A BROAD FORMULA FOR ESTIMATING EARTHQUAKE FORCES ON OSCILLATORS

By Frank Neumann*

The expression "earthquake forces" is used in the title of this paper because the ultimate objective of the investigation is to provide a formula for computing earthquake forces on buildings. The paper, however, will not be concerned with the masses involved in such forces but only with the accelerations to which they are subjected. To this extent the proposed formula serves the same purpose as those found in many building codes which attempt, usually through empirical approaches, to specify the lateral earthquake forces or accelerations that buildings and structural parts should be designed to resist. The proposed formula, however, will be based on seismological data exclusively and on purely seismological concepts.

The specific purpose is to provide a ready means of determining the maximum acceleration and displacement impressed on an oscillator of any period and damping by an earthquake of any intensity. Such a formula is obviously designed to estimate the deformation of an entire structure and the acceleration associated with it rather than to evaluate the linear accelerations experienced at any particular floor levels. It is based on a number of research projects that have extended over the past two decades but the details of these projects are too involved to cover in this brief paper. There is time only to summarize the results obtained and in some cases re-evaluate them. The intensity scale used is the Modified Mercalli Earthquake Intensity Scale of 1931.

EARTHQUAKE GROUND MOTIONS

The ground motion data used in this study were obtained exclusively from strong motion seismograph records of the U. S. Coast and Geodetic Survey (13). They are generally acceleration records. Years ago, while on the seismological staff of the Survey, the writer double integrated some of the more important records to obtain equivalent velocity and displacement curves (8). These curves revealed all of the periods in the ground motion and made it possible to construct graphs showing the maximum accelerations, velocities and displacements associated with the various periods. It was ultimately found that if the data were plotted on a 4-way logarithmic grid designed for period analyses a single curve would suffice because the acceleration, velocity and displacement of any point on such a curve can be measured by referring to the appropriate set of rectangular or diagonal coordinates. Examples of such grids and curves are found in Figs. 4 and 5. Such "ground spectra" are of vital importance in the current study because they provide a feasible means of comparing the outstanding characteristics of different earthquake motions regardless of their pattern or complexity.

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Earthquake intensity vs maximum vibrational velocity. In 1954 the writer published a booklet (9) in which an effort was made to associate the various grades of the MM (Modified Mercalli) Scale with specific ground accelerations taking into consideration the periods involved. The result was a rather complicated formula which has since been abandoned. It was subsequently concluded that the most feasible measure of earthquake intensity was the maximum vibrational velocity of the ground motion (10, 11). This was based largely on the following reasoning.

In the case of high frequency blasting vibrations a number of investigators (4, 5) have found that damage begins when building vibrations reach the 10 and 15 cm/sec. level. These are usually the maximum velocities to which near-by structures are subjected because such high frequencies are not likely to induce serious resonance effects. In the case of earthquakes MM-5 and MM-6 mark the onset of damage but the recorded ground velocities lie only between 2.5 and 4 cm/sec. It is common, however, for the upper portions of buildings to experience resonance effects during earthquakes and vibrate through as much as four times the amplitude of the ground. This would bring the maximum vibrational velocity of a building up to the same damaging 10 to 15 cm/sec. level found in high frequency blast vibrations. Because of the extremely wide differences in blasting vibration periods and earthquake periods it is felt that such an agreement, even though only approximate, provides rather convincing evidence that vibrational velocity, rather than acceleration or displacement, is the most acceptable measure of intensity.

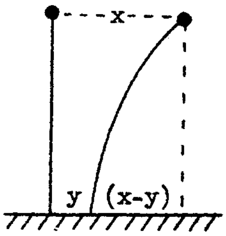
Another aspect of the problem is that the shorter duration of a blast vibration may require a greater velocity to reach the damaging stage than required in an earthquake motion that is equally damaging but of longer duration. The duration of any type of disturbance increases as the distance from its source increases. This principle seems to influence the intensity-velocity relationship at the greater epicentral distances as indicated in the insert note on the chart of Fig. 5. Further study of these factors is quite in order. It is possible that amplified building motions and duration may both play a part in 'efining ground vibrations that have just reached the potentially damaging stage.

A final relationship between intensity and vibrational velocity was based on two considerations. Since it was originally found (9) from a study of intensity and acceleration data that the acceleration doubled (on the average) for each grade increase in intensity (from MM-1 through MM-8) it was assumed that the same relationship would hold with respect to vibrational velocity. The relatively small period range found in waves of maximum acceleration is consistent with such an assumption. Secondly, a maximum vibrational velocity of 22 cm/sec. was accepted as corresponding to an epicentral intensity of MM-8.3 in the case of the El Centro record of 1940. This reduced to the following velocities for intensities MM-8 to MM-1 respectively: 18, 9, 4.5, 2.25, 1.12, 0.56, 0.28, and 0.14 cm/sec. (12). Such a relationship agreed within acceptable limits with the intensities observed and the velocities registered in other earthquakes. This adopted intensity-velocity relationship is shown graphically on all of the writer's 4-way logarithmic grids such as Fig. 5.

