EFFECT OF INELASTIC BEHAVIOR ON THE RESPONSE OF SIMPLE SYSTEMS TO EARTHQUAKE MOTIONS

by

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INTRODUCTION

The theoretical data available concerning the response of structures to earthquake motions are, with few exceptions, applicable only to elastic structures, although it is generally recognized that structures subjected to actual earthquakes can undergo deformations of relatively large magnitudes in the inelastic range before failure occurs. Structures are ordinarily designed for lateral forces considerably smaller than those which are computed by available theories for the particular earthquake motions that have been measured. Yet such structures ordinarily do not show evidence of the distress that one would expect if the design forces reached values comparable to the computed lateral forces. For example, for an earthquake comparable to the El Centro earthquake of May 18, 1940, for which the maximum ground acceleration is of the order of 0.32g and the maximum ground velocity is about 14 in. per sec., the lateral force coefficient for structures having natural periods of the order of 0.5 sec. are of the order of 0.6 times the weight, even for damping as high as ten percent critical. Yet structures in this range are commonly designed for 0.1g or less, and structures so designed have performed successfully under earthquakes of about the same order of magnitude.

It is the purpose of this paper to indicate by means of the analysis of relatively simple systems how inelastic behavior can effectively reduce the lateral force coefficients that may be used in design to values of the order of one-fourth or less of those which would be applicable for elastic systems. The results of this study are not directly applicable to design procedures. However, they suggest approaches which might be used to develop a rational design procedure for earthquake resistance.

It is recognized, of course, that other factors than inelastic energy absorption help to account for a reduced effect of actual earthquakes compared with the theoretical effect. These factors include ground coupling, and feed-back from the responding structure modifying the input motions for a particular structure. However, the phenomenon of inelastic energy absorption is so common and so important

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that it deserves major consideration in any rational design procedure. Consequently the results of the study reported herein are presented for consideration even though the studies are not yet completed for a wide range of conditions.

Some work has been done previously on the problem of inelastic response of systems subjected to earthquake motions. The first study of which we have a record was a dissertation by S. L. Pan (Ref. 1). This considered simple frames subjected to idealized ground motions. The results of the study indicated the major influence of plastic energy absorption in reducing the effects of dynamic motion.

Recent studies include those of Tanabashi (Ref. 2) and Housner (Ref. 3). The former concerns the response of systems subjected to idealized motions, and the latter proposes a design procedure based on energy considerations. A study by Bycroft, Murphy and Brown (Ref. 4) presents results of analog computer studies for an actual earthquake accelerogram. In a discussion of this paper, John A. Blume (Ref. 4a) described energy concepts of design for seismic motions for inelastic systems in general.

The study presented in this paper is based in part on a thesis by Sheth (Ref. 5). It is limited to elasto-plastic single-degree-of-freedom systems, both with and without damping, subjected to earthquake motions corresponding to the El Centro, California earthquake of May 18, 1940, and the Vernon, California earthquake of October 2, 1933. The first is one of the strongest earthquakes for which records of ground motion are available, whereas the second is a relatively weak earthquake. Numerical data are given for the maximum displacement, spectral velocity and spectral acceleration of systems covering a range of natural periods, damping and degrees of inelastic deformation. Particular emphasis is placed on determining the lateral strength required to limit the maximum inelastic deformation in a structure to a prescribed value for the conditions considered. Methods of drawing response spectra for elasto-plastic systems are discussed, and the implications of the results are described in terms of their possible application to design procedures.

METHOD OF ANALYSIS

System Considered. A single-degree-of-freedom is considered, of the type shown in Fig. 1. The mass m is connected to the ground by a weightless spring having a resisting force Q and by a dashpot exerting a force proportional to the relative velocity between the mass and the ground. The absolute displacement of the ground is designated by y and that of the mass by x. The relative displacement of the mass to the ground, or the spring deformation, is denoted by u.

The resistance-deformation relationship for the spring is shown in Fig. 2. This is a typical elasto-plastic resistance function with equal yield points in the two directions of displacement. Unloading, after yielding has occurred, is assumed to take place on a path parallel to the original loading curve. A typical loading, unloading, and reloading curve is shown in the figure. The spring constant for the spring, as long as it remains elastic, is designated by k. The yield point deformation is denoted by u_y and the maximum deformation by u_m. In an analogous manner, the yield resistance is designated by Q_y and the maximum resistance by Q_m. For deformations in excess of the yield point deformation, Q_m is equal to Q_y.

Analyses have actually been performed for more complex resistance deformation relationships. However only those applicable to the relation in Fig. 2 are presented in this paper.

Ground Motions. The ground motions considered for the two earthquakes include the two horizontal components for which strong motion accelerograms are available. However, only the results for the north-south component of the El Centro earthquake and for the nearly east-west component of the Vernon earthquake are described here because the other results are of generally the same type.

The characteristics of these motions are as follows: For the El Centro earthquake, the total duration of the recorded accelerogram is 29 sec. The maximum acceleration is 0.32g, the maximum ground velocity 13.7 in. per sec., and the maximum ground displacement is 8.3 in. These maximum values are reached at about 2.1 sec., 1.7 sec. and 6.2 sec. sec., respectively, from the beginning of the motion. For the Vernon earthquake, the duration is about 11 sec., the maximum acceleration is 0.12g, the maximum velocity 4.6 in. per sec., and the maximum ground displacement is 2.2 in. These maxima occur, respectively, at 1.2 sec., 0.8 sec., and 1.1 sec., from the beginning of the motion.

