Aseismicity of Suspension Bridges Forced to Vibrate Londitudinally.

by K.Kubo*

Chap. 1 Introduction

It is the usual aseismic design method of a bridge, to apply the statical load horizontally, the amount of which is equal to the weight of the bridge multiplied by the seismic coefficient.

It is very difficult and troublesome to design earthquake-proof bridges from the view point of dynamics, because in Japan there is no record of a strong motion earthquake, and the geological conditions are very complicated and the foundations are made of various kind of soils, and therefore a record of a strong motion earthquake at a certain place does not represent the characteristics of a strong earthquake at the another place. By reason of the fact stated above, in the dynamical analysis, the ground motion during an earthquake must be assumed to be given as a sinusoidal wave.

However, the damping constant of the suspension bridge is far smaller than the one of the other type of bridges, and the amplitude of forced oscillation of the vibrating body with a small damping constant becomes extremely large, when the period of the external force is the one of the vibrating body. In case of the suspension bridge, its vibrational amplitude during an earthquake might become much larger than estimated under the assumption that the seismic load is statically applied to the suspension bridge and it must be needed to perform dynamical calculation for design of a suspension bridge, or at least to check the seismic response of the suspension bridge designed statically, in both of which the suspension bridge is treated no more statically but dynamically.

In this paper, characteristics of forced vibration of the suspension bridges are described. In this case, the movement of the foundation is assumed to be represented by a sinusoidal wave. The investigation was done theoretically as well as experimentally. In the experimental analysis, vibration tests were carried out using the model which is 1/100 of the prototype in scale and the shaking tables, and checked the validity of the theory, which is given by the writer.

The damping coefficients of the actual suspension bridges whose span is more than 100 meters were measured and it is analyzed that the damping constants of the suspension bridges exist between 0.004 and 0.006.

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Chap. 2 Model Test

To obtain the general characteristics of the vibrating suspension bridge, the model test was performed , in the first place. In the model test the similarity between the model and the actual suspension bridge is the most important, and when the prototype is induced into the forced vibration by an earthquake, the vibrational amplitude, for instance, the deflection of the span center, can be determined by the elastic properties and the damping coefficient of the suspension bridge, and period and amplitude of the external force. It is very difficult for us to make the model, the similarity of which is sufficiently satisfied about elastic responce, stresses, damping, and the vibrational mode. And therefore in this research, only similarity of the vibrational mode of the suspension bridge is considered. The dimensions of the model and the method, by which the size and the weight of the model are decided, will be stated later.

The vibrations of the suspension bridge caused by an earthquake are classified into the following four types.

1) The vertical vibration due to a horizontal ground motion which moves parallel to the axis of the bridge.
2) The horizontal vibration, which is parallel to the axis of the suspension bridge, due to such a ground motion as stated in 1).
3) The vertical vibration due to a vertical movement of the ground.
4) The horizontal vibration perpendicular to the axis of the bridge due to a ground motion which moves perpendicular to the axis of the bridge.

In the above four types of vibrations, the second one is considered not to play an important part, and stress and deformation caused by the forth type of vibration may be no more than stress and deformation caused by typhoon. The stresses and deformations which are given rise to by typhoon, are already investigated and we can design a suspension bridge for typhoon. The third type of vibration as well as the first one must be studied in order to analyze the aseismisity of the suspension bridges.

As the analysis on the vertical vibration due to a vertical ground motion has not yet finished, the investigation on the third type of vibration will be explained in the followings.

The kinetic equation for the vertical movement of the suspension bridge is described as follows by Dr. F. Bleich,

$$\frac{m}{\sqrt{2}} Z \frac{Z+1}{\sqrt{Z-1}} \tan\frac{\mu}{2} - \frac{Z-1}{\sqrt{Z+1}} \tanh\frac{\nu}{2} - \frac{m^3 H_w^2 L Z}{32\sqrt{2}\, \omega l f A_c E}(Z^2-1) \doteq 0 \qquad (1)$$

(for symmetric vibration)

and

$$\frac{m}{2}\sqrt{Z-1} = 2n\pi \qquad (2)$$

(for unsymmetric vibration)

where

$$\mu = \frac{m}{\sqrt{2}}\sqrt{Z-1} \qquad \nu = \frac{m}{\sqrt{2}}\sqrt{Z+1}$$

$$m^2 = \frac{H_w l^2}{EI}$$

$$H_w = \frac{wl^2}{8f}$$

$$Z = \sqrt{1 + \frac{32f\gamma^2}{m^2 g}}$$

l = span
w = load per unit length
f = sag
r = circular frequency of the suspension bridge
L = total length of the cable
E = Young's modulus of the cable
A = cross-sectional area of the cable

As it is impossible to make the model so that the period of the model may be the one of the prototype, the ratio of the frequency of the prototype to the one of the model can not be equal to unit but equal to ξ , the value of which is smaller than unit. In this paper, we attach the suffix, p, to the prototype, and the suffix,m, to the model, respectively.

