DYNAMIC WATER PRESSURE ON DAMS DURING EARTHQUAKES

By
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SYNOPSIS

Theoretical solution is deduced on the dynamic water pressure on arch dams during earthquakes, by which the dynamic water pressure is calculated for various cases of central angle of arch, upstream radius of dams and intersection angle subtended by both banks, and furthermore, the result is testified by model experiment. In the next place, theoretical solution of dynamic water pressure caused by irregular earthquakes is evaluated for the case of gravity dam and its application to arch dams is suggested. The results show that the dynamic water pressure obtained by the author is much different from the usual value which has been hitherto adopted in the design of dams.

INTRODUCTION

In order to study the earthquake-proof properties of a dam, we must evaluate first the external forces upon dams during earthquakes, bring to light the vibration characteristics and investigate the distribution of dynamic stresses occurring in the dam body. The external forces on the dam in full reservoir are inertial force of dam body and dynamic water pressure. Among them the former is looked upon as proportioned to the earthquake acceleration and acting upon the dam by the phase equivalent to the earthquake acceleration. As to the latter, there is already Westergaard's theory(1), which is not only faulty mathematically but also inapplicable in case when the period of earthquake is shorter than the resonance period of dynamic water pressure. Accordingly, in later period, Zen-nosuke Anzo, Werner & Sundquist(2), Tadashi Hatano(3) etc. studied them to amend these deficiencies as well as to obtain right solution. As a result, theories on the dynamic water pressure have progressed and the phenomena have been greatly made known.

Since all of these theories assume the earthquake as stationary, simple harmonic motion, however, the dynamic water pressure becomes infinitely large when the earthquake period corresponds with the resonance period of dynamic water pressure. Despite the fact, in the design planned hitherto, the dynamic water pressure that does not resonate is adopted. On the other hand, for the dynamic water pressure acting upon such a curved surface as an arch dam, we must have a three-dimensional solution of dynamic water pressure. Whereas in the former design, usual equations based on a simple assumtion were used, depending upon Westergaard's solution of two-

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dimensional dynamic water pressure.

Much of the characteristics of dynamic water pressure was unknown before the author conducted the theoretical and experimental researches on those ambiguous points, giving an outlined report on the results thereby in this paper.

DYNAMIC WATER PRESSURE ON ARCH DAMS

The ground features of dam sites are generally too complicated to obtain any precise theoretical solution of dynamic water pressure on them. In this paper, therefore, the author will introduce a theory of dynamic water pressure on constant radius arch dams built in U-shaped canyons. Here is firstly taken up such circumstances in which the sections are symmetrical with both of the banks expanding straightly and infinitely, and the slopes of the both banks are so steep that the sections may be taken for as almost rectengular.

1. In case the both banks expand radially (Fig. 1-a)

The differential equation of dynamic water pressure will be expressed by the following equation.

$$\frac{\partial \sigma}{\partial r^2}+\frac{\partial \sigma}{r\partial r}+\frac{\partial^2 \sigma}{r^2\partial \theta^2}+\frac{\partial^2 \sigma}{\partial z^2}-\frac{w_0\,\partial^2 \sigma}{g E_v \partial t^2}=0 \quad \cdots\cdots\cdots\cdots(1)$$

Assuming the tremor as $(\alpha g/\omega^2)\sin \omega t$, the boundary condition will become as follows for the vibration in the direction of river course,

$$\left.\begin{array}{ll}(1) & \left(\dfrac{\partial \sigma}{\partial r}\right)_{r=r_e}=\alpha w_0 \cos\theta \sin\omega t \\ (2) & \left(\dfrac{\partial \sigma}{r\partial \theta}\right)_{\theta=0}=0,\ \left(\dfrac{\partial \sigma}{r\partial \theta}\right)_{\theta=\theta_a}=-\alpha w_0 \sin\theta_a \sin\omega t \\ (3) & \left(\dfrac{\partial \sigma}{\partial z}\right)_{z=0}=0,\ \left(\dfrac{\partial \sigma}{\partial z}+\dfrac{\partial^2 \sigma}{g\partial t^2}\right)_{z=h}=0\end{array}\right] \quad \cdots\cdots\cdots(2)$$

where, α is seismic coefficient, g acceleration of gravity, w_0 specific weight of water, E_v volmetric modulus of water, r, θ, z cylindrical coordinate, $2\theta_a$ central angle of arch and h height of dam.

The solution of equation (1) that satisfies boundary conditions (2) will be as follows,

$$\sigma=\sum_{n=0}^{\infty}\frac{8\alpha w_0 \sin\theta_a\, \varepsilon_n (-1)^n \sinh \lambda_0' h \cos\mu\theta \cosh\lambda_0' z}{\theta_a j_0' (\sinh 2\lambda_0' h+2\lambda_0' h)(1-\mu^2)(A_\mu^2+B_\mu^2)}$$

$$\times\left[\{A_\mu J_\mu(j_0' r)+B_\mu Y_\mu(j_0' r)\}\sin\omega t-\{A_\mu Y_\mu(j_0' r)-B_\mu J_\mu(j_0' r)\}\cos\omega t\right]$$

$$+\sum_{m=0}^{S-1}\sum_{n=0}^{\infty}\frac{8\alpha w_0 \sin\theta_a\, \varepsilon_n (-1)^n \sin\lambda_m h \cos\mu\theta \cos\lambda_m z}{\theta_a j_m' (\sin 2\lambda_m h+2\lambda_m h)(1-\mu^2)(A_{m\mu}^2+B_{m\mu}^2)}$$