Each accelerogram was approximated by a polygonal diagram having the same maximum and minimum points as the actual accelerogram. Certain corrections were made in order to arrive at input data with zero ground velocity at the end of the quake and reasonable values of ground displacement. These corrections had only a small influence on the maximum ground velocities and accelerations.

Method of Solution. The analysis was performed on the University of Illinois high-speed electronic digital computer, the ILLIAC, using a numerical method of step-by-step integration of the equation of motion with respect to time, as described by Newmark (Ref. 6).

PRESENTATION OF RESULTS

Typical Response Curves. In Figs. 3 and 4 are shown typical time histories of the response of elastic or elastoplastic systems having an undamped natural period of vibration, T, of 1.0 sec. The period is computed from the spring constant k, applicable to the initial elastic range of behavior.

In Fig. 3, which shows the response to the El Centro quake of a system having 10 percent critical damping, results are shown for both an elastic system and an elasto-plastic system. It is noted that the maximum relative displacement of the elastic system, u_o, is 3.28 in. The elasto-plastic system considered had a yield deformation of 1.64 in., or exactly half the maximum deformation of the elastic system, but the maximum relative displacement turned out to be exactly the same, namely 3.28 in. These peak values were reached at different times for the two cases. The diagram at the bottom of the figure shows the time intervals during which yielding took place. In the interval during which maximum responses were observed, an appreciable portion of the time involved plastic deformation, but outside of this interval, except for one minor interval at about 12 sec., no further yielding occurred.

The scale at the right-hand side of the elastic response curve shows the spring force in terms of the weight of the system W. For an elastic system without damping, the scale represents also the acceleration of the mass expressed as a fraction of the gravitational acceleration. The maximum value of this quantity is ordinarily referred to as the lateral force coefficient or the "spectral acceleration."

In Fig. 4 is shown the response of an elasto-plastic system without damping and a natural period of 1.0 sec., when subjected to the excitation from the Vernon earthquake. The yield deformation is 0.253 in., which is precisely one-third of the maximum displacement of an elastic system of the same period. In this case, the maximum deformation is determined to be 0.97 in., which is about 28 percent larger than the maximum deformation of the elastic system. In Fig. 4a there is shown the acceleration as a function of time and it is clear from the plotted results that the effect of the plastic behavior is to limit the acceleration to a value consistent with the maximum force that can be carried by the spring.

Yield Deformation and Maximum Deformation. Solutions were obtained for a number of values of period T and for a number of selected values of relative yield displacement u_y. Only a limited number of plots of the form shown in Figs. 3 and 4 were made. In most instances only the values of the absolute maximum response were determined. These calculations gave values of maximum deformation u_m for specified values of yield deformation u_y. From these values, by graphical interpolation, values were obtained of the required magnitudes of yield displacement to correspond to selected values of the ratio of maximum deformation to yield point deformation. This ratio, designated by μ, is defined as a "ductility factor" and indicates the relative magnitude of total deformation compared with the elastic range of deformation. Consequently, the quantity $(\mu-1)$ designates the plastic part of the total deformation relative to the elastic range of deformation.

The results of these interpolations are shown in Tables 1 and 2 for values of ductility factors of 1, 1.25, 2, and 4, for no damping, $\beta = 0$, and for 10 percent critical damping, $\beta = 0.10$, for selected values of period T. It is noted that values of the maximum relative displacement u_m can be obtained from the tabulated values by multiplying the tabulated quantities u_y by the ductility factor μ listed at the head of each column.

It is of some interest to compare the maximum relative displacement of an elasto-plastic system, u_m, with the maximum relative displacement u_o of an elastic system having the same slope in the elastic part of its load deflection curve. Values of u_m/u_o are shown in Figs. 5a and 5b for elasto-plastic systems having a ductility factor of 4, as a function of the undamped natural period of vibration, for the El Centro quake. Similar plots can be prepared for the Vernon earthquake from the data given in Table 2. In general these curves show that for no damping the values of u_m/u_o are generally less than 1.0 for the El Centro earthquake, and average about 1.0 for the Vernon earthquake, although in the latter case the values go as high as 1.4 or slightly more but oscillate between this value and values of the order of about 0.7.

For the systems with 10 percent critical damping, for both earthquakes, the values generally lie above 1.0, and are substantially above 1.5 over a portion of the range of periods considered, running to values as high as 1.8 for the Vernon earthquake and nearly that high for a somewhat more restricted range of periods for the El Centro earthquake, as indicated in Fig. 5b. It may be noted, however, that the values of maximum deformation for the damped systems, for all the values of ductility factor considered, are generally smaller than those for elastic systems with zero damping although they are often higher than the maximum deformations in the corresponding elastic system for the same amount of damping.

Another comparison of the maximum deformation in the inelastic system with that in a corresponding elastic system is shown in Figs. 6 and 7. Here the values of u_m/u_o are considered with regard to their variation with ductility factor, both for systems without damping in Fig. 6 and for systems with 10 percent critical damping in Fig. 7. The points plotted scatter pretty well over a range from 0.4 to 1.6 in Fig. 6 and from about 0.6 to 2.0 in Fig. 7, but without any systematic pattern. An attempt was made to study those points which appeared highest in the diagrams and some slight correlation was obtained over particular ranges of period of the systems considered. These ranges are designated by open circles in the plots and the particular ranges are indicated. One can conclude from Fig. 6 that for systems without damping, the maximum deformation of the elasto-plastic system is of about the same order of magnitude, in general, as the maximum deformation in the corresponding elastic system. Although one might draw the same general conclusion from the data in Fig. 7 also, there does appear to be a tendency for the ratios to rise with higher values of ductility factor.