Let γ_p/γ_m be ξ , then f_p/f_m must be $(1/\xi)^2$, in order to make the value of $f\gamma^2$ invariable. And we can make the value of m^2 invariable, by making $w_p l_p^4/8f_pE_pI_p$ to be equal to $w_m l_m^4/8f_mE_mI_m$, in which w_m and I_m are undecided parameters and the geometrical dimensions of the model are ξ times of that of the prototype. At last, all terms in Eq. (1) will be able to be maintained invariably, by making $l_pL_p/f_pE_{cp}A_{cp}$ to be equal to $l_mL_m/f_mE_{cm}A_{cm}$.

If the values, w_m , I_m , and A_{cm} of the suspension bridge are properly chosen, the circular frequency of the first mode of vibration of the model is given as ξ times of the one of the prototype, and the circular frequencies of the higher modes are decided by the same principle, too. Actually, one of the three values, w_m I_m and A_{cm} , can be peefectly arbitrarily determined when the similarity between the model and the prototype is confined within the frequency of the suspension bridge, and then in this research, the sectional area of the main cable, A_{cm} was decided in the first place, and in this case it is taken into account that the dimensions of the model do not become so large and the experimental work are easily done. The diameter of the main cable, which was piano wire, was 1 mm. And actually the value of ξ is one tenth. The dimensions of the model of the suspension bridge are shown in the followings.

$A_{cm} = 0.0079\ cm^2$	$w_m = 82.79\ g/cm$
$l_m = 364\ cm$	$h_m = 89.8\ cm$
$I_m = 0.083\ cm^4$	$I_{1m} = 0.083\ cm^4$
$f_m = 36.4\ cm$	

The area of the vibrating table on which a tower and an anchorage of the model were set, is 1m x 1.5m , and the table was shaked horizontally by the motor of 5 HP , the rotation speed of which was changeable. The other tower and the other anchorage of the model were set on the another table which can be moved with the various and different phase from the former table, and was shaked by the same motor of the former table. (see Photo. 1)

The deflections of the stiffening girder were measured at the points of 1/6, 1/3, 1/2, 2/3, and 5/6 of the span by the change of the electric resistance of the slide rheostat and the stresses of the main cable and the towers were measured by electric wire strain gauges.

The results obtained are shown in Fig. 1. Conclusions of the experiment are; 1) the vibration period of the tower is far smaller than the one of the super-structure of the bridge, 2) the fundamental equation on the vertical vibration of the suspension bridge is fairly good, 3) the vibration of the first symmetrical mode causes the most dangerous stresses of the main cable, 4) the period of the vibration of the first symmetrical mode is 0.310 sec. by the model experiment and as ξ is one tenth, the period of this mode of the prototype bridge is 3.10 sec., and 5) maximum deflection or maximum stress is given when the phase difference of the two shaking tables is equal to 180 degrees.

Chap. 3 Theoretical Analysis on Forced Vibration

The model test shows that the amplitude of the forced vibration of the suspension bridge, whose damping constant is very small, is not so severe except the case of resonance.

At first, it is assumed that a periodic ground motion is represented by $a \sin pt$, in which p is one of the circular frequencies of the suspension bridge, and each tower of the suspension bridge moves with a phase-lag of 180 degrees.

The fundamental equation which obtains the relation between the maximum deflection of the stiffening girder and the amplitude of the periodic ground motion, will be got in such a step as shown in the followings;

1) To get the additional horizontal reaction h_4 of the cable of

the side span due to the deflection of the top of the tower, the deflection of which is represented by Δ ,
2) To calculate the additional horizontal reaction of the cable of the main span, which is increased to $H_w + h_3$ by the deflection of the tower Δ .
3) To obtain the relation between the amplitude of the stiffening girder at its center, and the deflection of the tower Δ .
4) To represent the deflection of the tower Δ as the function of the amplitude, a , of the periodic ground motion.

The theoretical analysis will be stated according to the above procedure. The additional horizontal cable tension h_4 will be given as

$$h_4 = \Delta / K \tag{3}$$

where

$$K = \frac{L}{E_c A_c} - \frac{w^2 \ell^3}{H_w^3}\left\{ \frac{1}{m^2}\left(1 - \frac{2}{m}\tanh\frac{m}{2}\right) - \frac{1}{12}\right\}$$

Considering the fact that the frequency of the tower is much larger than the fundamental frequency of the vertical vibration of the suspension bridge, the deflection of the tower at its top is given by statical calculation. As $\Delta = h_t h^3 / 3EI_1$ and the additional horizontal cable tension h_3 of the main span is obtained by the equilibrium at the top of the tower, we get

$$h_3 = h_t + h_4 = 3EI_1 \Delta / h^3 + \Delta / K \tag{4}$$

The deflection curve η of the stiffening girder is assumed to be represented by

$$\eta = b \sin \frac{n\pi x}{\ell} \tag{5}$$

in which b is determined by the next equation

$$\frac{L_E h_3}{E_c A_c} + \int \eta \, dx = -2\Delta \tag{6}$$

And then

$$b = \left(2\Delta + 3EI_1\Delta/h^3 + \Delta/K\right)/2wl/n\pi H_w \tag{7}$$

We obtain the relation between a and b, using the condition that the energy which is lost by damping is equal to the energy which is supplied through the foundation of the suspension bridge.