$$\times\left[\{A_{m\mu}J_\mu(j'_m r)+B_{m\mu}Y_\mu(j'_m r)\}\sin\omega t-\{A_{m\mu}Y_\mu(j'_m r)-B_{m\mu}J_\mu(j'_m r)\}\cos\omega t\right]$$

$$-\sum_{m=s}^{\infty}\sum_{n=0}^{\infty}\frac{8\alpha w_0\sin\theta_a\,\varepsilon_n(-1)^n\sin\lambda_m h\,K_\mu(j_m r)\cos\mu\theta\cos\lambda_m z\sin\omega t}{\theta_a j_m(\sin 2\lambda_m h+2\lambda_m h)(1-\mu^2)\{K_{\mu-1}(j_m r_e)+K_{\mu+1}(j_m r_e)\}}$$

$$-\sum_{m=s}^{\infty}\frac{4\alpha w_0\sin\theta_a\sin\lambda_m h\{e^{-j_m r\sin(\theta_a-\theta)}+e^{-j_m r\sin(\theta_a+\theta)}\}\cos\lambda_m z\sin\omega t}{j_m(\sin 2\lambda_m h+2\lambda_m h)(1-\cos 2\theta_a e^{-j_m r_e\sin 2\theta_a})}$$

$$-\sum_{m=s}^{\infty}\sum_{n=0}^{\infty}\frac{8\alpha w_0\sin\theta_a\,\varepsilon_n I_{mn}\sin\lambda_m h\,K_\mu(j_m r)\cos\mu\theta\cos\lambda_m z\sin\omega t}{\theta_a j_m(\sin 2\lambda_m h+2\lambda_m h)(1-\cos 2\theta_a e^{-j_m r_e\sin 2\theta_a})\{K_{\mu-1}(j_m r_e)+K_{\mu+1}(j_m r_e)\}} \quad\ldots\ldots\ldots\ldots(3)$$

where

$\varepsilon_0=1,\quad \varepsilon_n=2$ (in case $n\neq 0$)

$A_\mu=J_{\mu-1}(j'_0 r_e)-J_{\mu+1}(j'_0 r_e)$, $B_\mu=Y_{\mu-1}(j'_0 r_e)-Y_{\mu+1}(j'_0 r_e)$

$A_{m\mu}=J_{\mu-1}(j'_m r_e)-J_{\mu+1}(j'_m r_e)$, $B_{m\mu}=Y_{\mu-1}(j'_m r_e)-Y_{\mu+1}(j'_m r_e)$

$j'_0=\sqrt{\lambda_0^2+C^2}$

$j'_m=\sqrt{C^2-\lambda_m^2}$ in case $C^2>\lambda_m^2$

$j_m=\sqrt{\lambda_m^2-C^2}$ in case $C^2<\lambda_m^2$

$C^2=\dfrac{w_0\,\omega^2}{g\,E_v}$

$\mu=\dfrac{n\pi}{\theta_a}$ ($n=0,\ 1,\ 2,\cdots$)

$$I_{mn}=-\int_0^{\theta_a}\{\sin(\theta_a-\theta)e^{-j_m r_e\sin(\theta_a-\theta)}+\sin(\theta_a+\theta)e^{-j_m r_e\sin(\theta_a+\theta)}\}\cos\mu\theta\,d\theta$$

and λ_0 is the root of $\coth\lambda' h=g\lambda'/\omega^2$, λ_m that of $\cot\lambda h=-g\lambda/\omega^2$

The solution will be obtained in the same way for vibration in the direction at right angle to the river.

The resonance period of dynamic water pressure is when $j_m=0$, similar to that of the two-dimensional dynamic water pressure, i. e.,

$$T_m=\frac{2\pi}{\lambda_m}\sqrt{\frac{w_0}{gE_v}}\quad\ldots\ldots\ldots\ldots(4)$$

2. In case the both banks are not in radial direction (Fig. 1-b)

Even in case that the intersection angle $2\theta_a'$ of both banks is smaller than the central angle $2\theta_a$ of arch, an approximate solution was obtained, firstly by evaluating solutions decomposing boundary conditions and ultimately by superposing these solutions, but any statement on the result will be omitted here.

3. Result of computation

Fig. 2 shows the result of evaluation of the vertical distribution of dynamic water pressure for the two points of crown and abutment made on the dam of $h = 100$ m, $r_e = 50$ m, $2\theta_a = 90°$ with both banks in radial direction, changing the earthquake period variously. As plain from the figure, the vertical distribution of dynamic water pressure is closely resembled to that of two-dimensional dynamic water pressure in the vibration in the direction of river course, while in the vibration in the direction at right angle to the river, it is considerably smaller than the usual value assumed from the two-dimensional dynamic water pressure.

Fig. 3 shows the horizontal distribution of dynamic water pressure at the bottom of dam with respect to various cases of upstream radius of dam, central angle of arch and intersection angle subtended by both banks. As plain from the figure, the dynamic water pressure by the author is strikingly different from the usual value. Namely, for the vibration in the direction of river course,

(1) Within the limit of real dam, the dynamic water pressure shows minimum value at crown, increasing toward abutment, independently of the intersection angle of both banks or the central angle of arch.

(2) When the width of river is extremely wide compared with the height of dam, the dynamic water pressure approaches nearer to the usual value.

(3) There is such a case where the dynamic water pressure becomes maximum in the half way between where the both banks stand in radial direction and where they are in parallel, but it seems there is no great difference in the value.

For the vibration in the direction at right angle to the river,

(1) The dynamic water pressure is much smaller than the usual value even in the case of both banks in radial direction, and becomes negative with the decrease of intersection angle of both banks. That the dynamic water pressure is negative means that it acts in the direction canceling the inertial force of dam body. It is therefore presumed that the dam bears considerable stability against the earthquake in this direction.