The solid curved lines sloping generally up to the right in Fig. 7 represent values of the ratio determined by setting the area up to the point of maximum deformation under the load-deformation diagram of the elasto-plastic system equal to the corresponding area for the elastic system. The results obtained by this equating of energies appear to represent reasonable upper bounds or limits to the values of u_m/u_o. The dependence of these limiting or upper bounds on the degree of damping involved is not clear from the available data. Additional studies of these relationships are currently in progress. It may be noted, however, that for ranges in ductility factor of practical importance, up to values of five to eight, these ratios are generally less than two.

It should be emphasized that, because of the way in which the calculations were made, the results in Tables 1 and 2 are subject to some inaccuracies and should be considered as approximate. However, the data plotted in Figs. 6 and 7 are not subject to the errors in interpolation from computed values since they represent the computed data directly.

RESPONSE SPECTRA

Acceleration Spectra. The maximum value of the spring force, Q_m, is expressed in the form

$$Q_m = CW \tag{1}$$

where C is the lateral force coefficient, or design load factor, and W is the weight of the system. For deformations in excess of the limiting elastic deformation, u_y, the spring force is equal to the yield force. As previously noted, the quantity C represents also the spectral acceleration expressed as a fraction of the acceleration of gravity.

In Fig. 8 are shown spectra of lateral force coefficient for elastic systems with various amounts of damping, for the El Centro earthquake. A semi-log plot is used to emphasize the differences between the various curves in the range of high natural periods for which the numerical values are relatively small. The influence of damping in reducing the design load factor is quite evident from this figure. It should be noted, however, that even for damping of the order of 20 percent critical, a structure having a natural period of about 0.5 sec. must still be designed for a lateral acceleration of about 0.45g to behave elastically during an earthquake of the intensity considered.

The effect of yielding is to reduce the value of the design loads below those required for elastic behavior, the magnitude of this reduction being a function of the degree of inelastic behavior that can be tolerated. Response spectra for elasto-plastic systems having 10 percent critical damping are shown in Fig. 9 for values of ductility factor of one (corresponding to an elastic system), 1.25, 2 and 4. These results were computed from the data presented in Table 1. It can be seen that even a relatively small amount of yielding, corresponding to values of μ of the order of 1.25 or 1.5 which ordinarily are considered as negligible, produce appreciable reductions in the value of the lateral force coefficient. The reductions from the values of the elastic systems are roughly 20 percent for $\mu = 1.25$, 50 percent for $\mu = 2$, and 75 percent for $\mu = 4$. By comparison of Figs. 8 and 9, it can be noted that increasing the damping coefficient from 10 percent critical to 20 percent critical has an effect of roughly the same order of magnitude as increasing the ductility factor from one to 1.25, with the damping coefficient remaining at 10 percent critical.

Velocity Spectra. Another method of presentation of the data is of some interest. This method involves the concept of a velocity spectrum for the elasto-plastic system. However, the velocity must be defined in a particular fashion in order that the quantities can be useful. The definition which seems most appropriate is to define the spectral velocity V by the relationship

$$V = pu_y \tag{2}$$

where p denotes the undamped circular natural frequency of the system. Then the quantity

$$\frac{1}{2}\, mV^2$$

represents the maximum recoverable strain energy of the system. It should be noted that this definition of the spectral velocity for the elasto-plastic system is consistent with that used for elastic systems, since in an elastic system the yield deformation may be considered to be equal to the maximum elastic deformation. Values of the spectral velocity so defined are shown in Fig. 10 for the El Centro earthquake for undamped elasto-plastic systems with ductility factors of 1, 1.25, 2 and 4. Here again the importance of the ductility factor in reducing the magnitude of the required resistance is clearly indicated.

Relation Between Input Motions and Response Values. It is convenient and desirable to plot velocity spectra on a four-way logarithmic grid, as shown in Fig. 11. Such a plot has the advantage that values of the maximum relative displacement and the spectral acceleration can be read directly from the diagonal scales on the figure.

The heavy solid lines in Fig. 11 represent response spectra for elastic systems with various amounts of damping for the El Centro earthquake. The curves have irregularities, as is common with response spectra, but in general these curves, as well as others that have been studied for simpler ground motions, have roughly a trapezoidal shape with a bound to the left corresponding to some constant times the maximum ground acceleration, a bound on the top corresponding to some constant times the maximum ground velocity, and a bound to the right corresponding to some constant times the maximum ground displacement. The maximum ground acceleration, the maximum ground velocity and the maximum ground displacement are shown in the figure by the dashed line. The ratios of the spectral values to the corresponding ground motion magnitudes are dependent on the damping coefficient. A summary of the ratios for velocity and acceleration is given in Table 3. Results are given both for the El Centro earthquake and the Vernon earthquake. From these and additional studies that have been made, it appears that for small amounts of damping, in the order of 2 to 5 percent, the average ratio of the maximum spectral velocity to the maximum ground velocity is of the order of 1.5, and the average ratio of the maximum spectral acceleration to the maximum ground acceleration is of the order of 2.5 to 3.0. For high degrees of damping, between 10 to 20 percent, the maximum value of the spectral velocity is about equal to the maximum ground velocity and the maximum value of the spectral acceleration is of the order of 1.5 to 2 times the maximum ground acceleration. Other studies indicate that the maximum spectral deformation, or relative displacement, is of the order of 1 to 2 times the maximum ground displacement. These general observations permit some conclusions to be drawn concerning possible design values.