The note on the Fig. 5 chart attempts to evaluate indirectly an over-all difference in ground motion noted when a given intensity is recorded instrumentally in an epicentral area and when the same intensity is recorded well outside this area. The formula given means that from a study of instrumental data and intensity distribution in the case of the Southern California (Bakersfield) earthquake of 1952 it was concluded that 150 miles from the epicenter the vibrational velocity for an MM-5 intensity was only one-half that for an MM-5 intensity in the central area. This would apply also to any other intensity. The longer duration of a disturbance experienced outside of a central area appears to compensate at least partly for decreasing ground motion in maintaining the same level of intensity. It appears that intensity is governed not only by the magnitude of the ground motion but also by its duration.

STEADY STATE FORCED VIBRATIONS

Before discussing the difficulties connected with determining oscillator responses to complex earthquake motions some features of steady-state vibrations and analyses will be briefly reviewed. This is desirable because many earthquake vibrations are of such uniform character that steady-state solutions might often be used in place of the more complicated procedures now being developed for complex and non-periodic ground motions. As it is very desirable to see how the two problems are related the same nomenclature will be used in both cases.



In all forced vibration studies interest is centered primarily on the relative motion between oscillator and ground as this measures the deformation of the oscillator at any instant. In the adjoining illustration this is indicated by (x-y) where (x) is the absolute motion of the oscillator and (y) the motion of the ground. Engineering interest is limited chiefly to the maximum deformation (x-y) experienced during a forced vibration and the acceleration $(\ddot{x}-\ddot{y})$ and velocity $(\dot{x}-\dot{y})$ associated with it. It is desired to know in particular just how (x-y) is related to (y).

For steady-state forced vibrations of simple harmonic character the relatively simple solution is found in all text books on vibration analysis. The so-called magnification curves in Fig. 2 are typical of the standard mathematical solution. Such curves show the relative motion (x-y) between the moving oscillator and the moving ground (y). The much-desired ratio (x-y)/(y) can be read from the graph if the period of the ground motion (T_e) and the period (T_o) and damping (h) of the oscillator are known. The curves readily show the responses of a single damped oscillator to steady-state ground vibrations of constant amplitude but variable period T_e, or the responses of a series of oscillators to a ground motion (y) of fixed period and amplitude. In Fig. 2 these are designated cases (a) and (b) respectively. To obtain the velocity $(\dot{x}-\dot{y})$ and the acceleration $(\ddot{x}-\ddot{y})$

associated with any of the relative displacements the period T_e must be used since the oscillator is forced to vibrate in unison with the ground regardless of its own period T_o.

The structural engineer's interest, however, is limited primarily to the maximum deformations associated with resonance, or near resonance, and low values of damping. This implies that he is not particularly interested in the small deformations associated with ground periods that are much greater or much less than the oscillator period. For this special case it may be assumed that since T_e and T_o are about equal one may compute the accelerations and velocities associated with the maximum resonant displacement by simply substituting T_o for T_e. This makes a calculation based on this assumption independent of the ground period. This point is emphasized because the equations take on the same form (as will be shown later) used in expressing the maximum response of an oscillator to a complex vibration in which the values of T_e are indefinite. In both cases

$$(\dot{x}-\dot{y})_{max.} \sim \frac{2\pi}{T_o}(x-y)_{max.}$$

and

$$(\ddot{x}-\ddot{y})_{max.} \sim \frac{4\pi^2}{T_o^2}(x-y)_{max.}$$

This provides an interesting analogy between the method used in determining (a) maximum oscillator responses to steady-state vibrations and (b) similar responses to complex earthquake vibrations.

The writer believes that virtually all earthquake vibrations can be treated basically as steady-state vibrations for the following reason. The basic element of an earthquake vibration is a relatively smooth wave train (9, pp. 19, 20). If an oscillator were to be subjected to such a wave motion its response in terms of (x-y)/(y) could be estimated quite well from standard magnification curves as illustrated in Fig. 2. Among seismologists application of this principle is standard practice where it is desired to compute the ground motion (y) from "smooth" types of ordinary seismograph records which provide the (x-y) data. The difficulty in strong motion records is that two or three or more wave trains may be impressing several types of wave trains on the seismograph pendulum simultaneously. If the (x-y) response of a particular oscillator could be computed for each wave train present, and all of the individual responses then combined, it is felt that the resultant motion would approximate that obtained when the (x-y) values are computed directly from strong motion seismograph records. This procedure will be discussed in the next section. The wave train concept is important and will later be shown playing an important part in the development of the proposed lateral force formula.