(2) In case that the width of river is extremely wide compared with the height of dam, the dynamic water pressure approaches nearer to the usual value.

(3) The magnitude of dynamic water pressure changes complicatedly according to the difference in the intersection angle

of both banks, the central angle and upstream radius of arch, but it is very small also in case when the both banks are in radial direction, which is most disadvantageous for dam.

In case the section of canyons are rectangular, the value of theoretical calculation has been confirmed to correspond to that of model experiment.

4. Effect of the section of canyon

When the section of canyon is a trapezoid or a triangle having a gentle slope of both banks, there is some difficulty in theoretical treatment. According to the results of model experiments for constant angle arch dam, however, it has been revealed that, in this case also, the dynamic water pressure is not very different from that of rectangular section and that the horizontal distribution of dynamic water pressure approaches nearer to uniform distribution in the vibration in the direction of river course, but extremely small in the vibration in the direction at right angle to the river. In the latter case, therefore, it seems unnecessary to take the trouble to calculate the stresses of arch dam.

DYNAMIC WATER PRESSURE DUE TO IRREGULAR EARTHQUAKE

Since the actual earthquake is the superposition of suddenly occurring vibrations with irregular period and amplitude, it is impossible to evaluate strictly the actual value of dynamic water pressure unless it is solved as an transient phenomenon of ever-changing dynamic water pressure. The propagation velocity of sound in water is approximately 1400 m/s, and as an earthquake keeps vibrating for a few seconds at least, reflected wave must be treated when the length of a reservoir is short. Since the form of an actual reservoir is not simple, however, this reflected wave will also dissipate in the course of propagation, and it will damp considerably before it reaches again to the dam site reflecting on the upstream boundary. In order to simplify the treatment of problems for these reasons, it was assumed here that the reservoir extended infinitely towards upstream direction, leaving out the treatment of the reflected wave.

1. Two-dimensional dynamic water pressure

As shown in Fig. 1-c, assuming x, z as rectangular coordinates, the dynamic water pressure due to irregular earthquake acceleration $\alpha g \psi(t)$ will be obtained by solving the differential equation (5) using the boundary conditions (6).

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}-\frac{w_0}{gE_v}\frac{\partial^2\phi}{\partial t^2}=0 \qquad \text{.....................(5)}$$

$$\left.\begin{array}{ll}(1) & \left(\dfrac{\partial\phi}{\partial z}\right)_{z=0}=0, \quad \dfrac{w_0}{g}\left(\dfrac{\partial\phi}{\partial t}\right)_{z=h}=0 \\ (2) & \left(\dfrac{\partial\phi}{\partial x}\right)_{x=0}=\alpha g\displaystyle\int_0^t\psi(\tau)d\tau\end{array}\right| \qquad \text{.....................(6)}$$

(3) $(\phi)_{t=0}=0\,, \quad \frac{w_0}{g}\left(\frac{\partial\phi}{\partial t}\right)_{t=0}=0$

$$\sigma=\frac{w_0}{g}\left(\frac{\partial\phi}{\partial t}\right) \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(7)$$

This solution will be obtained as follows,

$$\sigma=\sum_{m=0}^{\infty}\frac{4\alpha w_0(-1)^m v\cos\lambda_m z}{(2m+1)\pi}\int_{\frac{x}{v}}^{t}\psi(t-\tau)J_0\left(\lambda_m\sqrt{v^2\tau^2-x^2}\right)d\tau \qquad \cdots\cdots(8)$$

$$\left(t>\frac{x}{v}\right)$$

where,

$$\lambda_m=\frac{(2m+1)\pi}{2h}\,,\quad v=\sqrt{\frac{gE_v}{w_0}}=\text{velocity of sound in water}$$

The resonance period will be obtained by the same equation as (4).

Considering now the case where the earthquake suddenly occurs with $(\alpha g/\omega^2)\cos\omega t$ and separating the dynamic water pressure into stationary term and transient term, the following equation will be obtained.

$$\sigma=-\sum_{m=0}^{S-1}\frac{4\alpha w_0(-1)^m\cos\lambda_m z}{(2m+1)\pi\sqrt{C^2-\lambda_m^2}}\sin\left(\omega t-x\sqrt{C^2-\lambda_m^2}\right)$$
$$-\sum_{m=S}^{\infty}\frac{4\alpha w_0(-1)^m\cos\lambda_m z}{(2m+1)\pi\sqrt{\lambda_m^2-C^2}}\,e^{-x\sqrt{\lambda_m^2-C^2}}\cos\omega t$$
$$+\sum_{m=0}^{\infty}\frac{4\alpha w_0(-1)^m v\cos\lambda_m z}{(2m+1)\pi}\int_{t}^{\infty}\cos\omega(t-\tau)J_0\left(\lambda_m\sqrt{v^2\tau^2-x^2}\right)d\tau \quad \cdot\cdot(9)$$

The first and the second term of equation (9) are the stationary terms and the third term transient one. When the period of tremor is longer than any resonance period of dynamic water pressure, the first term disappears, becoming the dynamic water pressure of the same phase with the tremor. When the period of tremor is shorter than some of the resonance periods of dynamic water pressure, the first term also remains, giving rise to such dynamic water pressure delayed from the phase of tremor by 90°.