Then the energy loss which is consumed by damping during one cycle will be

$$W = \int F_d d\eta = \int_0^l dx \int_0^{2\pi/p} pbc \sin\frac{n\pi x}{l}\cos pt \sin\frac{n\pi x}{l}\cos pt\, dt$$

$$= \frac{\pi b^2 pcl}{2} \tag{8}$$

where F_d means damping force, that is, $C\, d\eta/dt$, and C is a damping coefficient.

On the contrary, the energy (V) which is supplied through the foundation is

$$V = 2\int_0^{2\pi/p} h_3 \cos pt\, d\eta = 2\int_0^{2\pi/p} ap \cos pt\, h_3 \cos pt\, dt$$

$$= 2a\pi\left\{3EI_1/h^3 + 1/K\right\} \tag{9}$$

since the vibration of the ground is represented by

Then we get Eq. (10), from the condition W = V,

$$\Delta = \frac{16aw^2l^2}{n^2\pi^4H_w^2pcl} \times \frac{3\frac{EI_1}{h^3} + \frac{1}{K}}{\left[2 + \frac{L}{E_cA_c}\left\{3\frac{EI_1}{h^3} + \frac{1}{K}\right\}\right]^2} \tag{10}$$

Substituting Δ into Eq. (4), we have

$$h_3 = \frac{16w^2la}{n^2\pi^2H_w^2pc} \times \frac{\left(3\frac{EI_1}{h^3} + 1/K\right)^2}{\left[2 + \frac{L}{E_cA_c}\left\{\frac{3EI_1}{h^3} + \frac{1}{K}\right\}\right]^2} \tag{11}$$

and

$$b = \frac{n\pi H_w\Delta}{2wl}\left[2 + \left\{\frac{3EI_1}{h^3} + \frac{1}{K}\right\}\frac{L}{E_cA_c}\right] \tag{12}$$

Eq. (11) and Eq. (12) are derived under the assumption that the forced vibration of the suspension bridge is stationary. However, the periodic ground motion does not continue for ever, but the main and strong ground motion during an earthquake will be over, after it continues for several cycles. Therefore it is reasonable to solve the problem of earthquake motion as the one of a transient phenomenon. In the first place, we estimate the vibrational amplitude, $y_{t=t_0}$, which grows during the time from $t=0$ to $t=t_0$, and the amplitude excited by external force whcih vibrates eternally, that is, $y_{t\to\infty}$.

For the sake of simplicity, we substitute the vibration of a structure by the vibration of an one mass system. Then the equation of motion is given by the next equation.

$$\ddot{y} + 2pq\dot{y} + p^2 y = 0 \tag{13}$$

And the solution will be easily found.

$$y = y_s \left\{ \frac{1}{\sqrt{1-q^2}} e^{-pqt} \cos\left(\frac{2n\pi p'}{p} - \varepsilon \right) - \cos 2n\pi \right\} \tag{14}$$

where

$$p' = p\sqrt{1-q^2}$$

$$\varepsilon = \tan^{-1}\left(q/\sqrt{1-q^2} \right)$$

y_s = statical deflection

Let k be the growth coefficient whcih is defined by the ratio, $y_{t=t_0}/y_{t\to\infty}$,in which $y_{t=t_0}$ means the amplitude of the mass which is enlarged by the forced oscillation from $t = 0$ to $t = t_0$.

Then we must multiply the values of Δ and b obtained in the above by a growth coefficient , k, in order to get a reasonable response of a structure to an earthquake.

In the followings, the results of the numerical calculation of the Wakato Bridge are shown. The Wakato Bridge is the largest suspension bridge in Japan, and now under construction. Its main span is 364 m in length and the each span of the two side span is 90 m in length.

$E_c A_c$ = 2.22 x 10 ton
EI = 1.05 x 10 ton-m^2
w = 1.8 ton-m^{-1}
h = 82.3 m
I_o = 1.16 m^4
f = 36.4 m

Table 1 shows the results obtained by the numerical calculation about the prototype, namely, the Wakato Bridge, under the assumption that the seismic wave is sinusoidal and continues for some time, for example, 30 seconds.

Chap. 4 Damping Coefficient of Suspension Bridge

As stated in the above, damping coefficient is very important for the dynamical analysis of suspension bridges, and there are some researches performed theoretically as well as experimentally. But judging from the results obtained by many researchers, the values of the damping coeficients of suspension bridges are distributed over a considerably wide range.