The same type of logarithmic plot can be used for elasto-plastic systems, provided the quantities are interpreted in the following way:

(1) The spectrum deformation plotted is the yield deformation of the spring

(2) The spectral velocity plotted is given in Eq. 2,

$$V = pu_y$$

(3) The spectral acceleration plotted is given by the equation

$$A = p^2u_y \tag{3}$$

DESIGN IMPLICATIONS

The results of the study reported herein, and of the observations which have been made in the preceding sections of this paper, suggest an approach to design which is summarized in this section. It appears that the response spectra for an elasto-plastic system can be related to the spectra for a corresponding elastic system with a reasonable degree of approximation. The data discussed indicate two possible approaches, for which no valid distinction can yet be made until additional studies are completed. One of the possibilities is to relate the spectrum for the elasto-plastic system to that for the corresponding elastic system by considering the maximum relative displacements for the two systems to be equal. The other approach is to compute the displacements for the elasto-plastic system by equating the energy corresponding to the maximum deformation of this system to the maximum strain energy in the corresponding elastic system. The two procedures give nearly identical results for small values of ductility factor. However, for the larger values the differences are appreciable.

With the first suggested approach, namely considering the maximum deformations to be the same, the value of the lateral force coefficient for the elasto-plastic system is obtained from the corresponding value of the elastic system by dividing the latter by the value of the ductility factor for which the design is to be made. The values of the yield point deformation, u_y, and the spectral velocity, V, are obtained in a similar manner from the corresponding values of the elastic system. If the spectra are plotted on the logarithmic plot used in Fig. 11, whether or not this is approximated by a trapezoidal diagram, the spectrum for the elasto-plastic system is obtained by drawing a curve similar in shape to that for the elastic system, but displaced downward by the ratio $1/\mu$. Under these conditions, the design values can be read from the chart in terms of displacement, velocity, or acceleration, by using the corresponding lines on the diagram.

For example, in the region where the response velocity is nearly constant, for an earthquake similar in character to the El Centro earthquake, one would have an elastic response velocity of the order of 20 in. per sec. for a damping coefficient of about 10 percent. For a design ductility factor of 4, the design value of spectral velocity for the elasto-plastic system would be 5 in. per sec. For a structure having a period of 1.6 sec., the spectral acceleration would be 0.05g, the yield deflection 1.3 in., and the maximum deflection 4 times 1.3 in., or 5.2 in.

For the second alternative, namely equating the energies, the same technique can be used except that the spectrum is displaced downward by the ratio

$$\frac{1}{\sqrt{2\mu - 1}}$$

instead of the quantity $1/\mu$. This is a somewhat more conservative approach since in general the value obtained from the quantity which equates the energy is larger than $1/\mu$. For example, when $\mu = 2$, the ratio for equality of energy is 0.577 whereas $1/\mu$ is 0.500. This is a difference of the order of 15 percent, which is negligible. For a value of μ of 4, the quantity $1/\mu$ is 0.25, and the quantity which equates the energy is 0.378 which involves a difference of the order of nearly 50 percent.

From the results shown in Figs. 6 and 7, it appears that the approach which equates the maximum displacement is the most reasonable for relatively small magnitudes of damping, probably of the order of less than 5 percent, and the approach which equates energies seems to be most reasonable for larger magnitudes of damping factor. Incidentally, it should be pointed out that Housner (Ref. 3) has presented a procedure which is equivalent to the procedure described herein, involving the preservation of the energy.

It is appropriate to consider the choice of the ductility factor for which the design is made. For most structural materials, including steel or reinforced concrete, plastic deformations of the order of three times the elastic limit deformation, or total deformation corresponding to $\mu = 4$, involve no serious distortions or undue amounts of damage. The yielding in most structures is of a local nature and does not affect the general appearance of the structure. Consequently, it appears reasonable to use in design for earthquake resistant structures a value of ductility factor of the order of 4. Possibly higher values might be reasonable under some circumstances, but they would require more careful attention to details of construction in order to insure the development of the plastic deformation required. A design for a ductility factor in the range recommended would permit the structure to behave elastically or nearly elastically for most earthquakes except those which are nearly as intense as the design earthquake.

SUMMARY

The studies reported herein appear to indicate that the response of elasto-plastic systems can be related to the response of corresponding elastic systems having the same initial slope of the load deformation curve. The maximum accelerations in the elasto-plastic systems, and consequently the design load factors for such systems, can be stated in terms of the corresponding quantities for elastic systems multiplied by a reduction factor which is related to the degree of plastic deformation which is permissible. If ductility factors of the order of magnitude of about 4 are used in the design, the design load factors are consistent with values approximately one-fourth those which are computed for elastic systems with moderate degrees of damping, and

appear to be reasonably close to the values currently used in design, when one takes account of the relationship between working stresses and actual yield values of materials.

