Earthquake Responses of a Long Span Suspension Bridge

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INTRODUCTION

This paper concerns primarily with the earthquake responses of a long span suspension bridge. Problems on aerodynamic stability of suspension bridges have been investigated by many specialists during the past few decades, but there are few investigations and technical papers on earthquake responses of suspension bridges. In this paper, a method of analysing the earthquake responses of a long span suspension bridge is presented, and the fundamental dynamic characteristics of the suspension bridge to earthquake are investigated.

Because of the complexity of the structure, it is seldom possible to obtain an exact solution of the problem. In this investigation, the suspension bridge will be simplified into a physically analogous system to which the theory of finite degrees of freedom system can be applied. The earthquake motions being also quite complicated are assumed to be of simple shapes. Only the effects of ground motions acting in the direction of the bridge axis will be treated in this paper. In the analysis of earthquake responses of the suspension bridge, the effects of the stiffness and masses of the towers are of significant importance, and both effects are taken into account in the analysis.

Numerical calculation will be done on the AKASHI Strait Bridge which is now being planned by Kobe City Authority.

METHOD OF ANALYSIS

In this chapter, a method of analysis of a suspension bridge subjected to earthquakes is presented using a simplified analogous system.

Physical System Considered

The system considered is as shown in Fig.1. Stiffening frames of the suspension bridge consist of rigid bars connected with elastic hinges. Elastic constants of the hinges are so selected that the analogy of bending characteristics to the original stiffening frame is satisfied. Dead weight of the stiffening frames, floor systems, and cables are assumed to be concentrated at the hinged points considered.

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Each hinged point has a freedom of motion to the vertical direction specified by a deflection y_r.

Towers of the bridge are also replaced by the same physically analogous system. Only the horizontal motion is assumed to be allowable to the points in the tower. Axial forces and horizontal forces acting to the top of the towers are taken into consideration.

Ordinary assumptions in the analysis of suspension bridges, such as suspenders being inextensible, are assumed in this analysis. Vibration damping of the structure is omitted in the analysis.

Using such simplified system, theory of systems having finite degrees of freedom can be applied to the problem.[1] When the suspension bridge is devided into fairly large number of segments, a good approximation is obtained. Only high speed computers can execute numerical calculations for the analysis of such system.

Differential Equation of Motion of a Point in Stiffening Frame

A point considered here, is the elastic hinged point of the system specified above and the dead load is assumed to be concentrated in this point. In Fig.2, the three adjoining points in the stiffening frame are shown, and the equilibrium of the point r will be discussed here. Solid lines on Fig.2 show the condition of static equilibrium due to dead load. As the stiffening frames of suspension bridges carry no dead weight, the following static equilibrium is derived regarding the point r.

$$\tan \alpha^{\circ}_{r,r+1} - \tan \alpha^{\circ}_{r,r-1} = \frac{W_r}{H_w} \quad \cdots\cdots\cdots\cdots (1)$$

where

H_w = horizontal component of the cable tension due to dead load

W_r = dead load concentrated at the point r.

Using cable sags of three points, Eq. (1) is

$$\frac{f_{r-1} - 2f_r + f_{r+1}}{a} = -\frac{W_r}{H_w} \quad \cdots\cdots\cdots\cdots (2)$$

Two kinds of internal reactions take place with the displacement of the point. One is due to bending moment of stiffening frame, and the other is due to increment of the cable tension.

Reaction due to the bending moments of stiffening frame is

$$R_r^B = \frac{1}{a}(M_{r-1} - 2M_r + M_{r+1}) \quad \cdots\cdots\cdots\cdots (3)$$

where

M_r = bending moment at the point r in the stiffening frame

R_r^B = reaction due to bending moment of the stiffening frame.

Moment M_r is assumed to be expressed by the deflections of three adjoining points in the stiffening frame as,

$$M_r = \left(-\frac{B_r}{a}\right)(y_{r-1} - 2y_r + y_{r+1}) \quad \cdots\cdots\cdots\cdots (4)$$

where

B_r = elastic constant, selected to satisfy the physical conditions.

With the increment in cable stress due to vibration, horizontal component of cable stress increases from H_w to H_w+h. Reaction due to cable tension H_w+h is,

$$R_r^{H_w+h} = (H_w+h)(\tan\alpha_{r,r+1} - \tan\alpha_{r,r-1}) \quad \cdots\cdots (5)$$

in which

$$\left.\begin{aligned} \tan\alpha_{r,r-1} &= \frac{(f_r+y_r)-(f_{r-1}+y_{r-1})}{a} \\ \tan\alpha_{r,r+1} &= \frac{(f_{r+1}+y_{r+1})-(f_r+y_r)}{a} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (6)$$

Substituting Eq. (6) into Eq. (5), and using the relation of Eq. (2), the reaction due to increment of cable tension is

$$R_r^h = \frac{h}{a}(f_{r-1}-2f_r+f_{r+1}) + \frac{H_w+h}{a}(y_{r-1}-2y_r+y_{r+1}) \quad \cdots\cdots (7)$$