NON-PERIODIC FORCED VIBRATIONS

This section concerns the theoretical (x-y) response of an oscillator of given period (T_o) and damping (h) to a given earthquake motion such as registered on a strong-motion seismograph. One may disregard the wave train concept just advanced as a feasible approach to the oscillator response problem because a very direct solution was proposed by M. A. Biot (2) in the early thirties. His mathematical solution for obtaining maximum (x-y) from earthquake ground motion is now widely known but for some years application of his equation to practical problems was delayed because of the labor involved in solving the integral portion of the equation. The writer independently suggested the use of a torsion pendulum for computing oscillator responses of this kind (7). In 1943 Biot reported the results of comprehensive torsion pendulum studies made at the California Institute of Technology (3). Subsequently Alford, Housner and Martel found the electrical analog computer better adapted to this purpose and after adding a damping factor to Biot's equation (6) proceeded to calculate a large number of so-called "earthquake spectra." These curves show the maximum (x-y) responses of groups of damped oscillators to many strong earthquake motions registered on Coast and Geodetic Survey strong motion seismographs (1). The following form of equation was used in these analyses:

$$(x-y) = \frac{T}{2\pi}\int_0^t \ddot{y}(\tau)e^{\frac{2\pi h}{T}}\sin(t-\tau)dt = \frac{T}{2\pi}S \qquad (\dot{x}-\dot{y}) = S \qquad (\ddot{x}-\ddot{y}) = \frac{2\pi}{T}S$$

The oscillator spectra for the El Centro record of May 18, 1940, together with the corresponding ground spectrum determined by the writer, are shown in Fig. 3.

The problem confronting engineers is to utilize these spectra to the best possible advantage. It was previously stated that while maximum (x-y) values were important it was necessary to associate them with the corresponding maximum values of (y) if a solution similar to that for steady-state vibrations is to be developed. It appears that this may be possible.

GROUND SPECTRA VS OSCILLATOR SPECTRA

It was previously shown how a ground spectrum can indicate the maximum amplitude (acceleration, velocity or displacement) associated with each of the ground periods in a particular earthquake motion (y). It was later shown how, from that same motion (y), it is possible to compute the maximum (x-y) responses of a series of multiperiod oscillators having different degrees of damping. The (x-y) responses, or oscillator spectra, for the El Centro record of 1940 are shown in Fig. 3 for four degrees of oscillator damping. On the same grid is the ground spectrum (y) calculated from that record with data from the Helena (Montana) record of October 30, 1935, covering the high frequency end of the spectrum. The important question is: Does this set of spectra, and similar sets for other earthquake motions, indicate any systematic relationship between the oscillator spectra (x-y) and the ground spectrum (y)? If so, there would be available an (x-y)/(y) ratio similar to the ratio available for forced steady-state vibrations which permits (x-y) responses to be expressed in terms of (y). While the five curves in Fig. 3 are obviously not parallel, the writer believes that from theoretical and other considerations such curves are basically parallel so that one may assume that approximately fixed ratios exist between the smoothed axes of the various oscillator spectra (x-y) and the ground spectra (y).

The theoretical aspect of the question has been covered in part in the preceding section on steady-state forced vibrations. It was indicated there that because of the much greater (x-y) amplitudes found in the zone of resonance the responses in other period zones were relatively unimportant. This virtually answers the important question: What ground waves in an earthquake motion impress the greatest (x-y) motion on an oscillator of given period and damping? The answer is that if only low damped oscillators are considered the ground wave impressing the greatest (x-y) motion is the wave having the same period as the oscillator. If one accepts the wave train concept of earthquake ground motion, assumes that the wave train impresses a brief and virtually steady-state vibration on the oscillator, and from the magnification curve in Fig. 2 computes the (x-y) motions impressed on a particular oscillator by waves of any period and amplitude shown in the ground spectrum (in this case the El Centro ground spectrum) it will be found that the ground waves having the same period as the oscillator produce the dominant values of (x-y). It goes without saying that ground waves of other periods will modify this dominant response, and it is for this reason primarily that the (x-y) curves in Fig. 3 take on such variegated patterns as compared with the ground spectrum. But there seems ample reason to believe that the general level of the (x-y) responses is controlled primarily by resonance effects. When resonance does not occur the general level naturally drops.