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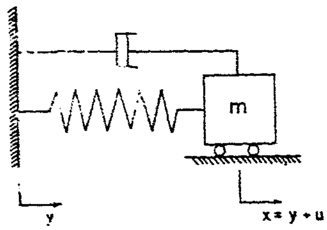

Fig. 1 System Considered

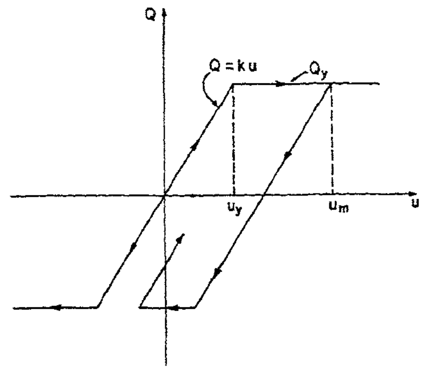

Fig. 2 Resistance-Deformation Relation

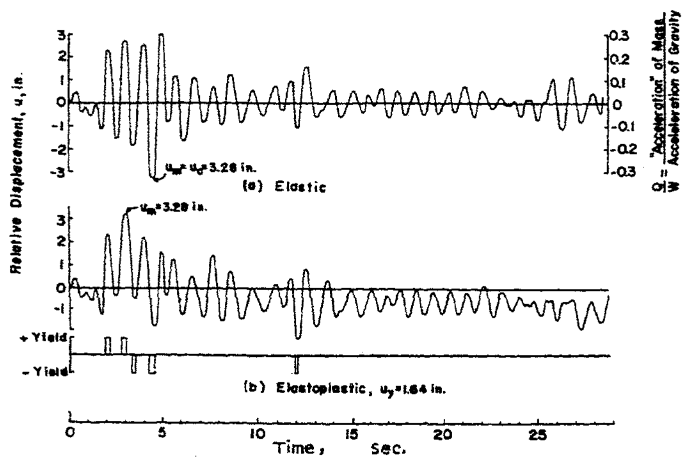

Fig. 3 Response of Systems with T = 1.0 sec., β = 0.10
El Centro, California Earthquake, May 18, 1940,
N-S Component

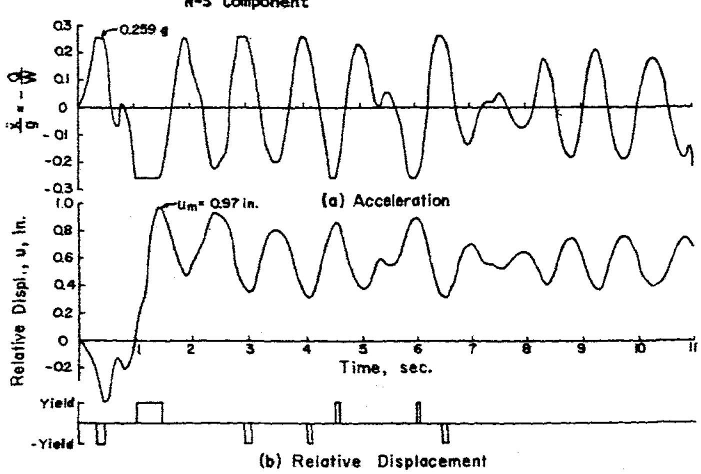

Fig. 4 Response of Elasto-Plastic System with T = 1.0 sec.,
$\beta = 0$, $u_y = \frac{1}{3} u_o = 0.253$ in.
Vernon, California Earthquake, Oct. 2, 1933, S 82°E
Component

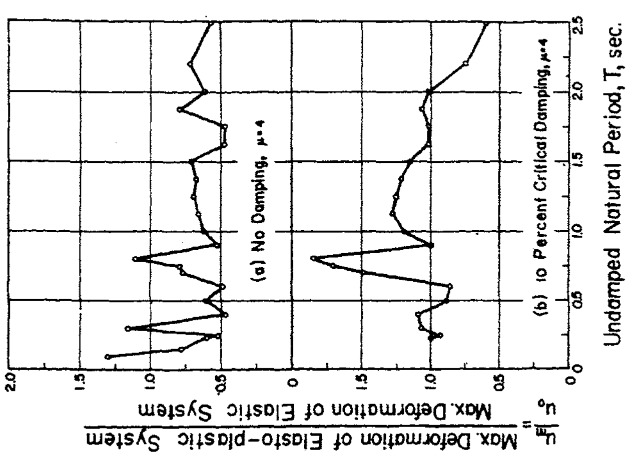

Fig. 5 Comparison of Maximum Relative Displacements of Elasto-Plastic and Elastic Systems as a Function of Natural Period -- El Centro Earthquake

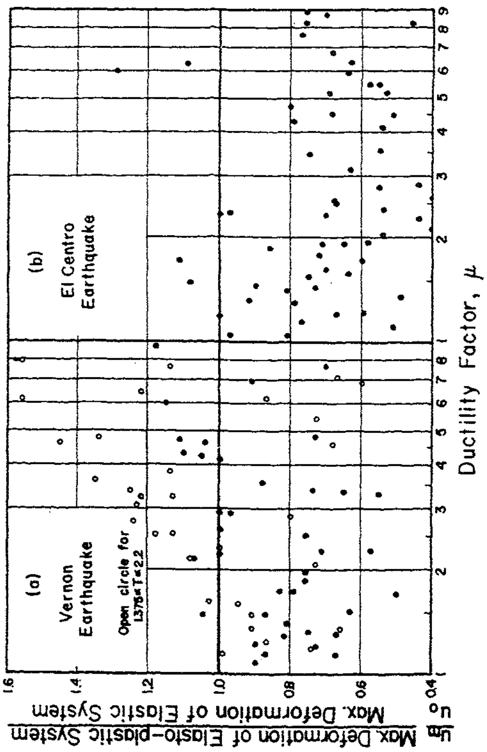

Fig. 6 Comparison of Maximum Relative Displacements of Elasto-Plastic and Elastic Systems as a Function of Ductility Factor -- Systems Without Damping