Total reaction at the point r due to the deflections of the stiffening frame is, therefore,

$$\left.\begin{aligned} R_r &= R_r^B + R_r^h \\ &= \frac{1}{a}(M_{r-1}-2M_r+M_{r+1}) + \frac{h}{a}(f_{r-1}-2f_r+f_{r+1}) \\ &\quad + \frac{H_w+h}{a}(y_{r-1}-2y_r+y_{r+1}) \end{aligned}\right\} \cdots (8)$$

No external forces being applied to the point r, the equation of motion of the point is given as

$$\frac{W_r}{g}\ddot{y}_r = R_r \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (9)$$

Cable Equation

The increment of cable tension, h, in Eq. (8) is the function of deflections of the stiffening frame and it can be obtained from so-called cable equation.

Assuming that the cable element r, r+1, moves to the position r′, r+1′, as shown in Fig. 3, due to the increment of the cable tension, the following relation is given neglecting small quantities of higher orders.

$$(u_{r+1}-u_r)\cos\alpha_{r,r+1}^0 + (y_{r+1}-y_r)\sin\alpha_{r,r+1}^0 = \frac{t_{r,r+1}L_{r,r+1}}{E_c A_c} \quad \cdots\cdots (10)$$

where

$t_{r,r+1}$ = the increment of cable stress of the cable element r, r+1
$L_{r,r+1}$ = length of the cable element r, r+1
E_c = Young's modulus of the material of the cable
A_c = cross section area of the cable
u_r = horizontal component of the displacement of the point r
y_r = vertical component of the displacement of the point r

The horizontal component of the elongation of the cable element r, r+1 is

$$\left.\begin{aligned} \Delta u_{r,r+1} &= u_{r+1}-u_r \\ &= \frac{t_{r,r+1}L_{r,r+1}}{E_c A_c \cos\alpha_{r,r+1}^0} - (y_{r+1}-y_r)\tan\alpha_{r,r+1}^0 \end{aligned}\right\} \cdots\cdots (11)$$

Substituting the relations

$$t_{r,r+1} = \frac{h}{\cos\alpha^{\circ}_{r,r+1}}, \quad L_{r,r+1} = \frac{a}{\cos\alpha^{\circ}_{r,r+1}}$$

one obtains

$$\Delta u_{r,r+1} = \frac{ha}{E_c A_c}\frac{1}{\cos^3\alpha^{\circ}_{r,r+1}} - (y_{r+1} - y_r)\tan\alpha^{\circ}_{r,r+1} \quad \cdots\cdots\cdots (12)$$

Total elongation of the cable is

$$\left.\begin{aligned} u &= \sum \Delta u \\ &= \frac{h}{E_c A_c}\left(\sum \frac{a}{\cos^3\alpha^{\circ}_{r,r+1}}\right) + \sum y_r(\tan\alpha^{\circ}_{r,r+1} - \tan\alpha^{\circ}_{r,r-1}) \end{aligned}\right\} \cdots (13)$$

Using Eq. (2), Eq. (13) will be

$$u = \frac{h}{E_c A_c} L_E - \frac{W_r}{H_w}\sum y_r \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (14)$$

where

$$L_E = \sum \frac{a}{\cos^3\alpha^{\circ}_{r,r+1}} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (15)$$

Eq. (14) must be satisfied in each span of the suspension bridge, and the different values of h are given in different spans.

Equation of Motion of Towers

Towers of the suspension bridge are also devided into small segments which consist of rigid bars and elastic hinges. Fig. 4 shows the tower subjected to ground motions at the base and the horizontal and vertical forces at the top.

The same method of derivation of equation of motion as before is employed, and defining

P_w = axial force due to dead load acting at the tower
P = increment of the axial force due to inertia force
Δh = horizontal force acting at the top of the tower during vibration, due to the difference of cable tension
W_r = dead weight of the tower concentrated at the point r,

the equations of motion of inner point of the tower are obtained as

$$\frac{W_r}{g}\ddot{y}_r = \frac{1}{b}(M_{r-1} - 2M_r + M_{r+1}) - \frac{1}{b}(P_w + P)(y_{r-1} - 2y_r + y_{r+1}) \cdots (16)$$

At the top of the tower the following equation is to be applied as horizontal force Δh is acting there.

$$\frac{W_n}{g}\ddot{y}_n = \frac{1}{b}(M_{n-1}) - \frac{1}{b}(P_w + P)(-y_n + y_{n-1}) + \Delta h. \quad \cdots\cdots\cdots (17)$$

Linearization of the Theory

The equation of motion derived are originally depending upon the deflection theory of suspension bridges and have non-linear characteristics. When the increment of the cable tension and of the axial force due to the inertia force are small compared to those due to dead load, the terms $H_w + h$, and $P_w + P$, in Eqs. (9) and (16) are assumed to be H_w, and P_w, and the quantities having non-linear properties are ignored. Equations obtained correspons to so-called linearized deflection theory of suspension bridge.(2)

System of the Fundamental Equation of Motion

Using the linearized theory, a system of fundamental differential equations of motion of the following form can be derived.

$$[A]\,(\ddot{y}_r)+[B]\,(y_r)+(P_r(t))=0 \qquad \cdots\cdots\cdots\cdots (18)$$

In Eq. (18) [ ], ( ) show a square symmetric and a vector matrix respectively , and

y_r = displacement of a point in the stiffening frame or the tower

$P_r(t)$ = external force due to ground motion.