In examining the (x-y) spectra in Fig. 3 one can only speculate on the magnitude of the errors that might creep into their computation, and on whether sufficient oscillator responses were computed, especially for long period oscillators, to establish reliable spectral curves covering all oscillator periods. The integration of seismograph records, which generally involves arbitrary choices of axes, always seems to call for a consideration of probable errors. It is a matter which should be given further attention and discussion in future work of this kind since it is directly related to the problem under discussion.

It is for the reasons just given that the writer believes that oscillator spectra for various degrees of damping can be considered approximately simple multiples of the corresponding ground motion spectra throughout the entire range of recorded periods. In other words, the spectra can furnish approximate values of (x-y)/(y) that correspond to the mathematically correct ratios found in steady-state forced vibration analyses. One should recognize the fact, however, that there may be wide variations from this simple ratio especially in the case of low-damped oscillators.

It is of particular interest to compare the resonant amplifications obtained for various degrees of damping (a) in the case of a simple sustained forced vibration and (b) the case of partial resonance resulting from a complex earthquake motion. For the four cases of $h = 0.0$, 0.02, 0.2, and 0.4 the amplifications for a simple sustained vibration are infinity, 25, 2.5

and 1.25 respectively. In the case of the El Centro, 1940 analysis they are approximately 8±5, 6, 2, and 1.5 respectively. It seems safe to assume that the El Centro case represents an extreme in low values for the amplification factors (x-y)/y. The record is extremely complex and it was obtained close to the epicenter. It would seem safe to presume that as other records are analyzed in a similar way these factors may increase as the records grow less complex with increasing epicentral distance. The factors given for the case of a simple steady-state vibration might be looked upon as limiting values.

DETERMINATION OF ACCELERATIONS AND DISPLACEMENTS IMPRESSED ON OSCILLATORS

From what has been previously stated there is obviously more than one way to make the computation indicated in this heading. The writer strongly endorses the use of a 4-way logarithmic grid as shown in Fig. 5. The essential data are the period and damping of the building or oscillator under consideration, and the intensity of the earthquake. The first step is to determine from the grid the amplitude of the ground wave having the same period as the oscillator as this is the wave that causes the greatest (x-y) deformation. The El Centro ground spectrum is placed on the grid to show what might be considered a representative spectrum for an earthquake of intensity MM-8.3. Unlike most other ground spectra (see Fig. 1) maximum amplitude waves are shown in all period categories so that the El Centro graph serves as an envelope that includes the maximum amplitudes associated with all ground periods. For the time being the writer feels that this curve can be used to show the period-amplitude distribution in earthquakes of other intensity by simply raising or lowering the entire curve to the intensity level desired remembering that the curve shows period-amplitude distribution for intensity MM-8.3 only. The intensity symbols at the appropriate velocity levels help to simplify this procedure.

A few notations on the grid illustrate how one estimates the maximum possible ground amplitudes in the case of an earthquake of intensity MM-5 for the ground periods 0.5, 0.3 and 0.15 sec. On the ordinates showing these periods a pair of dividers may be used to lower the points of intersection between the coordinates and the El Centro curve by an amount equivalent to 3.3 grades of intensity. This is the difference between the El Centro intensity MM-8.3 and MM-5. The desired ground amplitudes are indicated by the points a, a', and a", which correspond to the values of (y) previously discussed.

The (x-y) values associated with a, a', and a" are governed by the damping of the oscillator. The insert chart in Fig. 5 shows the increase of (x-y) intensity over the (y) intensity for different values of h based on the El Centro spectra in Fig. 3. This is equivalent to an (x-y)/(y) ratio if the intensities are expressed in terms of their equivalent vibrational velocities. If h is 0.1 the intensity difference to be added to a, a' and a" is 1.4 grades which places the (x-y) intensities at points b, b' and b". These are the maximum velocities of the (x-y) motions. The corresponding accelerations and displacements can be read from the diagonal coordinates as illustrated at the point b. This is the maximum displacement of the center of oscillation relative to the ground. The corresponding acceleration is the acceleration required to obtain that displacement of the oscillator.

The graph obviously may be used for problems in which the ground motion is not expressed as an earthquake intensity. The method here described presupposes earthquake conditions in which resonance is assumed between the oscillator and the ground. Furthermore, the normal amplification expected in the case of steady-state resonance is greatly reduced because of the complexity of earthquake motions as indicated in the insert diagram in Fig. 5.