In this paper, the field survey of the vibration of the suspension bridges whose span length is more than 100 m and the model suspension bridge, which is built in the ground of our institute, will be described.

Damping ratios and damping constants, which are defined by the percentage of the critical damping, are obtained from the records of free damped oscillation written by seismograph.

In order to get the initial deflection of a suspension bridge, portable oscillation generator was used, whose centrifugal force was not so large that the generator could be stopped easily, after some amplitude had been obtained on account of the resonance phenomenon.

The portable oscillation generator is about 70 kg in weight, eccentric mass is 25.3 kg in weight, and the diameter of the wheels is 58.8 cm ,and each wheel runs in the inverse direction respectively, and the force applied on a suspension bridge by this generator is upward and downward and periodic, and its centrifugal force is equal to $mr\omega^2$.

The main purpose of this measurement consists in surveying the damping coefficient of the actual suspension bridges, whose main span is more than 100 m in length and the next 7 bridges are selected. the dimensions of which are shown in Table 2 .

These suspension bridges have special characteristics in construction , namely;

1) Stiffening girder (Yagumo Bridge) and stiffening truss (the other bridges)
2) Main cable is used as the upper chord of the stiffening truss of the Tabisoko Bridge, instead of steel or wooden members.
3) Only the Miyoshi Bridge has its side spans.
4) The bridge has the different flexural rigidity of the stiffening truss, or girder, each other.
5) Some have the storm cable and the other none.

The relation between damping constant and the above characteristics in construction, span length and the vibration amplitude are the principal problems of this investigation.

Before the damping constant is described, the writer states briefly the period of vibration obtained by use of three methods, i.e. elastic theory, approximate calculation, and experiment.(see Table 3)

In the approximate calculation, the effective mass of the suspension bridge is given under the assumption that the deflection curve is represented by the curve of $A \sin \pi x / l$, where A is the maximum amplitude, and l, span length.

There is some difference between the experimental period and the approximate one of the Yagumo Bridge, and the difference can be explained by the fact that the Yagumo Bridge of stiffening girder has smaller stiffness than the other bridges of stiffening truss.

From the results obtsined, it is concluded that the damping constants of suspension bridges may be distributed between 0.004 and 0.006 as for the first symmetrical mode, and there may not exist exactly clear relation between the damping constants and the amplitude of the vibration. Generally speaking, the damping constant of the suspension bridge is much smaller than the one of the ordinary bridges, whose damping constants are distributed between 0.1 and 0.05.

In order to know more exactly the damping characteristics of the suspension bridge, the small suspension bridge was made in the ground of our institute.(see Fig. 3) The main purpose of the test is to make study on the relation between damping constants and such factors as;

1) the vibrational amplitude
2) the dead load
3) the bearing friction at the supports
4) the flexural rigidity of the stiffening girder
5) unequal cable tension of the hangers

As the dead load, sand bags were used, and tests were pursued, when the stiffening girders had such flexural rigidities as $188.9 cm^4$ and $2175 cm^4$,respectively. In order to investigate the vibrational characteristics of the suspension bridge with unequal hanger tension, the hanger at the point of 1/4 of the span was shortened successively by 1 cm, 2 cm, and 3 cm, and the other hangers were remained unchanged.

The results obtained by the test are explained briefly in the followings. The damping constants are given as a linear function of the amplitude, and become larger and larger as the amplitude, in so far as the test was pursued. It seems that the dead load as well as the lack of uniformity in hanger tension, has few influence upon the damping constant of suspension bridges.

When the flexural rigidities of both the girder bridge and the stiffening girders are exactly alike, the ratio of the damping constant of the suspension bridge to the one of the girder bridge is approximately unit in case of [-100x50x5 , but one half, in case of I-200x100x7 . Bearing friction which is increased by enlarging the reaction force, has few effect upon the damping of the suspension bridge, since the support is roller bearing and the energy lost at the support may be negligible.

At the end, the writer shows the results which was obtained at the Harada Bridge. From the record of the damped free vibration, which was taken by the seismograph, (magnification factor = 5, and vibrational period = 1.5 sec.) we can obtain the damping constant at a certain amplitude, and Fig. 4 shows that the damping constant is not independent on the amplitude, but is given by a linear function of amplitude. The maximum amplitude of the damped free oscillation is 10 mm . Due to the errors in reading of the amplitude, the damping constants at a small amplitude are not accurate, and therefore not plotted.

Chap. 5 Conclusions

The suspension bridge whose damping constant is much smaller than the bridges of the other type, must be designed by the theory of dynamics if it is necessary to check its aseismicity, because there is a fear that the vibrational amplitude becomes larger due to resonance.

In case of dynamic analysis, the ground motion during an earthquake must be needed, but in Japan there is no record of a violent earthquake, and therefore it must be considered as an inevitable consequence of the circumstances to assume the earthquake motion as a periodic ground motion, although the above assumption is questionable.