Matrix [A] is a diagonal matrix with the diagonal element $a_{rr}=\frac{W_r}{g}$
Matrix [B] is stiffness matrix. Vector matrix of external forces $P_r(t)$ is the function of the ground motion Z_A, Z_B, Z_C, and Z_D.

Z_A = displacement of the left side anchorage
Z_D = displacement of the right side anchorage
Z_B = displacement of the left side tower base
Z_C = displacement of the right side tower base

The problem expressed by Eq. (18) is physically the vibration problem with multi-degrees of freedom and can be effectively solved by modal analysis of vibration if the natural frequencies and modes of vibration are obtained.

Natural Frequencies and Modes of the System

Frequency equation of the system is

$$|[B]-\lambda[A]|=0 \qquad \cdots\cdots\cdots\cdots (19)$$

where

$\lambda=\omega^2$ = characteristic values
ω = circular frequencies of the system

The characteristic roots and vectors of this determinantal equation represent the natural frequencies and modes of the system.

Matrices [A] and [B] in Eqs. (18) and (19) can be obtained if the simplification of the suspension bridge and the structural constants concerned are given.

As the first approximation, the system shown in Fig. 5 is adopted. Dimensions and dead loads of the system are selected refering to the values of the AKASHI Straits Bridge and are,[3]

$a = L/8 = 1300/8 = 162.5$ m
$b = L_T/4 = 200/4 = 50.0$ m
$H_w = 19560$ ton
$W_r = 1625$ ton (r = 1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 19, 20, 21)
$W_4 = W_{18} = 340$ ton, $W_5 = W_{17} = 1165$ ton
$W_6 = W_{16} = 1.778$ ton, $W_7 = W_{15} = 2521$ ton
$B_r = 5.169\times10^5$ t.m (r = 8, 9, 10, 11, 12, 13, 14)
$B_r = 6.462\times10^5$ t.m (r = 1, 2, 3, 19, 20, 21)
$B_5 = B_{17} = 46.158\times10^5$ t.m, $B_6 = B_{16} = 107.52\times10^5$ t.m
$B_7 = B_{15} = 216.05\times10^5$ t.m, $B_B = B_C = 391.3\times10^5$ t.m

The stiffening frames are considered to have uniform cross section for each span, and towers values to have varying cross sections. Numbers of the points are given in Fig. 5.

Using these, square matrices [A] and [B] with 21 × 21 elements are obtained. Tables of these matrices are omitted here as they take much space.

Because the system of Fig. 5 is symmetric with respect to the center of the system, the natural modes of vibration are either symmetric or asymmetric, and it becomes convenient to evaluate the natural frequencies and modes in two separate groups.

Values of λ and characteristic vectors for the symmetric and asymmetric modes are given in Tables 1 and 2. Configurations of the modes are shown in Figs. 6 and 7. They were obtained on the high speed digital computer.

For the sake of convenience of the later analysis, the natural modes of the system, shown in Tables 1 and 2, are normalized so as to satisfy the condition

$$\sum_{r=1}^{21} \frac{W_r}{g} (Y_r^{(i)})^2 = 1 \qquad \text{........................ (20)}$$

(i = 1, 2, ..., 11 for symmetric modes)
(i = 1, 2, ..., 10 for asymmetric modes)

EARTHQUAKE RESPONSES

Application of Modal Analysis

Natural frequencies and modes of the system obtained in the preceding section will be utilized in the following analysis[4].

The displacement y_r of the system can be expressed as follows employing the normal modes obtained and new time functions q_1, q_2, ..., q_n.

$$y_r = q_1 Y_r^{(1)} + q_2 Y_r^{(2)} + \cdots\cdots + q_n Y_r^{(n)} \qquad \text{....... (21)}$$

(r = 1, 2, 3, ..., n)

where $Y_r^{(i)}$ = amplitude of ith mode free vibration

Substituting Eq. (21) into Eq. (18), one obtains, after some calculation

$$\ddot{q}_l + \lambda_l q_l + \sum_{i=1}^{n} Y_i^{(l)} P_i(t) = 0 \qquad \text{................ (22)}$$

(l = 1, 2, 3, ..., n)

providing all the modes of vibrations are normalized.

Each equation of the system of differential equations (22) has single dependent variable q_s and can be solved by ordinary method. If all q_s are obtained for the specific external force $P_r(t)$, the displacement can be obtained from Eq. (21).