The notations E.R. 1, 2, etc. on the chart indicate vibrational velocities that may be considered either safe or potentially damaging. E.R. refers to an "energy ratio" indicated by the square of the acceleration in ft./sec^2 divided by the square of the frequency in cycles per second. They refer only to high frequency waves found in blasting operations. Some state statutes consider E.R. 1 or E.R. 2 "safe blasting vibrations." Crandell (4) designates E.R. 3 to 6 as a "caution zone."

GRAPHICAL REPRESENTATION OF BUILDING CODE FORMULAE

Fig. 4 illustrates how building code formulae covering lateral force provisions can be shown graphically on the 4-way logarithmic grid. It provides a ready means of comparing different formulae and evaluating their probable effectiveness. To aid in this there will be seen at the top of the grid an effort to relate the period of a building to its height in stories. This, of course, is subject to wide variation and is placed on the chart with the intention of serving only as a very rough guide. It is based on data published in reference (13).

The line marked DD defines the accelerations called for in the so-called "Joint Committee" (San Francisco) code (14a). Line CC represents the lateral force provision in the Uniform Building Code (14b) of 1958 and some years prior to that. AA represents a proposed formula based on the equation

$$C = \frac{0.05}{\sqrt[3]{T}}$$

in which C is the acceleration factor and T the building period (14c). It is expected that the latter formula will soon replace the two others.

The remaining lines on Fig. 4 illustrate an approach suggested by the writer based on the assumption that the deflection of a structure should never exceed a displacement in excess of .001 the height of the structure; otherwise damage may be expected. If one assumes certain heights for multi-storied buildings, this ratio can be shown on the chart since the diagonal displacement grid can be used as a measure of (x-y) deformations. The line BB shows this deformation pattern based on the writer's choice of building heights. Since one is concerned primarily with the motion of the center of oscillation of a structure, not the top floor, the BB values are reduced one-half, to line

B'B', to indicate very approximately the deformation (x-y) of the center of oscillation of the building. This brings the curve down to roughly double the values indicated by the new code formula mentioned in the preceding paragraph. One need only apply a safety factor of two (2) to the code formula to obtain virtual agreement with this purely seismological approach. It is clear that the new empirically developed code formula is in closer agreement with this seismological concept than the formulae indicated by lines CC and DD.

CONCLUSIONS

There are some aspects of this proposed formula that are encouraging and others that some may consider discouraging. On the optimistic side may be listed the ultimate simplicity in its application to specific problems. If one is willing to accept the idea that the formula furnishes the best estimate that can be made with the information available he need not be concerned with the many technicalities involved in its development. He may find room for optimism in the apparent agreement between this purely seismological solution and one of the most recent engineering solutions as revealed in the new building code formula discussed in the preceding paragraph.

On the pessimistic side one may question the accuracy of a formula based on certain relationships that are not too well established. For this reason the formula must be looked upon as furnishing an estimate of the maximum possible lateral acceleration rather than a value that cannot be questioned. It is difficult to envision a precise earthquake force formula. Further research for more information on the over-all damping effect of different patterns of earthquake motion will probably alter the damping formula shown in the insert of Fig. 5. The degree of resonance attained will always, it seems, be largely a function of the complexity of the earthquake motion. So far as other basic concepts are concerned, it may take years to establish their validity beyond question. This will require an almost endless comparison between predicted effects and actual earthquake effects on structures unless one is willing to accept empirical building code formulae as representing factual information based on studies of structural damage.

The formula and the reasoning behind it seem quite consistent with the selective character of earthquake damage. If a building sways in resonance with a dominant ground wave it may suffer damage. If there happens to be no ground wave with a period similar to that of the building, the building will be shaken only slightly. One of the major fields for study is to obtain more data on the factors that govern dominant ground wave periods in a given locality. Local amplification factors depending almost solely on geological factors also require further study.

With the information now becoming available on building and ground motions it is difficult to see why there should be in the future such great variations in the estimates of earthquake forces impressed on buildings as recently pointed out by Prof. J. K. Minami in "Structural Design for Dynamic Loads." It is felt that the discrepancies are due largely to deficiencies in the seismological rather than the engineering phases of the problem. Seismologists as a whole have failed to focus attention on the needs of earthquake engineering research.

It is hoped that the ideas presented in this paper, new findings revealed in the various references, and recent contributions of Japanese investigators, especially in ground motion research, will soon result in greatly reducing or completely eliminating this deficiency.

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NOMENCLATURE

x - Broadly used to connote the continuing absolute oscillator displacements resulting from a given earthquake motion (y). $\dot{x}$ and $\ddot{x}$ indicate corresponding velocity and displacement.

y - Broadly used to connote the continuing ground displacement in a given earthquake motion. $\dot{y}$ and $\ddot{y}$ indicate corresponding velocity and displacement.