Fig. 4 is an example of earthquake records. In Fig. 5 is given the result of calculation made on the dynamic water pressure caused by such earthquake using equation (8). The uppermost row is the value when the dynamic water pressure by Westergaard's theory is assumed to be generated in proportion to the earthquake acceleration by the minute by the phase equivalent to the earthquake acceleration, where the maximum value was assumed as 1. From Fig. 4 the following fact is seen.

(1) The dynamic water pressure at a given time is not always proportioned to the corresponding earthquake acceleration, but different in time of resonance and in magnitude according to the relation between the resonance period of dynamic water pressure and the earthquake period.

(2) In either case of height of h = 100 m and 75 m, the maximum value of dynamic water pressure is considerably larger than that based on the Westergaard's equation.

(3) When the earthquake period is longer than the resonance period of dynamic water pressure, the phase of the dynamic water pressure is equivalent to that of earthquake acceleration, whereas when the earthquake period is shorter than the resonance period of dynamic water pressure, the dynamic water pressure is delayed from that of earthquake acceleration by 90°.

(4) Even after the suspension of tremor, the dynamic water pressure is not reduced to zero all of a sudden, but gradually diminishes, acting upon the dam with its eigen period for some time.

(5) In the whole dynamic water pressure, greater part of them is occupied by the pressure of primary mode of vibration.

(6) According to the former theory, it was impossible to explain the phenomena when the earthquake period has agreed to the resonance period of dynamic water pressure, but the author's theory enables to calculate the magnitude of it even in such a case.

2. Effect of the section of canyon

As it is very difficult to try theoretical solution for a given section, the solution was obtained only for the case of fan-shaped section or narrow triangular section, as shown in Fig. 1-d~e. The results show that the magnitude and the primary resonance period of dynamic water pressure become smaller as the section of canyon approaches nearer to the triangular form from rectangular one, and that the value in case of triangular section becomes 70 % of that in case of rectangular section, but that there are no great difference in other points from the rectangular section.

3. Application to the section of general form

From these results, the dynamic water pressure for the given section of canyon will be expressed approximately by the following equation,

$$\sigma = \alpha w_0 h \sqrt{1-\frac{z}{h}}\,\frac{4v}{\pi h}\int_0^t \psi(t-\tau)\, J_0\!\left(\frac{2\pi\tau}{T_1}\right) d\tau \qquad \text{........(10)}$$

where, T_1 is the primary resonance period of given section and will be expressed approximately by the following formula,

$$T_1 = T\sqrt{\frac{h_m}{h}} \qquad \text{.........................(11)}$$

in which, h is the maximum depth of water of given section; h_m mean depth of water of given section, T two-dimensional primary resonance period for the maximum depth.

4. Magnifying power of dynamic water pressure

In designing a dam, Westergaard's equation has been used, which was formulated on the assumption that the earthquake is a simple harmonic motion and the earthquake period T is larger than the resonance period T_1 of dynamic water pressure. According to the latest earthquake survey, however, since the earthquake eminent period at dam site is close to the resonance period of dynamic water pressure, the value of the dynamic water pressure caused then becomes considerably larger than that of Westergaard's equation. Fig. 6 shows the result of calculation on the maximum value of the dynamic water pressure caused when dams of various height have suffered the earthquake acceleration shown in Fig. 4 using equation (10), where the dynamic water pressure by Westergaard's equation was assumed as 1. As it is plain from the figure, for a specially high or low dam, the dynamic water pressure sometimes gives smaller value than that of Westergaard's equation, but for a dam about 100 m, the magnifying power of dynamic water pressure is as large as approximately 1.6.

5. Application to arch dam

For an arch dam it is almost impossible to explain theoretically the magnitude of dynamic water pressure caused by irregular earthquake. Accordingly, if the result obtained by the stationary vibration theory is adopted as it is for vertical and horizontal distribution of the dynamic water pressure upon arch dams and if equation (10) is adopted for its magnitude, the result from Fig. 6 can be applied to arch dam in that condition. It is, however, expected that if the intersection angles subtended by both banks increase, time function will be different, and we have to depend upon future studies for further details.

CONCLUSION

Summing up the above, we can draw conclusions on dynamic water pressure acting upon dams as follows.

1. For vibration in the direction of river course

The dynamic water pressure acting upon a gravity dam or an arch dam will be given by the following equation,

$$\sigma = \alpha w_0 h \sqrt{1-\frac{z}{h}} \cdot k \quad \ldots\ldots (12)$$

where, k is the magnifying power of dynamic water pressure, which will be expressed by the following equation,

$$k=\frac{4v}{\pi h}\int_0^t \psi(t-\tau)J_0\left(\frac{2\pi\tau}{T_1}\right)d\tau \qquad \text{.................(13)}$$

The magnitude of k is considered to take the value of about 1.5, though it will be clarified when numerous records of earthquake acceleration at dam site have been obtained.

When the earthquake period is shorter than the resonance period of dynamic water pressure, the dynamic water pressure is delayed from the phase of inertial force of dam body by $90°$, and the dynamic water pressure caused by the elastic displacement of dam is delayed by $90°$ from the phase of the elastic displacement, acting upon the dam as a kind of damping forces. In such a case, it seems that dams are comparatively safe. Accordingly, we have only to deal with the case where the earthquake period is longer than the resonance period of dynamic water pressure.

2. For vibration in the direction at right angle to the river

The dynamic water pressure acting upon an arch dam is extremely small even in the case where the both banks extend radially and becomes negative pressure when the intersection angle of both banks is small. Since the damping force due to viscosity of water in full reservoir is large, it is not necessary to calculate the stresses of the arch dam for the earthquake in this direction.