Assumption of Ground Motion

Because of the great complexity of the earthquake motion, an exact solution of the earthquake responses of the suspension bridge is seldom possible, and only an approximate solution of the suspension bridge to some idealized ground motions are possible. In this paper, the ground motion is, as the first stage, assumed to be a displacement with a simple harmonic shape given as

$$\left.\begin{aligned} Z &= A\left(1-\cos\frac{2\pi}{T}t\right) & (0\leq t\leq T) \\ &= 0 & (T<t) \end{aligned}\right\} \quad \ldots.\ (23)$$

and shown in Fig. 8. Responses to other kinds of distrubances can be obtained by this analysis, and it will be done in future work. In Numerical analysis of this paper, the amplitude is assumed as A = 1m, then the maximum displacement of the ground motion is 2m. Because of the linearity of the system considered, responses to any amplitude can be obtained by linear reduction.

Responses to the Ground Motion

In the design purpose, responses of the bending moments in the system are much more significant than those of the displacement. The responses of the bending moments on the following sections will be duscussed in this paper: (1) the tower base, (2) the center of the tower, and (3) the center of the center span stiffening frame.

The ground motion of Eq. (23) can be applied to any points connecting the structure to the ground, those are left and right side cable anchorages and two tower bases, and each ground motion has a individual effect on the structure. Because the structure has long span lengths, any phase differences between each disturbance is possible. The responses in this chapter are obtained by adding the effect of each disturbance graphically so as to make the resultant bending moment maximum.

Fig. 9 (a) and (b) show the time-bending moment curves to the point of the center of the tower due to the disturbances with different durations of ground motions. T = 0.125, 0.25, 0.375, (Fig.9 (a)), 0,50, 0.75, 1.00, 1.50 (Fig. 9 (b)) in sec. Fig.10 shows the time-bending moment curves for the center of the center span due to the same disturbances. In Fig. 10, are given fairly different response characteristics from Fig. 9, and the maximum moment is much less than that of the tower.

Fig. 11 shows the spectra for the maximum bending moment at the center of the tower resulting from the disturbance of eq.(23). Fig. 12 shows the spectra for the maximum bending moment at the tower base, and Fig. 13 shows the spectra at the center of the center span stiffening frame. The spectra of Figs. 11 and 12 have their maximum values at about the value of T = 0.25 sec. and Fig. 13, at about T = 4 sec.

Patterns of the relations of the amplitudes and periods of earthquakes have to be determined in order to get clear understanding on response spectra. If the amplitude and period in Eq. (23) are given, the response to the given external disturbance can be obtained considering the linearity of the system.

CONCLUSION

The responses of the suspension bridge due to ground disturbances are analysed by adopting the simplified structural system with finite degrees of freedom, and some response spectra are obtained.

The main conclusions are:
(1) Vibration modes of the system, Figs. 6 and 7, can be separated into two groups with different characteristics. In one group, the displacement of the stiffening frames are predominent, such as the 1st through the 7th symmetric and the 1st through the 6th asymmetric modes, and in the other group, displacements of the towers are predominent, as the 8th through the 11th symmetric and the 7th through the 10th asymmetric modes.
(2) As it is clear from the response spectra, motions of the towers subjected to an earthquake are more significant than those of the stiffening frames, and the stiffness and masses of the towers must be taken into account in the analysis of earthquake responses of suspension bridges.
(3) Two vibration modes of the tower, such as the 8th symmetric and the 7th asymmetric modes, are almost the same as shown in Figs. 6 and 7 excepting that they are symmetric and asymmetric. This means that in experimental and theoretical investigations of earthquake responses, a partial model where only the tower and physically equivalent effects of cables and stiffening frames are considered is approximately applicable.

Only the fundamental characteristicswere obtained, in this analysis, then the following experimental and theoretical investigations are necessary to obtain clear understanding on the problem and material on practical design.
(1) The same numerical analysis for the simplified system with better approximation than the system of Fig. 5 must be done to obtain design data for the suspension bridge. Natural modes and frequencies for the system having eight segments in the tower and the same number of segments as in Fig. 5 in the stiffening frames were obtained, and the dynamic responses are now being calculated.
(2) The effects of the higher mode vibration to the bending moment of the tower were significant according to the numerical calculation done. Damping of the higher mode vibrations have to be clarified to obtain better information on higher mode responses because the damping effect is considered to be more effective to the higher mode vibrations than to the lower modes.
(3) Model tests on the tower has to be done to obtain the experimental results on earthquake responses and damping characteristics.
(4) Analysis upon the deflection theory and the earthquake responses with plastic deformations of the structure must be done, and the allowable plastic deformations of the suspension bridge must be made clear.
(5) Responses due to earthquake with any directions of motion must be investigated.