$(x-y)$ - The continuing differential displacement between moving oscillator and moving ground. The displacement of the ground subtracted from the absolute displacement of the oscillator. Because of frequent repetition of the maximum value of $(x-y)$, the only value in which interest is centered, this expression is generally used in the text to indicate the maximum even though it is not so designated.

$(\dot{x}-\dot{y})$ - Generally indicates the maximum differential velocity when an oscillator is subjected to a given earthquake motion. See $(x-y)$.

$(\ddot{x}-\ddot{y})$ - Generally indicates the maximum differential acceleration when an oscillator is subjected to a given earthquake motion. See $(x-y)$.

T_e - Period of ground waves in seconds.

T_o - Undamped period of oscillator in seconds.

h - Fraction of critical damping.

ϵ - Damping ratio.

t - Time in seconds.

τ - Time parameter in integration.

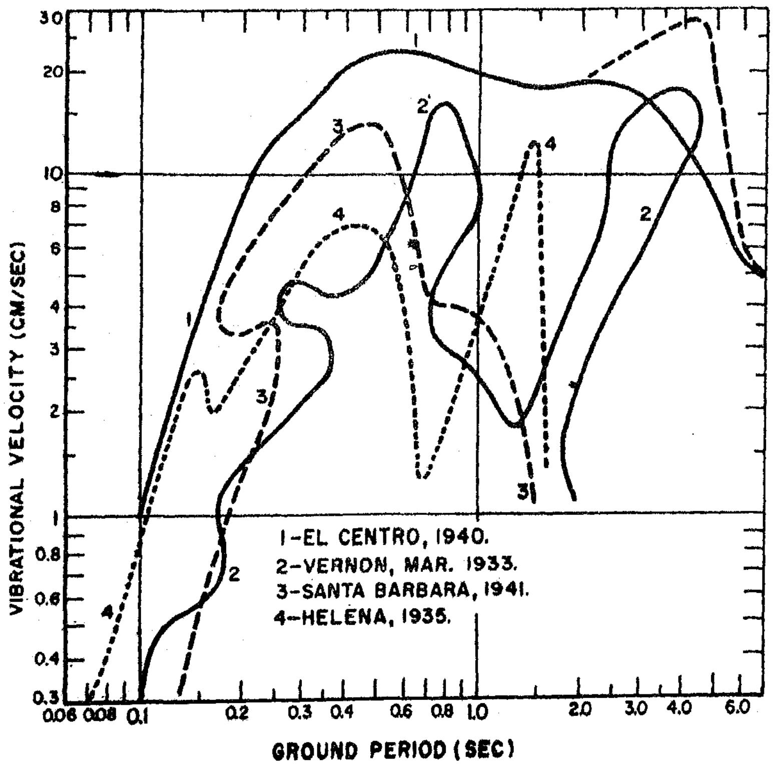
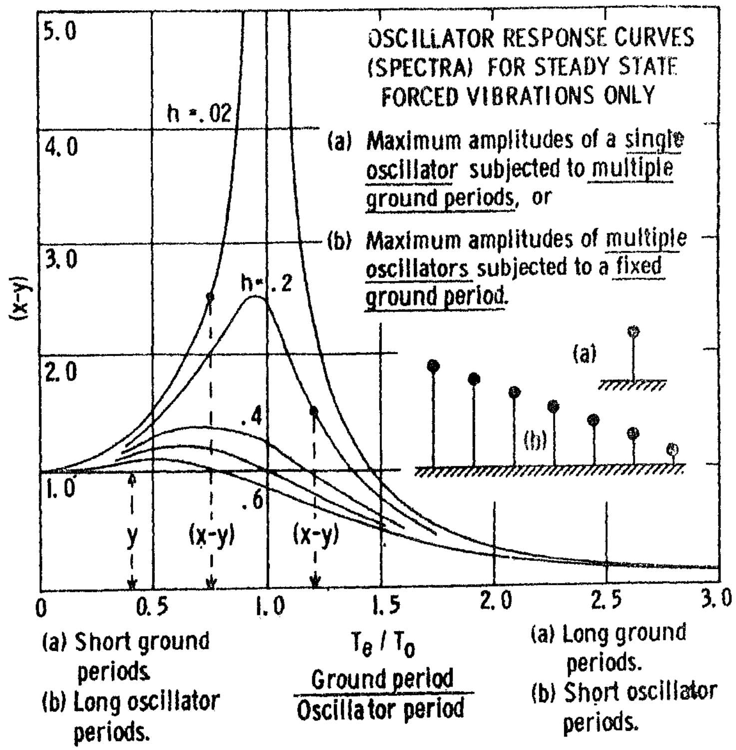


FIGURE 1. VELOCITY-PERIOD ENVELOPE FOR 4 EARTHQUAKES.



$$(x-y) = 1 \Big/ \sqrt{\left(\frac{Te^2}{To^2} + 1\right)^2 + 4\,\frac{Te^2}{To^2}\,(h^2 - 1)}$$

$$h = \log_{10} t \Big/ \sqrt{1.862 + (\log_{10} t)^2}$$

Figure 2. Magnification Curves For S.H.M.