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NOMENCLATURE

E_v = Volmetric modulus of water

Q = Coefficient of dynamic water pressure

T = Period of earthquake

T_m = Resonance period of dynamic water pressure

g = Acceleration of gravity

α = Seismic coefficient

w_0 = Specific weight of water

h = Height of dam or depth of water

h_m = Mean depth of water

r, θ, z = Cylindrical coordinate

r_e = Upstream radius of arch

θ_a = One-half of arch central angle

t = Time

τ = Time parameter in integration

v = Velocity of sound in water

x, z = Rectangular coordinate

ω = Angular velocity of earthquake

σ = Dynamic water pressure

ϕ = Velocity potential

ρ, θ, x = Cylindrical coordinate

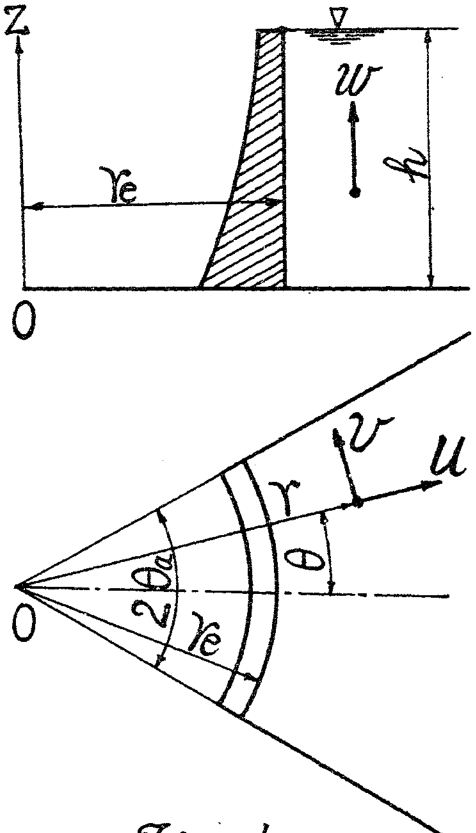


Fig. 1-a

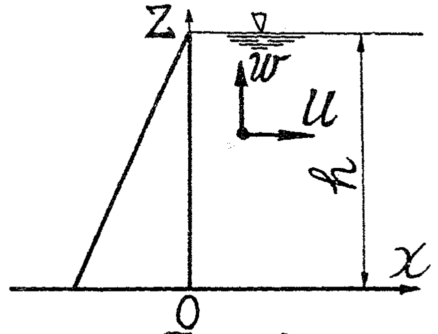