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(4) Timoshenko, S., Vibration Problems in Engineering, Second Edition, p. 209, 1939

NOMENCLATURE

$[A]$	= a diagonal matrix with element $a_{rr} = W_r/g$	
A_c	= cross section area of a cable	m^2
a	= length of one devided segment of stiffening frames	m
$\alpha^{o}_{r,r+1}$	= declination of cable element r, r+1 in static equilibrium	
$\alpha_{r,r+1}$	= declination of cable element r, r+1 during the motion	
$[B]$	= stiffness matrix, square and symmetric	
B_r	= elastic constant, selected to satisfy the physical conditions	t.m
b	= length of one devided segment of the tower	m
E_c	= Young's Modulus of cable material	t/m^2
f	= cable sag at the center of the bridge	m
f_r	= cable sag at the point r	m
g	= acceleration of gravity, 9.8 m/sec^2	
H_w	= horizontal component of the cable tension due to dead load	ton
h	= horizontal component of increment of cable tension due to vibration, different in each span	ton
Δh	= horizontal force acting to the top of the tower	ton
L	= length of the center span, 1300m in the Akashi Strait Br.	
$L_{r,r+1}$	= length of cable element r, r+1	m

L_T = height of the tower, 200m in the Akashi Straits Br.

λ = characteristic values

M_r = bending moment at the point r — t.m

$(P_r(t))$ = vector matrix of external forces due to ground motion

P_w = axial force due to dead load, acting at the top of the tower — ton

P = increment of axial force due to vibration — ton

q_i = time function for the ith mode vibration

R_r = total reaction at the point r due to deflection of the stiffening frame — ton

R_r^B = reaction due to bending moment of the stiffening frame — ton

R_r^h = reaction due to increment of cable tension — ton

T = duration of ground motion — sec

$t_{r,r+1}$ = increment of cable tension of the element r, r+1 — ton

u = total elongation of cable in horizontal direction — m

u_r = horizontal component of displacement at the point r in cables — m

$\Delta u_{r,r+1}$ = horizontal component of elongation of the cable element r, r+1 — m

W_r = dead load of tower or stiffening frames, concentrated to the point r — ton

Y_r = maximum displacement of the point r — m

$Y_r^{(i)}$ = amplitude of ith mode free vibration, normalized — m

y_r = displacement of the point r — m

Z_A, Z_D = horizontal displacements of the left and right side anchorage, respectively — m