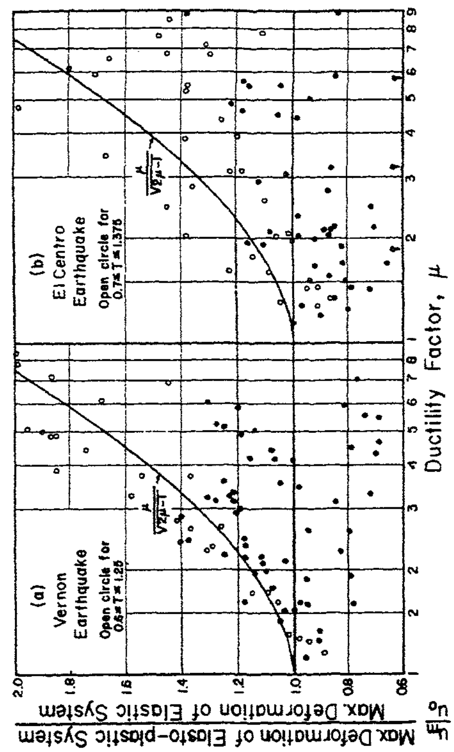

Fig. 7 Comparison of Maximum Relative Displacements of Elasto-Plastic and Elastic Systems as a Function of Ductility Factor -- Systems with 10 Percent Critical Damping

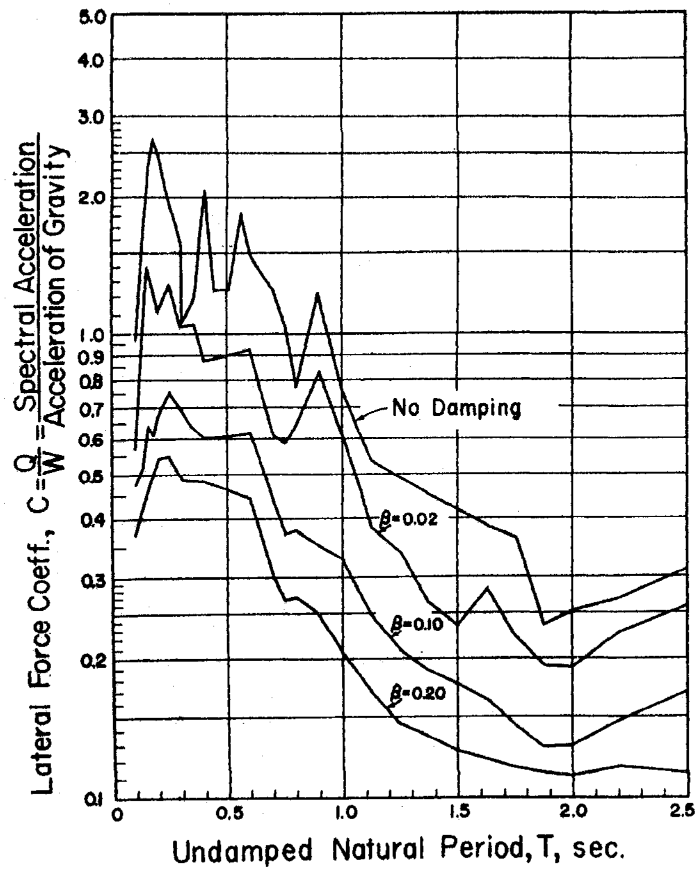

Fig. 8 Acceleration Spectra for Elastic Systems -- El Centro Earthquake

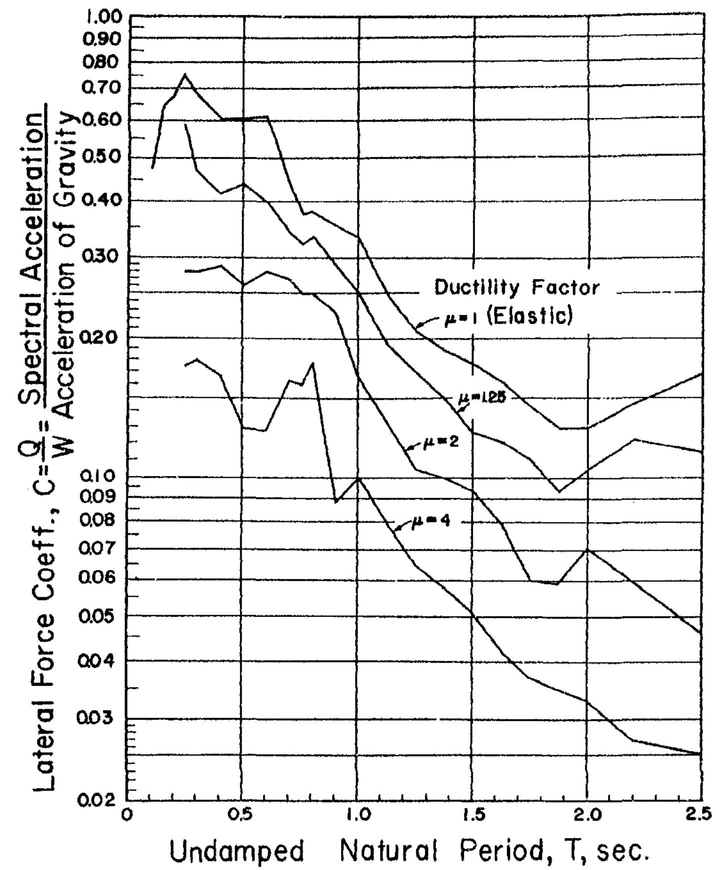

Fig. 9 Acceleration Spectra for Elasto-Plastic Systems with 10 Percent Critical Damping -- El Centro Earthquake