Fig. 1-c

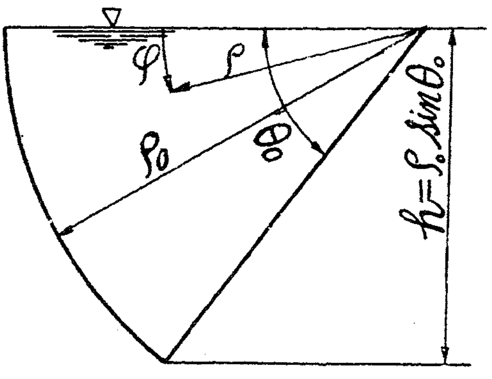


Fig. 1-d

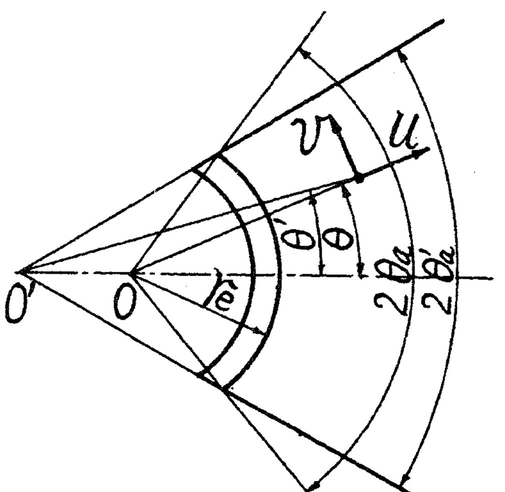


Fig. 1-b

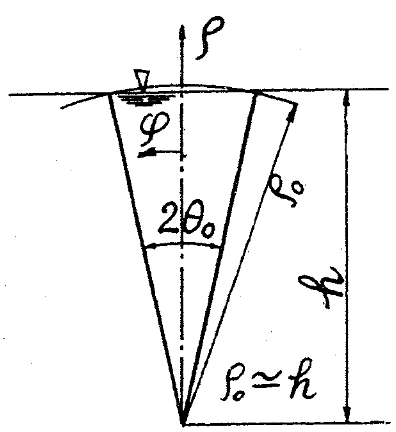
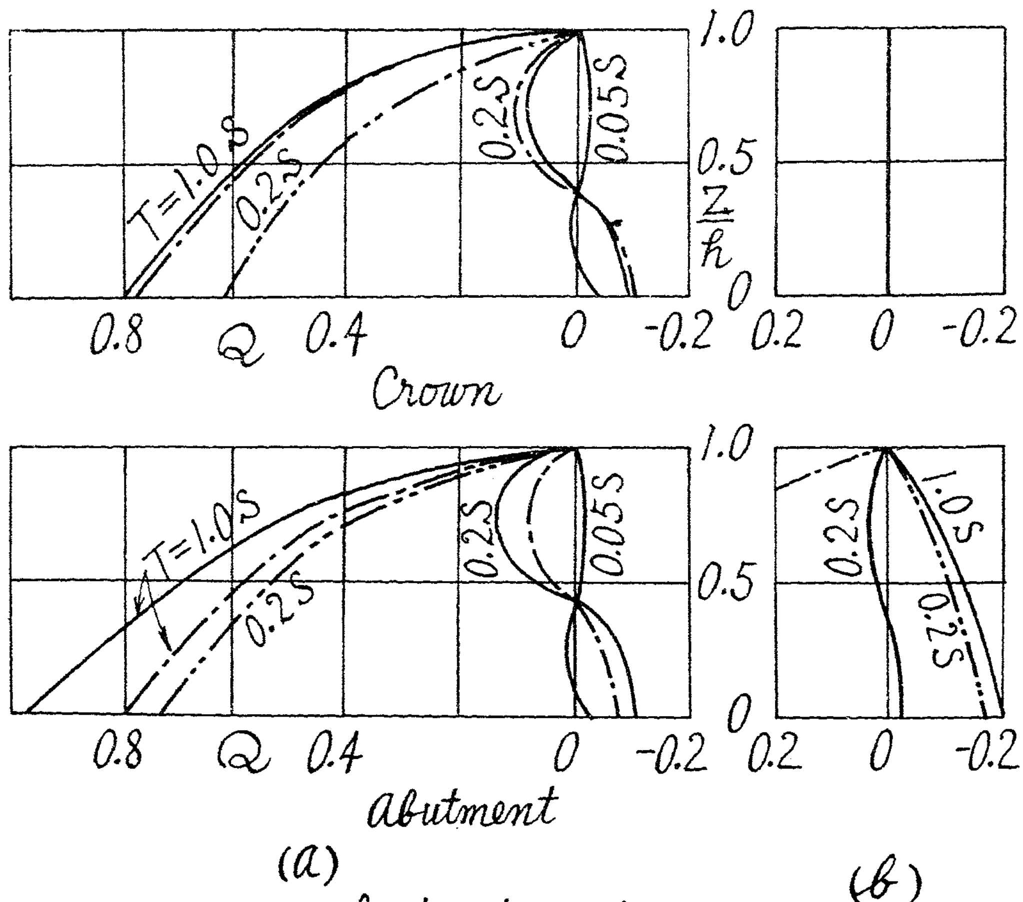


Fig. 1-e



(a) Vibration in the direction of river course
(b) Vibration in the direction at right angle to river

$\sigma = \alpha w_0 h Q$ ($h = 100m$, $r_e = 50m$, $2\theta_a = 90°$)

——— Dynamic water pressure by the author's theory
– – – Dynamic water pressure delaying by 90° from earthquake acceleration
—·— Dynamic water pressure by Westergaard's equation
------ Dynamic water pressure due to vibration of only dam

Fig. 2 Vertical Distribution of Dynamic Water Pressure

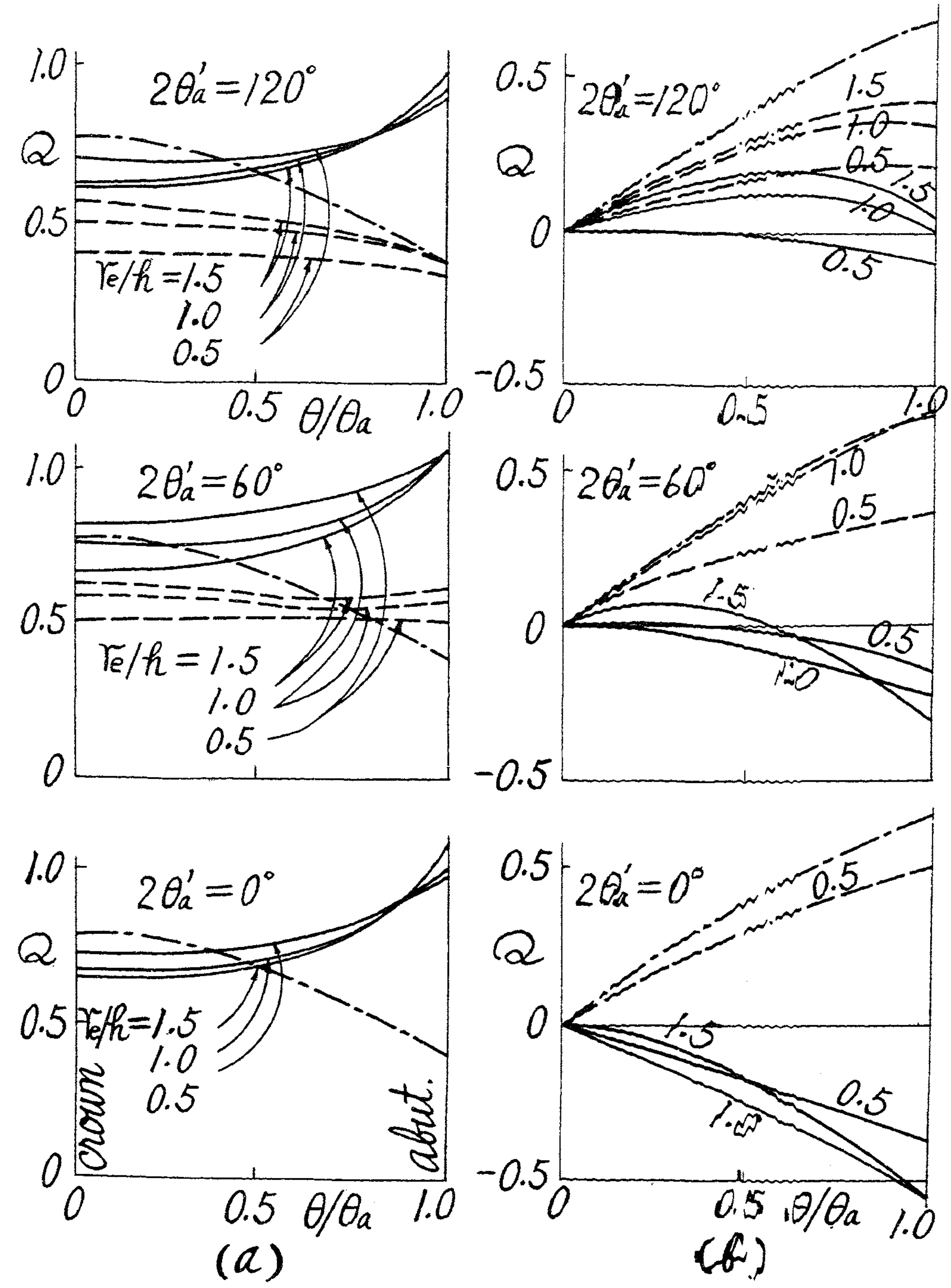
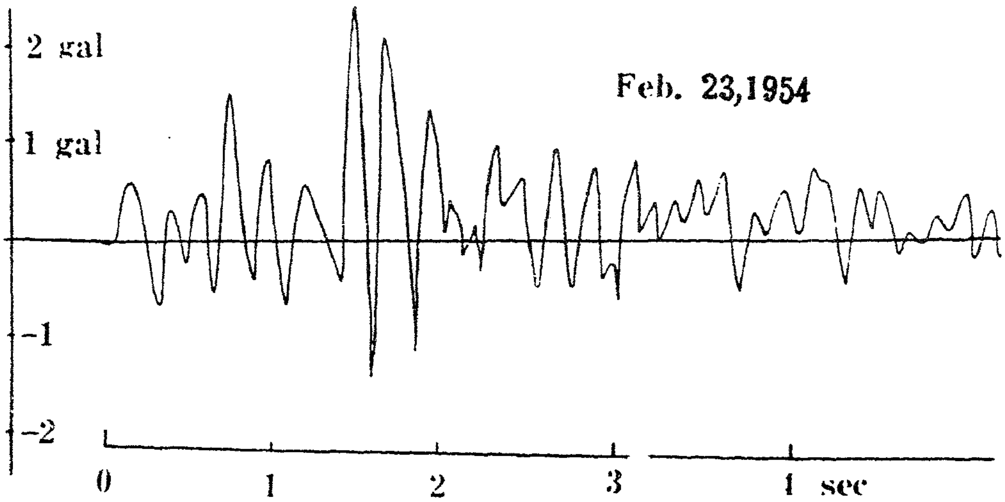


