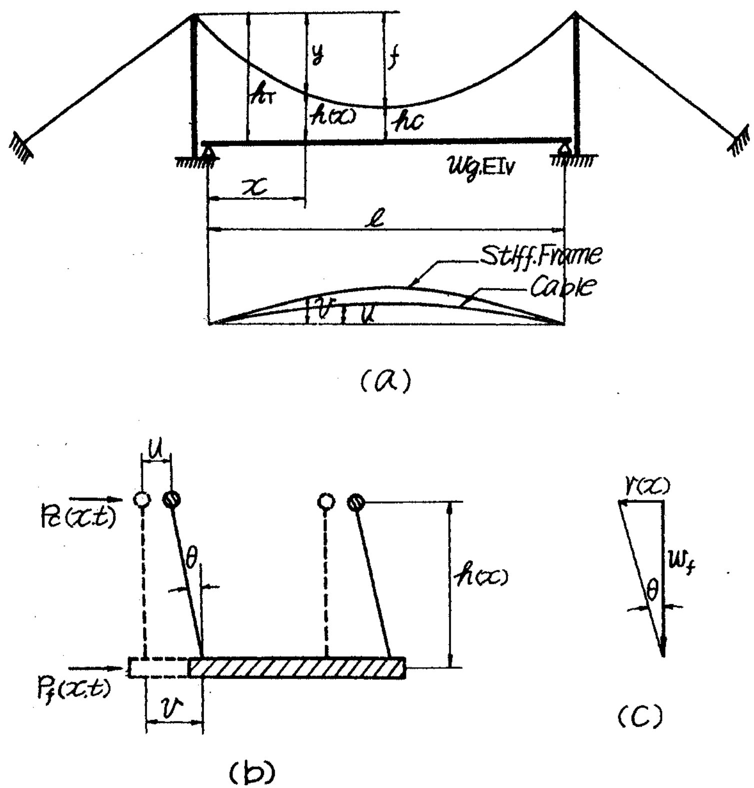

Fig. 1 Small Lateral Displacement of a Suspension Bridge

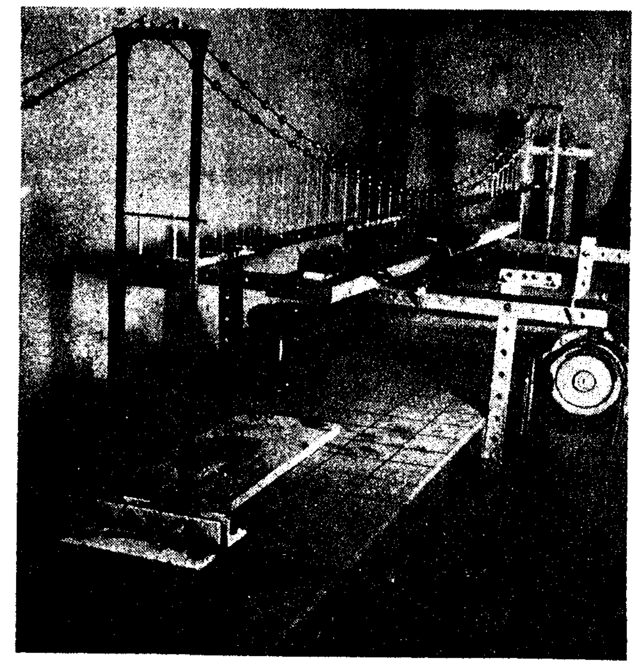

Fig. 3 Model Test of the Lateral Vibration of a Suspension Bridge

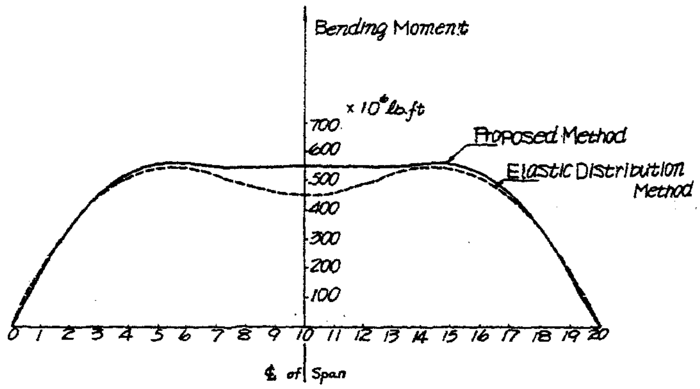

Fig. 2 A Numerical Example of the Bending Moment induced in Stiffening Frame of a Suspension Bridge subjected to Lateral Statical Forces