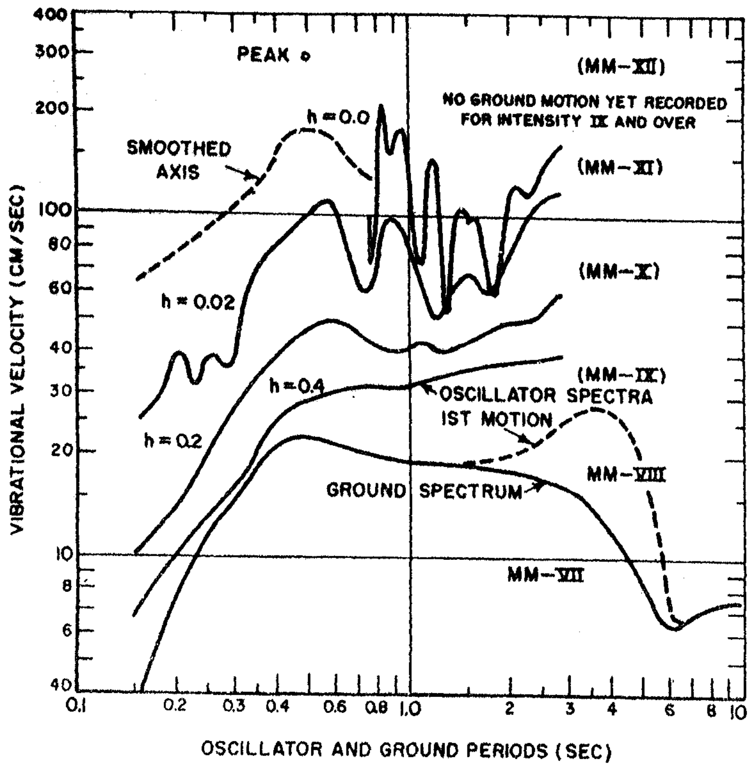


FIGURE 3. OSCILLATOR AND GROUND SPECTRA DETERMINED FROM EL CENTRO RECORD OF MAY 18, 1940 EARTHQUAKE.

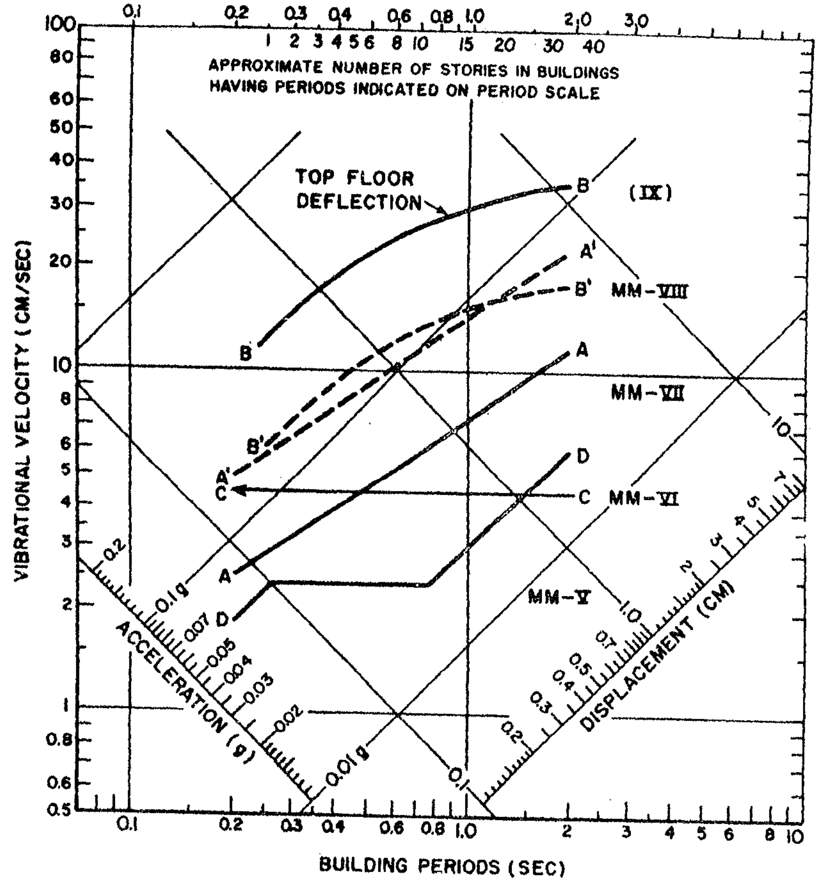


FIGURE 4. BUILDING CODE FORMULAE ON 4-WAY LOG CHART.