Fig. 3 Horizontal Distribution of Dynamic Water Pressure (h=100m, $2\theta_a$=120°, T=1.0S)



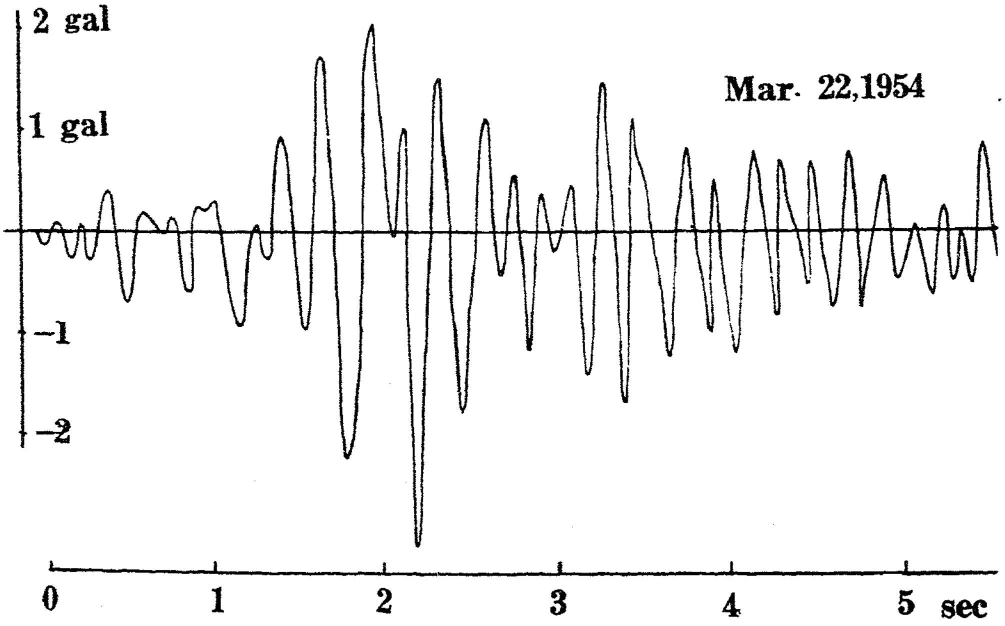


Fig. 4 Earthquake Records

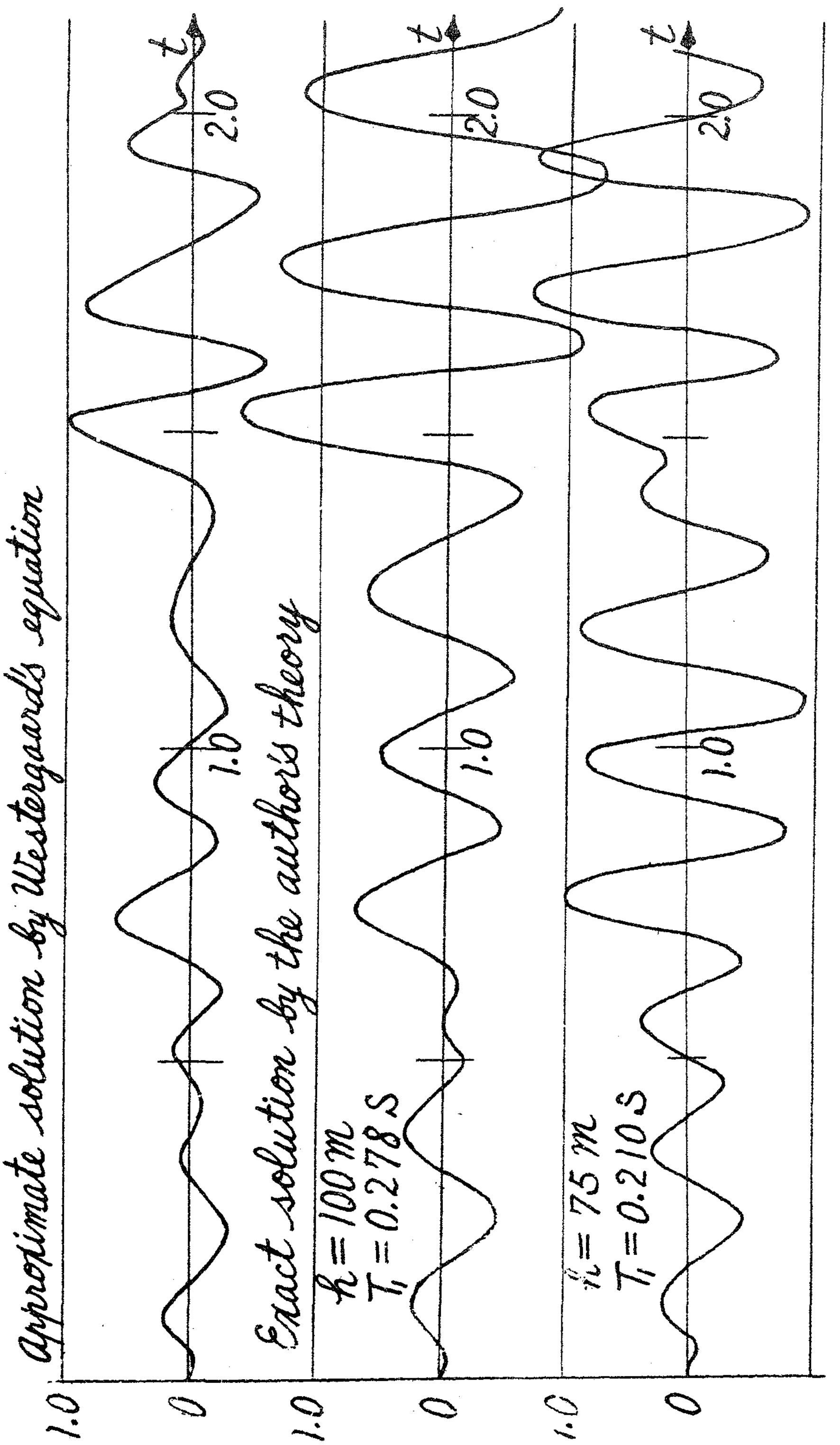