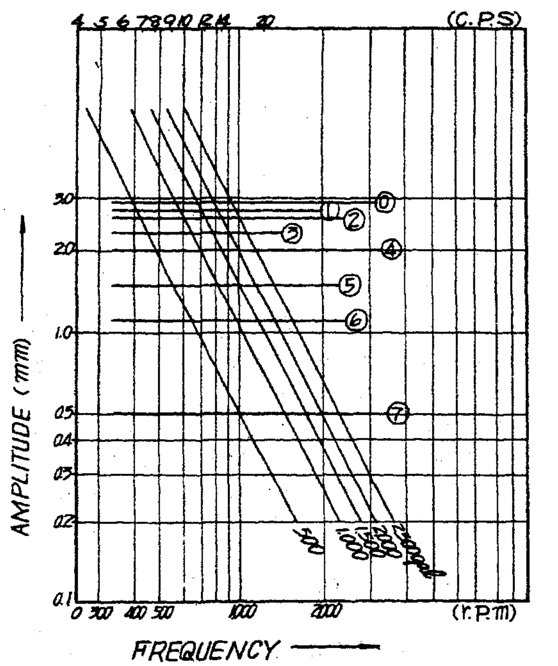

Fig. 4 The Characteristic-Diagram of the Vibration-Table used in Tests

Note: Number in circle indicates the degree of unbalance of the counter-weights.

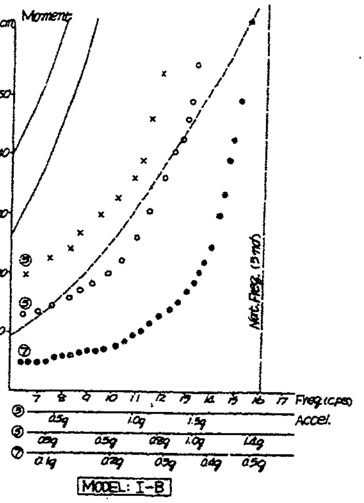

Fig. 5 Test Results of Dynamic Response (Model I-B)

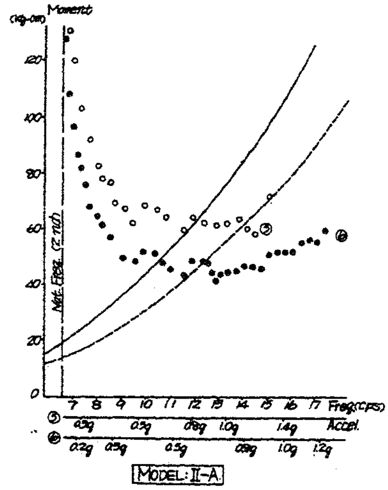

Fig. 6 Test Results of Dynamic Response (Model II-A)

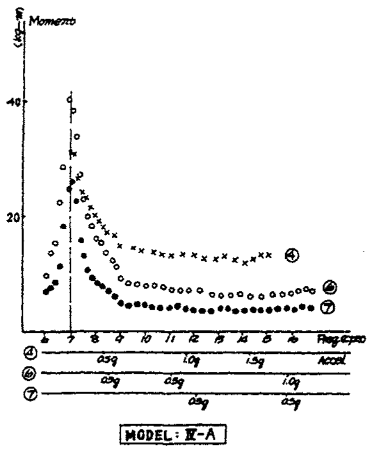

Fig. 7 Test Results of Dynamic Response (Model IV-A)

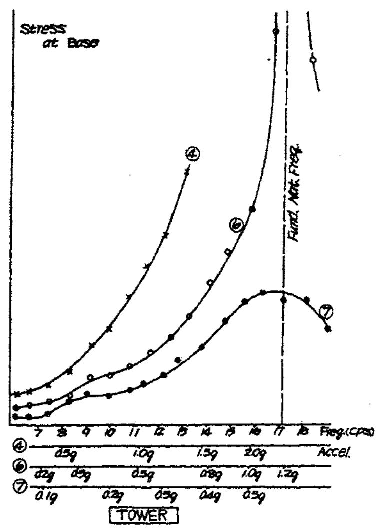

Fig. 8 Test Results of Dynamic Response (Tower)

Note: In Figs.5, 6, 7, the curves indicate the calculated results under the assumption that the lateral forces proportional to the given acceleration were applied to the span.