By proper estimation of the duration of the main part of an earthquake, the theory stated by the writer will be considered to investigate the response of a suspension bridge to a strong earthquake well enough.

Finally, the followings will be concluded;

1) The vibration period of the suspension bridge is correctly calculated by the theory which was given by F. Bleich.
2) The vibration of the suspension bridge caused by the horizontal ground motion which moves parallel to the bridge axis, is solved by the theory stated in this paper and the validity of the theory is investigated by the model test.
3) Owing to resonance, the suspension bridge is suffered most seriously by the earthquake whose period is equal to the period of the fundamental symmetric vibration of the suspension bridge, even if the acceleration of the earthquake is not so large.
4) The damping constants of the suspension bridge are distributed between 0.004 and 0.006 , according to the results of field experiments by the writer. And it is given as a linear function of amplitude, as long as the amplitude is not so large.

5) The aseismicity of the Wakato Bridge is checked, where the maximum acceleration of the horizontal periodic ground motion is assumed to be 30 gal, and the duration of the ground motion is 30 sec, which is the longest duration of the main part of the earthquake recorded in the past.

The results of checking is shown in Table 1

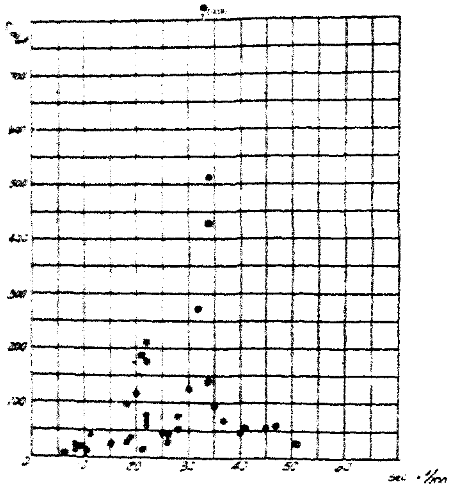

Fig. 1a Stress of the main cable

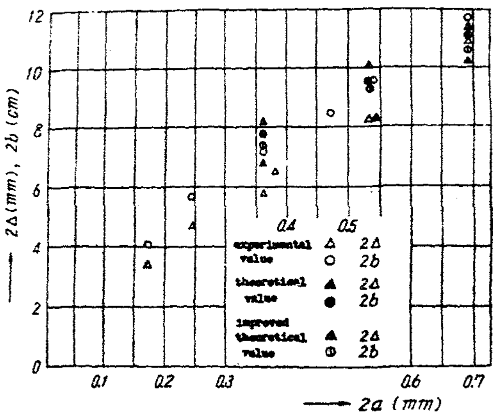

Fig. 1b Relation between ground motion and response

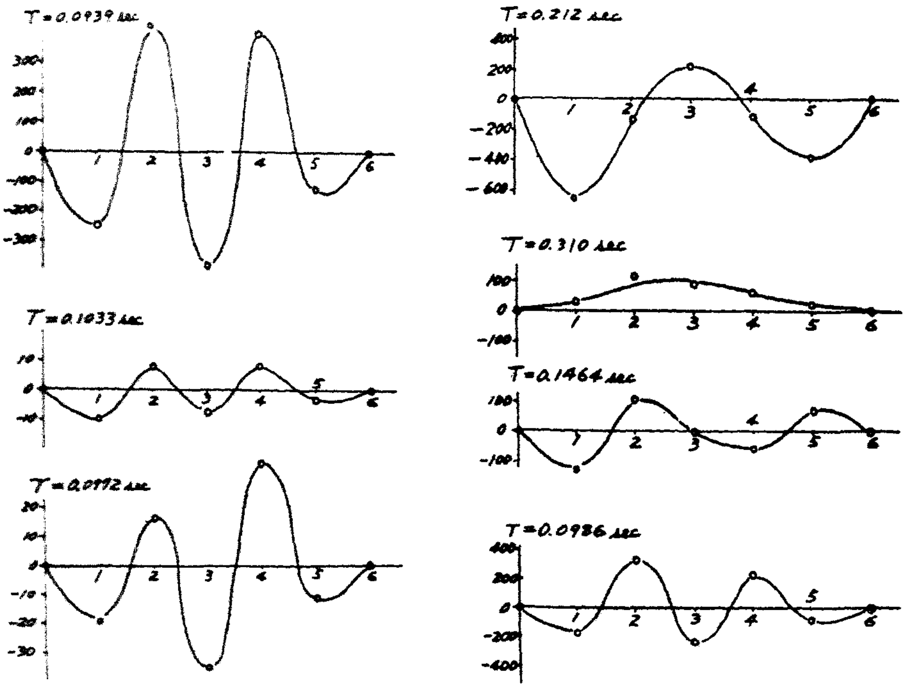

Fig. 2 Deflection curves

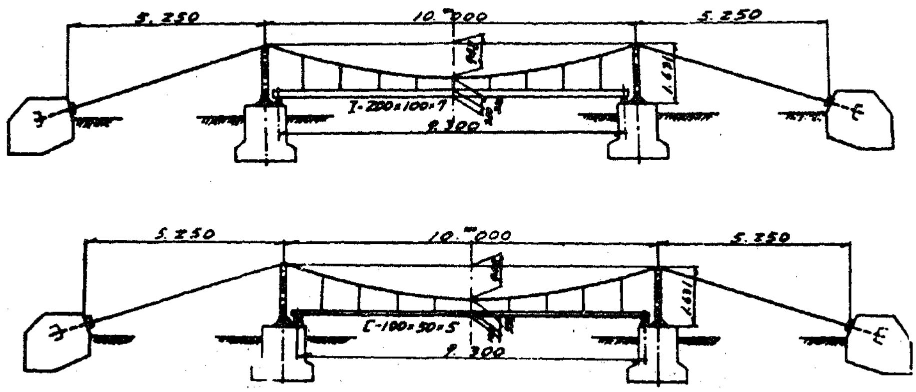

Fig. 3 Suspension bridges of small scale

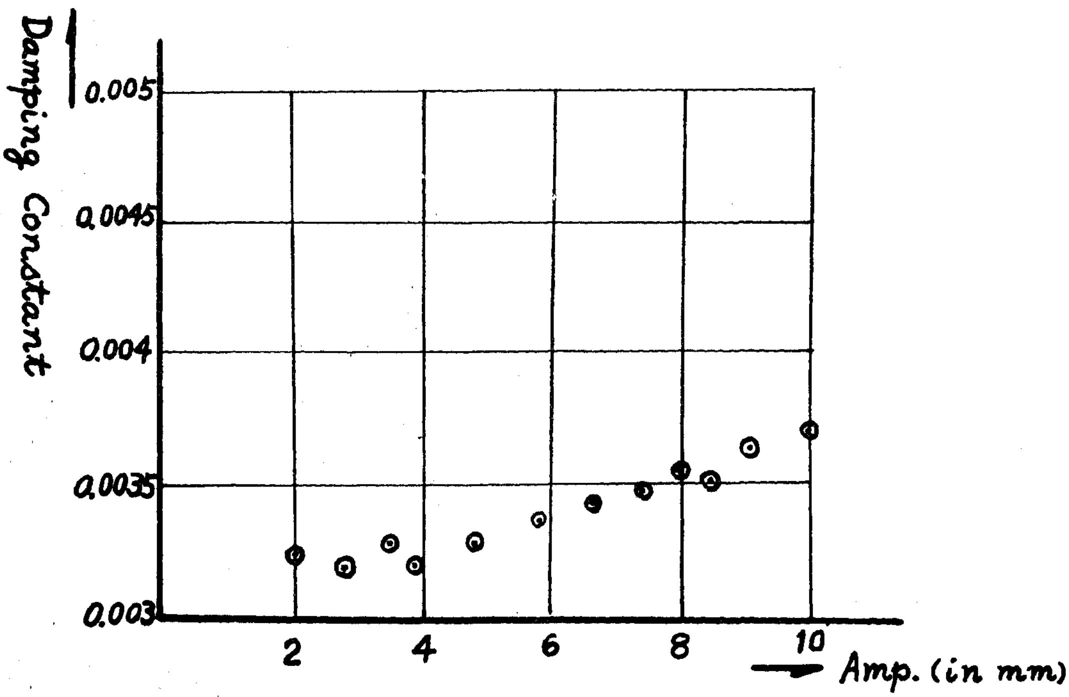

Fig. 4 Relation between amplitude and damping constants