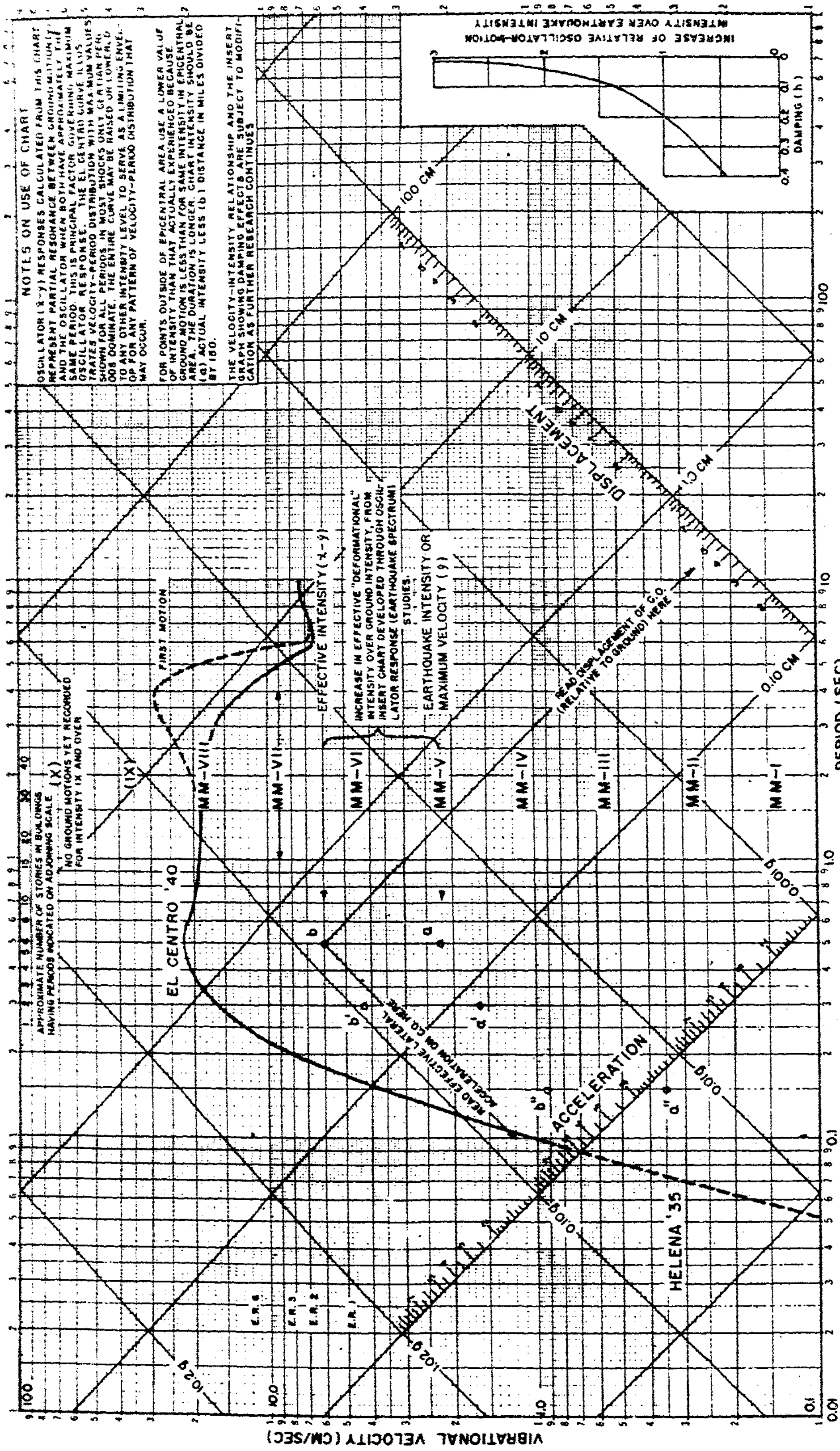


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CHART FOR DETERMINING EARTHQUAKE ACCELERATIONS AND DISPLACEMENTS IMPRESSED ON OSCILLATORS

FIGURE 5