Z_B, Z_C = horizontal displacements of the tower base B and C respectively — m

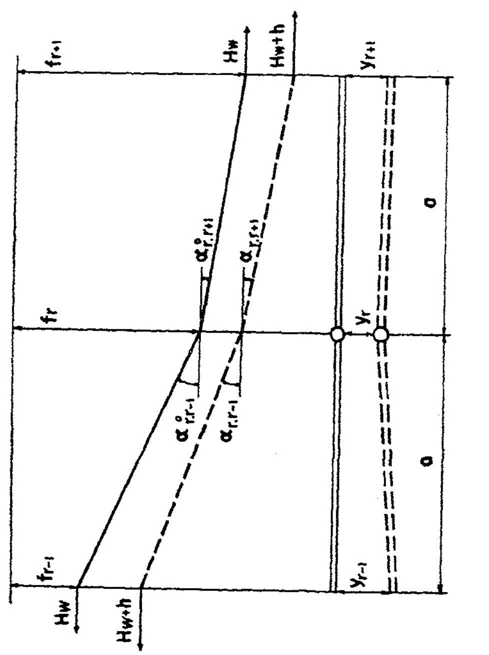


FIG. 2

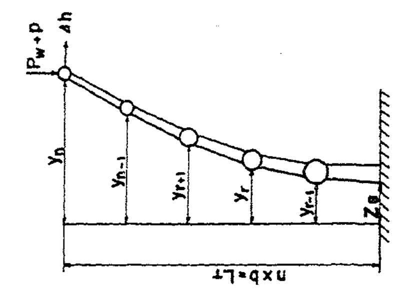


FIG. 4

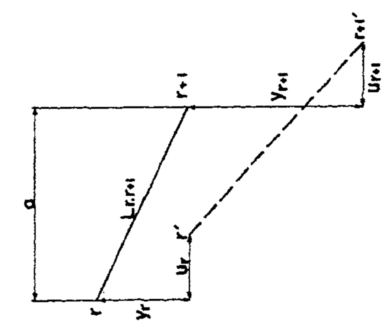


FIG. 3

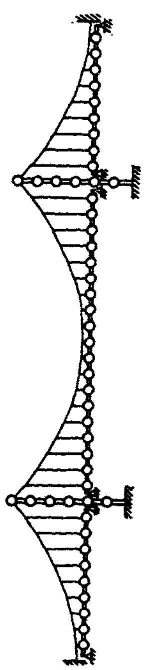


FIG. 1

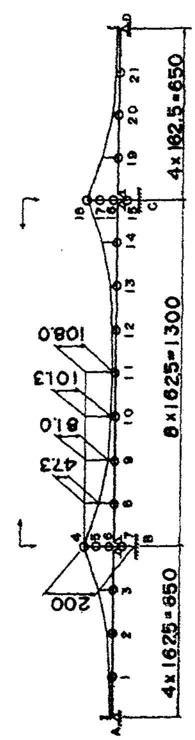
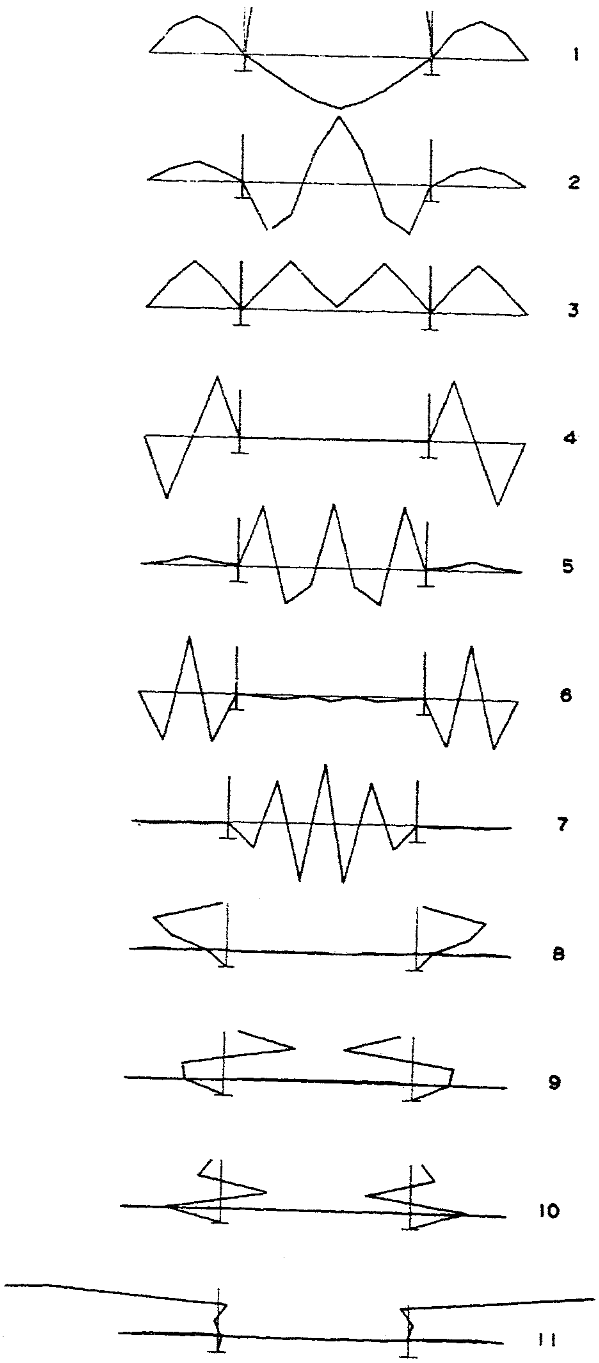
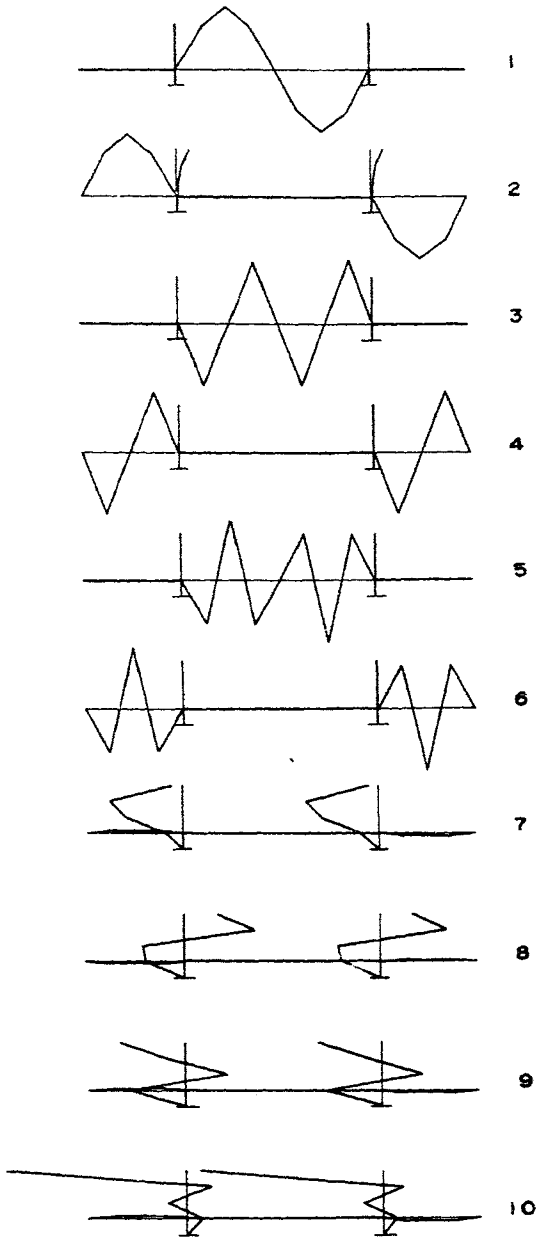