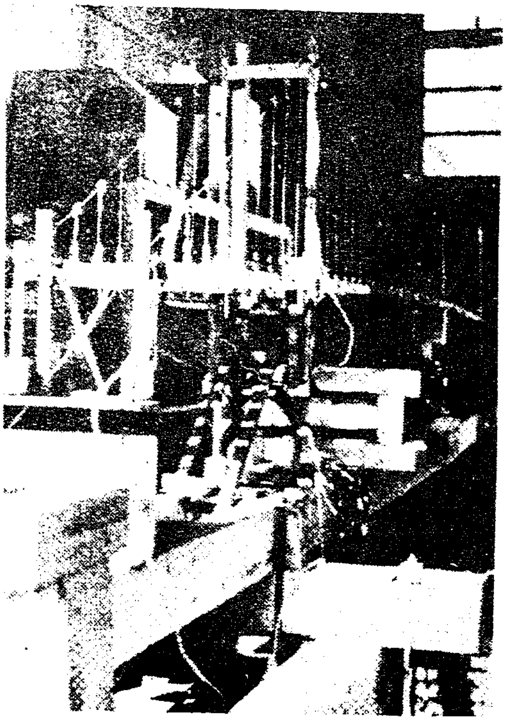

Photo 1 Shaking table and the model

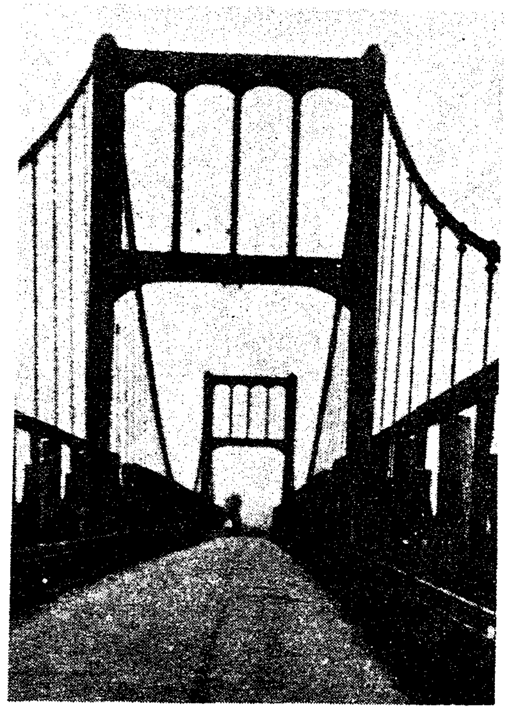

Photo 2a Sakae Bridge

Photo 2b Miyoshi Bridge

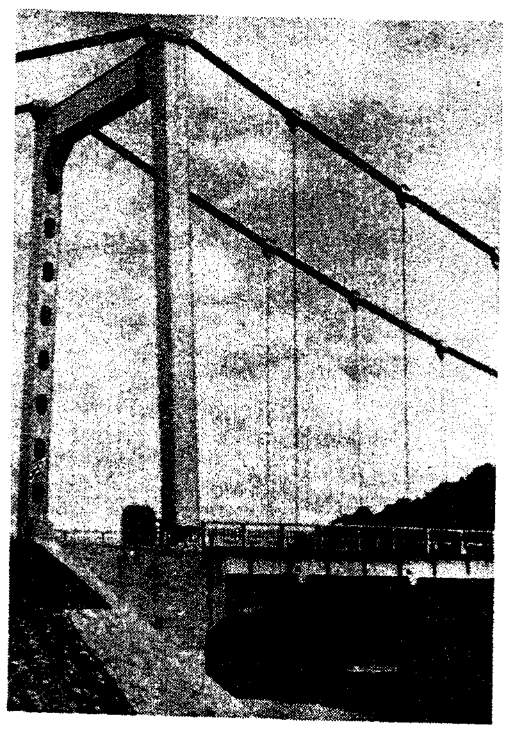

Photo 2c Yagumo Bridge

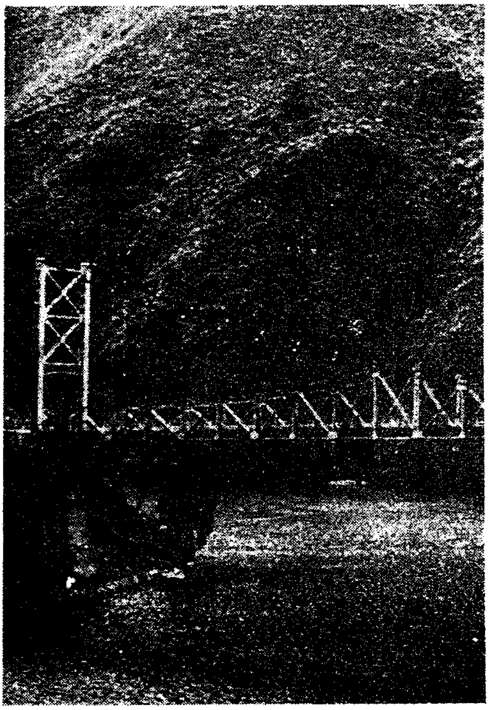

Photo 2d Tobisoko Bridge

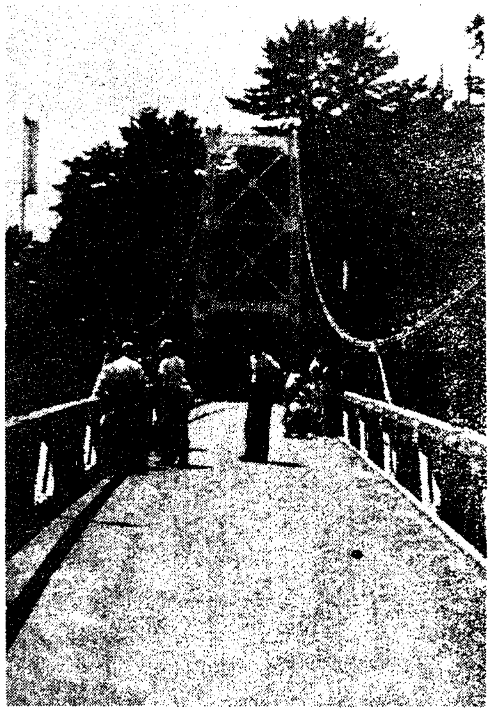

Photo 2e Seto Bridge

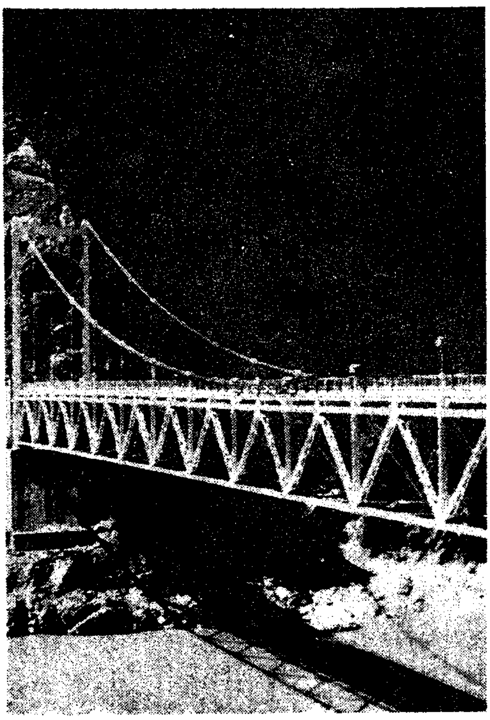

Photo 2f Harada Bridge

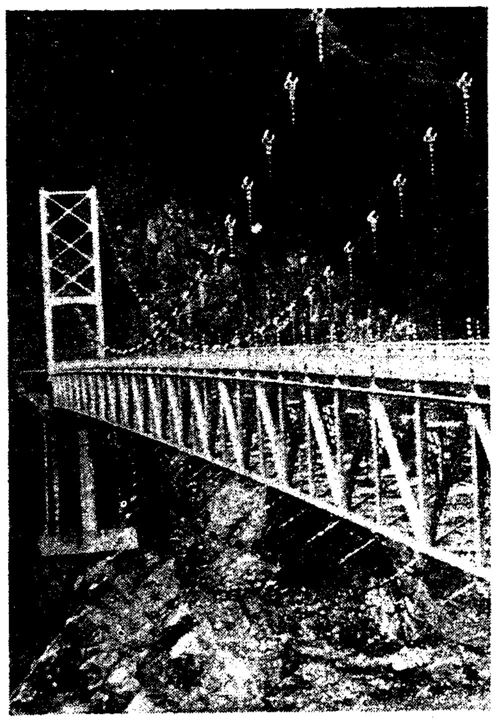

Photo 2g Takanosu Bridge

Table 1

Vibration mode	Period	Acceleration	Amplitude	k	$\Delta(m)$	$\Delta_u(m)$	$b(m)$	$b_u(m)$
1st. Symmetric	3.3 sec.	27.3 gal	7.53 cm	15%	0.60	2.40	7.22	7.30
2nd. Symmetric	2.1 sec.	42.8 gal	4.70 cm	20%	0.03		0.72	1.62
1st. Asymmetric	4.0 sec	22.0 gal	9.40 cm	11%			0.26	
	For towers and stiffening truss							
					$T(ton)$		$T_u(ton)$	
1st. Symmetric	3.3 sec	27.3 gal	7.53 cm	15%	14,100		18,000	
	For main cable							