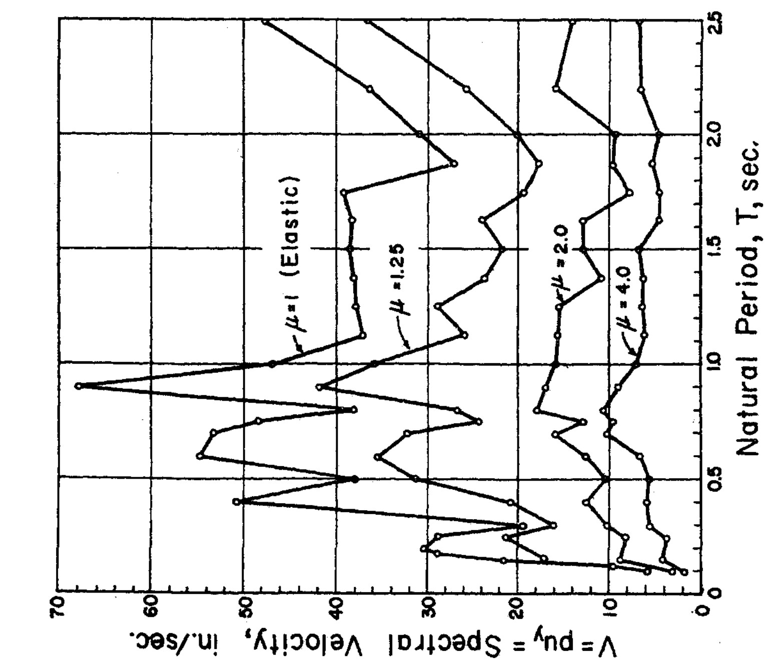

Fig. 10 Velocity Spectra for Undamped Elasto-Plastic Systems -- El Centro Earthquake

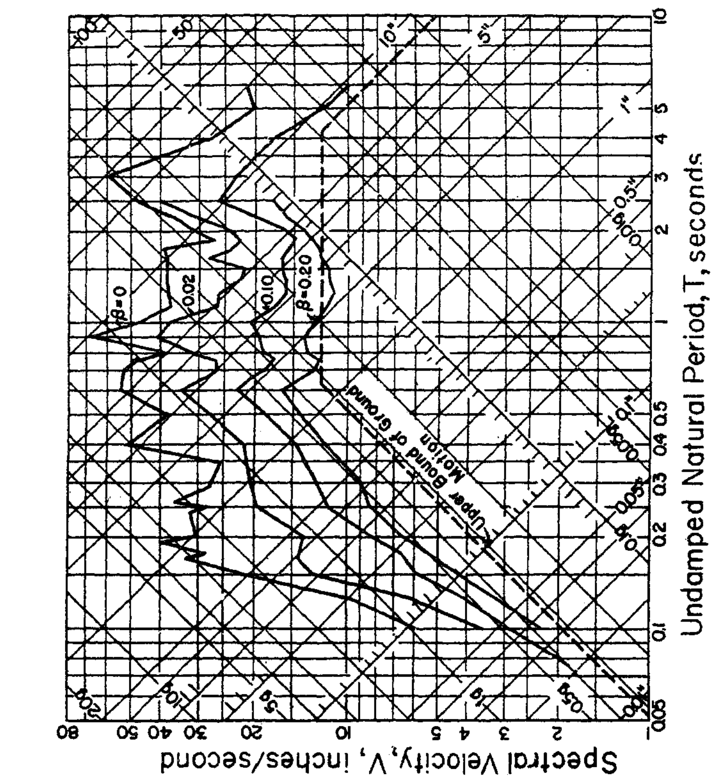

Fig. 11 Response Spectra for Elasto-Plastic Systems -- El Centro Earthquake

TABLE 1

YIELD POINT DEFORMATION FOR ELASTO-PLASTIC SYSTEMS

El Centro Earthquake, May 18, 1940, N-S Component

Tabulated values of u_y in in. Maximum deformation, u_m, is given by the relation