FIG. 5



SYMMETRIC MODES

FIG. 6



ASYMMETRIC MODES

FIG. 7

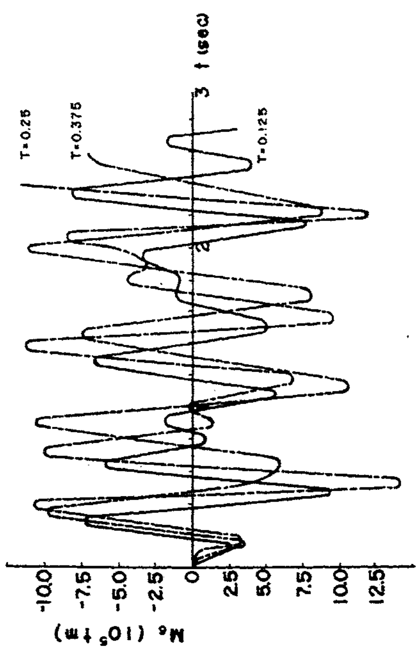


FIG. 9 (a)

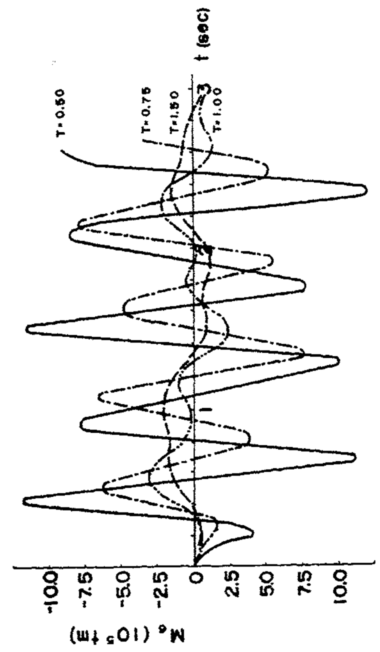


FIG. 9 (b)