					$T(ton)$		$T_u(ton)$	
1st Symmetric	3.3 sec	27.3 gal	7.53 cm	15%	244		401	
	For hangers							

Table 4

Name of Bridge	Damping Constant	Max. Amplitude
Sakae	0.0037	1.49 mm
Miyoshi	0.0063	2.09
Yagumo	0.0048	6.21
Tabisoko	0.0059	3.48
Seto *	0.0036	1.56
Harada	0.0073	0.98
Harada	0.0037	54.2
Takanosu *	0.0122	0.58

* vibration of second mode

Table 2.

Name of Bridge	Span Length(in m)	Sag(in m)	W(in kg/m)	I(in cm^4)
Sakae	37.2+98.2+98.2+37.2	11.0	1744	3.01×10^6
Miyoshi	31.5+139.9+31.5	16.8	2930	5.75×10^6
Yagumo	114.0	12.0	2718	3.78×10^5
Tabisoko	114.0	11.2	1300	4.39×10^6
Seto	125.0	10.4	958	2.19×10^6
Harada	137.6	15.0	2250	7.04×10^6
Takanosu	163.0	18.0	1777	5.49×10^6

Table 3

Name of Bridge	Experimental	Theoretical	Approximate
Miyoshi	1.20	1.24	1.29
Yagumo	1.51	1.44	1.93
Tabisoko	0.93	0.97	0.94
Seto	0.53 *	0.50	1.86
Harada	1.25	1.43	1.36
Takanosu	0.63 *	0.71	1.58

* second mode of symmefrical vibration