$$u_m = \mu\, u_y$$

Period T Sec.	No Damping				10 Percent Critical Damping			
	$\mu = 1$	$\mu = 1.25$	$\mu = 2$	$\mu = 4$	$\mu = 1$	$\mu = 1.25$	$\mu = 2$	$\mu = 4$
0.10	0.095	0.083	0.049	0.031	0.046	-	-	-
0.15	0.514	0.41	0.21	0.10	0.141	-	-	-
0.228	1.13	0.64	0.31	0.17	0.265	0.28	0.16	0.082
0.25	1.15	0.85	0.33	0.15	0.459	0.36	0.17	0.107
0.30	0.93	0.77	0.49	0.27	0.600	0.41	0.25	0.16
0.40	3.24	1.33	0.80	0.38	0.948	0.66	0.45	0.26
0.50	3.03	2.5	0.83	0.46	1.49	1.19	0.64	0.33
0.60	5.24	3.4	1.2	0.64	2.18	1.40	1.00	0.47
0.70	5.94	3.6	1.8	1.15	2.09	1.65	1.23	0.78
0.75	5.79	2.9	1.6	1.15	2.06	1.77	1.4	0.88
0.80	4.87	3.4	2.3	1.35	2.38	2.09	1.57	1.10
0.90	9.72	6.0	2.5	1.3	2.80	2.30	1.63	0.70
1.00	7.47	5.8	2.6	1.15	3.28	2.45	1.63	0.98
1.125	6.65	4.6	2.8	1.1	3.07	2.4	1.63	0.98
1.25	7.55	5.7	3.1	1.3	3.18	2.6	1.60	1.00
1.375	8.37	5.2	2.4	1.4	3.51	2.8	1.84	1.07
1.50	9.21	5.2	3.1	1.65	3.90	2.8	2.05	1.13
1.625	9.94	6.2	3.3	1.2	4.23	3.1	2.04	1.08
1.75	10.91	5.4	2.2	1.3	4.32	3.3	1.80	1.10
1.875	8.13	5.3	2.9	1.6	4.48	3.2	2.05	1.20
2.00	9.88	6.4	3.0	1.5	5.08	4.1	2.8	1.3
2.20	12.79	9.0	5.6	2.3	6.96	5.8	2.8	1.3
2.50	19.00	14.6	5.7	2.7	10.39	7.0	2.8	1.55

TABLE 2

YIELD POINT DEFORMATION FOR ELASTO-PLASTIC SYSTEMS

Vernon Earthquake, October 2, 1933, S82°E Component

Tabulated values of u_y in in. Maximum deformation, u_m, is given by the relation

$$u_m = \mu\, u_y$$

Period T Sec.	No Damping				10 Percent Critical Damping			
	$\mu = 1$	$\mu = 1.25$	$\mu = 2$	$\mu = 4$	$\mu = 1$	$\mu = 1.25$	$\mu = 2$	$\mu = 4$
0.10	0.054	0.030	0.020	0.011	0.016	-	-	-
0.15	0.092	0.075	0.054	0.039	0.054	-	-	-
0.20	0.217	0.157	0.105	0.050	0.104	-	-	-
0.228	0.33	0.195	0.13	0.061	0.123	0.100	0.074	0.036
0.25	0.22	0.14	0.11	0.057	0.124	0.102	0.075	0.042
0.30	0.26	0.21	0.12	0.070	0.164	0.120	0.085	0.048
0.40	0.60	0.41	0.17	0.089	0.292	0.215	0.120	0.072
0.50	0.63	0.51	0.24	0.15	0.410	0.34	0.17	0.10
0.60	0.85	0.47	0.34	0.22	0.382	0.31	0.23	0.16
0.70	0.78	0.47	0.29	0.21	0.390	0.29	0.23	0.16
0.75	1.03	0.67	0.27	0.19	0.395	0.30	0.23	0.17
0.80	0.81	0.40	0.28	0.18	0.391	0.30	0.23	0.17
0.90	0.80	0.51	0.28	0.20	0.420	0.33	0.26	0.20
1.00	0.76	0.51	0.35	0.25	0.520	0.43	0.32	0.23
1.125	1.20	0.85	0.50	0.34	0.726	0.58	0.43	0.28
1.25	2.28	1.25	0.70	0.42	0.950	0.76	0.53	0.33
1.375	1.58	1.25	0.87	0.53	1.149	0.96	0.63	0.37
1.50	1.99	1.40	1.02	0.64	1.32	1.05	0.71	0.40
1.625	2.85	1.55	1.03	0.61	1.45	1.17	0.80	0.43
1.75	2.29	1.60	1.06	0.60	1.55	1.25	0.85	0.43
1.875	2.13	1.74	1.20	0.70	1.64	1.33	0.91	0.45
2.00	2.21	1.75	1.20	0.80	1.70	1.37	0.94	0.39
2.20	2.68	1.90	1.37	0.70	1.76	1.45	1.00	0.39
2.50	2.93	1.93	1.40	0.40	1.84	1.51	1.05	0.32

TABLE 3

AVERAGE SPECTRAL VALUES FOR ELASTIC SYSTEMS

Quantity	Earthquake	Percent of Critical Damping				
		0	2	5	10	20
V_{av}, in./sec.	El Centro	42.8	28.8	22.9	17.6	13.3
	Vernon	7.8	5.8	5.2	4.5	3.7
$\frac{V_{av}}{\dot{y}_o}$	El Centro	3.12	2.10	1.67	1.28	0.97
	Vernon	1.70	1.26	1.13	0.98	0.80
A_{av}, in g's	El Centro	1.84	1.05	0.83	0.60	0.48
	Vernon	0.43	0.31	0.26	0.21	0.16
$\frac{A_{av}}{\ddot{y}_o}$	El Centro	5.7	3.3	2.6	1.9	1.5
	Vernon	3.6	2.6	2.2	1.8	1.3

V_{av} is average spectral velocity for range 0.4 sec. $\leq T \leq$ 2.5 sec.

A_{av} is average spectral acceleration for range 0.1 sec. $\leq T \leq$ 0.4 sec.

$\dot{y}_o$ is maximum ground velocity

$\ddot{y}_o$ is maximum ground acceleration