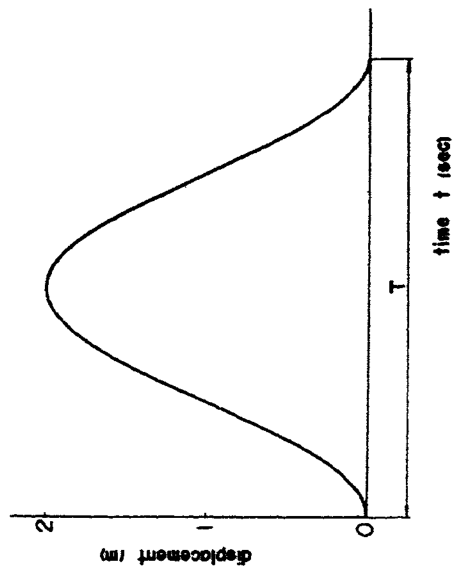


FIG. 8

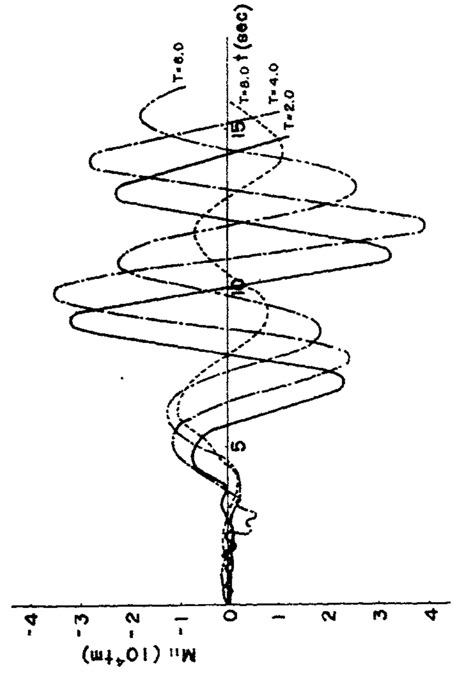


FIG. 10

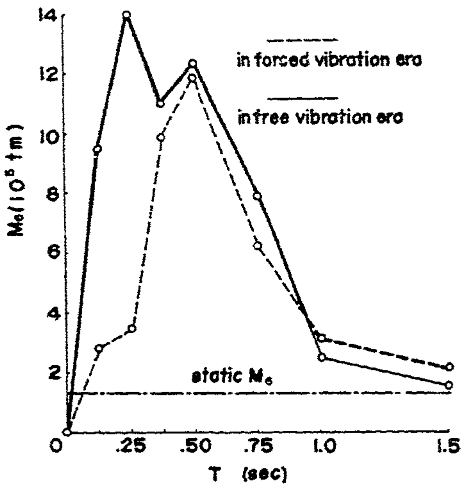


FIG. 11

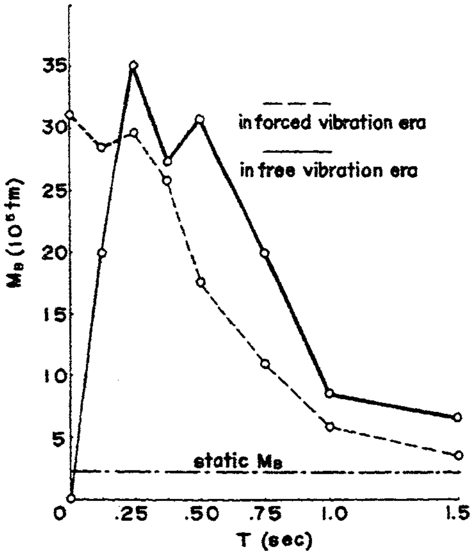


FIG. 12

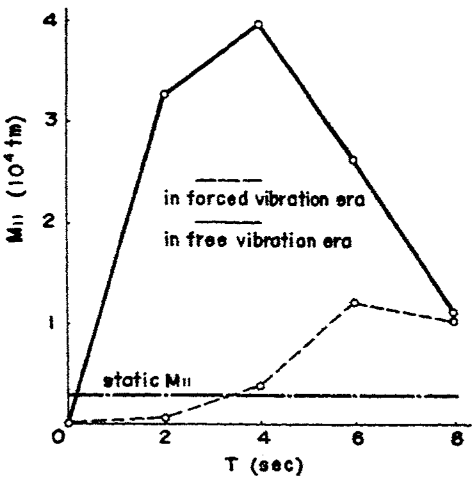


FIG. 13

TABLE 1 SYMMETRIC MODES

Modes	1	2	3	4	5	6	7	8	9	10	11
	Characteristic Values										
λ_i	.2930	1.0208	1.9928	2.0421	2.9491	4.2385	4.5435	21.734	129.83	358.37	781.86
T_i	11.608	6.2191	4.4511	4.3969	3.6588	3.0519	2.9478	1.3477	.5514	.3319	.2247
	Natural Modes, Normalized $(10^{-1}m)$										
$Y_1=Y_{21}$	-.1734	-.0834	-.1697	.3883	-.0203	.2901	.0122	-.0106	.0044	-.0013	-.0097
$Y_2=Y_{20}$	-.2413	-.1251	-.3042	0	-.0583	-.3626	-.0058	-.0099	.0044	-.0013	-.0097
$Y_3=Y_{19}$	-.1734	-.0834	-.1697	-.3883	-.0203	.2901	.0122	-.0106	.0044	-.0013	-.0097
$Y_4=Y_{18}$	.0547	.0156	-.0006	0	-.0059	-.0063	.0015	-.0321	.0881	-.0720	-1.193
$Y_5=Y_{17}$	.0274	.0080	-.0003	0	-.0033	-.0037	.0009	-.4501	.4377	-.1504	.0549
$Y_6=Y_{16}$	.0105	.0031	-.0001	0	-.0014	-.0016	.0004	-.3529	-.2623	.2854	-.0266
$Y_7=Y_{15}$	.0022	.0006	0	0	-.0003	-.0004	.0001	-.1110	-.2371	-.3546	.0070
$Y_8=Y_{14}$	.0934	.3100	-.1564	0	-.3845	.0208	.1543	.0107	-.0044	.0013	.0097
$Y_9=Y_{13}$	.2005	.1991	-.3092	0	.2284	.0243	-.2705	.0101	-.0044	.0013	.0097
$Y_{10}=Y_{12}$	.2864	-.2078	-.1755	0	.1195	.0027	.3620	.0102	-.0044	.0013	.0097
Y_{11}	.3192	-.4313	-.0335	0	-.4076	.0355	-.3827	.0102	-.0044	.0013	.0097

TABLE 2 ASYMMETRIC MODES

Modes	1	2	3	4	5	6	7	8	9	10
					Characteristic Values					
λ_i	.4658	.5011	1.9241	2.0421	3.8546	4.1984	21.167	125.39	350.79	430.40
T_i	9.2067	8.8771	4.5297	4.3969	3.2003	3.0666	1.3657	.5611	.3354	.3028
					Natural Modes, Normalized (10^{-1}m)					
$Y_1=-Y_{21}$	0	-.2731	0	.3883	0	.2744	-.0219	.0101	-.0072	-.0164
$Y_2=-Y_{20}$	0	-.3871	0	0	0	-.3884	-.0202	.0100	-.0072	-.0164
$Y_3=-Y_{19}$	0	-.2731	0	-.3883	0	.2744	-.0219	.0101	-.0072	-.0164
$Y_4=-Y_{18}$	0	.0763	0	0	0	-.0134	-.0640	.1960	-.3978	-1.111
$Y_5=-Y_{17}$	0	.0385	0	0	0	-.0078	-.4544	.4255	-.1064	.1421
$Y_6=-Y_{16}$	0	.0149	0	0	0	-.0034	-.3486	-.2689	.2595	-.1192
$Y_7=-Y_{15}$	0	.0031	0	0	0	-.0008	-.1087	-.2330	-.3467	.0895
$Y_8=-Y_{14}$	-.2746	0	.3883	0	.2746	0	0	0	0	0
$Y_9=-Y_{13}$	-.3883	0	0	0	-.3883	0	0	0	0	0
$Y_{10}=-Y_{12}$	-.2746	0	-.3883	0	.2746	0	0	0	0	0
Y_{11}	0	0	0	0	0	0	0	0	0	0