Fig. 5 Dynamic Water Pressure due to Irregular Earthquake (Feb. 23, 1954)

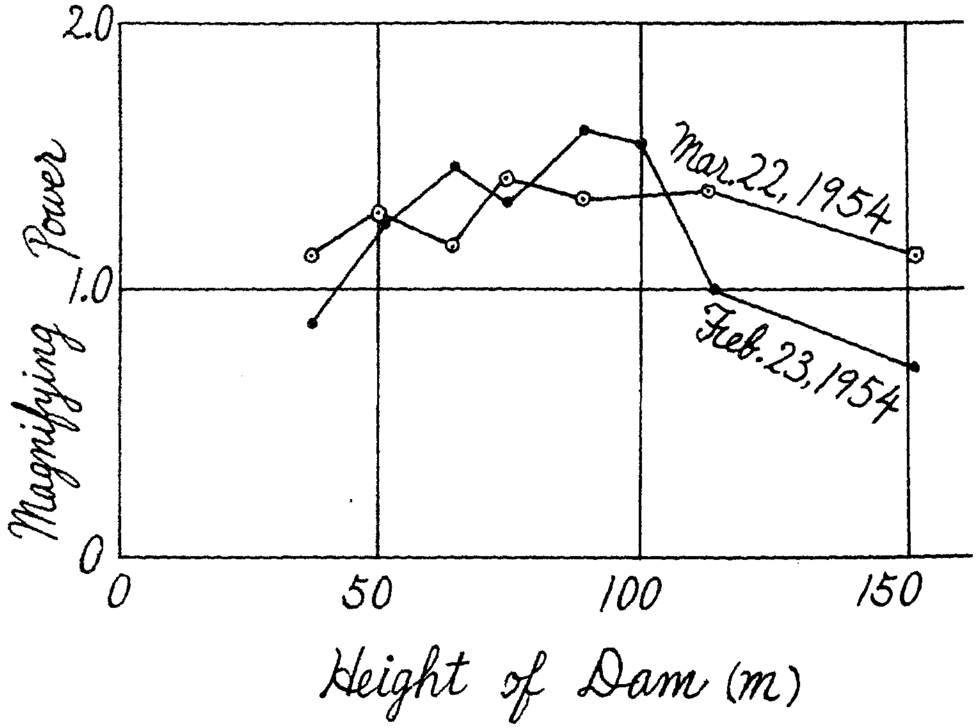


Fig. 6 Magnifying Power of Dynamic Water Pressure