ELASTIC RESPONSE OF MULTI-STORY SHEAR BEAM TYPE STRUCTURES SUBJECTED TO STRONG GROUND MOTION

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1. ABSTRACT

The elastic response of a multiple degree of freedom system vibrating as a base excited shear-beam is analyzed by superposition of normal modes. Absolute viscous damping is included in the theoretical treatment, both for modal analysis and for direct numerical integration of the governing equations of motion. The two methods of analysis are used to compute the undamped response of three multi-story structures to 12 recorded earthquakes. The results of the two methods are compared and conclusions drawn about the use of modal analysis to compute the maximum response of structures to time-varying ground displacements.

2. INTRODUCTION

The total effect of an earthquake on a structure with an infinite number of degrees of freedom is complex. To obtain a useful solution it is necessary to make simplifying assumptions about the structure and the energy input to the structure.

If the ground motion is essentially horizontal, and the structure is of rigid frame construction it is not unreasonable to treat the system as a compound oscillator with rigid masses connected by shear springs. With these assumptions the response of the structure can be analyzed mathematically.

Other authors have made modal analyses of undamped structures treating them as cantilever rods vibrating either in flexure[1,2], or in shear. The question as to whether the structure will vibrate in shear, flexure or a combination of both is not easy to predict, although some relative measurement of this is determined by the rigidity of the structure[3].

It is the intent of this paper to develop the modal analysis of damped structures vibrating in shear modes only. Absolute viscous damping is considered. The modal analysis of a two degree of freedom shear beam with intermass (relative) damping has been developed independently of this investigation.[4] The results are generally of the same nature as those presented herein.

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3. THE MECHANICAL ANALOGY

The system to be analyzed is shown schematically in Fig. 1. To treat a structure as a "shear beam", it is assumed that:

(a) The distributed mass of the structure is concentrated at equidistant floor levels. This is M_i.

(b) The masses (floor slabs) are infinitely rigid and do not rotate during deformation.

(c) The entire shear stiffness at any column level is concentrated in one linear elastic shear spring with stiffness K_i.

(d) The structure has linear absolute viscous damping measured with reference to the absolute velocity--not the velocity relative to the ground. (See Fig. 1)

(e) There is no foundation rotation.

With these assumptions the complete motion of the i^{th} mass of the structure is governed by one second order linear ordinary differential equation:

$$M_i \ddot{x}_i + K_i (x_i - x_{i-1}) - K_{i+1} (x_{i+1} - x_i) + \delta_i (\dot{x}_i) = 0 \;\ldots\ldots \;(1)$$

with the initial conditions that the velocity and displacement of each mass are zero at the beginning of the excitation. The dots indicate time derivatives, while the subscripts designate the mass or spring considered (See Fig. 1). Note that the acceleration, velocity and displacement of each mass are measured with respect to a fixed datum and not the base of the structure.

The magnitude of the absolute viscous damping for each mass depends on the viscous constant δ_i. By definition δ_i is chosen to be

$$\delta_i = \beta\, \delta_{cr} = 2\beta\, M_{av}\, \omega_1 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots \;(2)$$

where ω_1 is the circular frequency of the first mode.

This arbitrary choice of the viscous constant assumes that the fundamental mode dominates in the general vibration.

There is one equation similar to equation (1) for each of the n masses. In effect there are n equations in (n+1) unknowns, the absolute displacements x_i. The $(n+1)^{ST}$ unknown is the absolute ground displacement, x_o, supplied as a boundary forcing function. The base acceleration, $\ddot{x}_o$, is integrated twice to obtain x_o thus reducing the problem to the solution of n simultaneous differential equations in n unknowns.

Equation (1) may be solved by numerical integration or modal analysis. The modal analysis is now developed.

4. GENERAL COMMENTS ON MODAL ANALYSIS

The theory of modal analysis is based on the assumption that modes exist for the structure being treated. A mode is a harmonic, or damped harmonic, vibration in which all parts of the system oscillate in phase or in anti-phase, reaching their maximum displacements simultaneously and maintaining a fixed ratio between displacements during each cycle of vibration. For an undamped system of n masses, consideration of conservation of energy shows that modes do exist, and that there are n different types of modes each having, in general, a different natural frequency. These are known as principal modes.

The undamped structure can be forced to vibrate in any one of its n principal modes by proper choice of initial displacements. The system will continue to oscillate in this one mode indefinitely until external excitation modifies the vibration. This characteristic of the undamped structure is caused by the fact that undamped modes are not coupled. Again from conservation of energy it may be shown that these uncoupled modes may be superposed.

However for a damped system, with certain exceptions discussed below, the damped modes will be coupled, and free vibration cannot take place in any one mode shape. The general damped mode will decay exponentially[5] and phase differences will occur between the masses when vibrating in that "mode". This phase difference in the mode itself makes a general modal analysis impossible. However, if the damping coefficients for each mass of each damped mode are properly chosen, the mode may be forced to decay in such a fashion that all masses remain in phase[6]. Then a modal analysis is possible.

Thus we see that if we "uncouple" the damped modes by a selective choice of the damping coefficients a modal analysis is permissible. First the undamped modal analysis is developed, and then this method is extended to develop the necessary relations for a damped modal analysis.

5. UNDAMPED MODAL ANALYSIS

Any deflection of an undamped elastic system may be resolved into a series of components related to the principal modes by modal participation constants. Thus we assume the displacement of the i^{th} mass relative to the base of the structure to be:

$$u_i = x_i - x_o = \sum_{j \text{ modes}} c_j \, f_{i,j} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where c_j is the timewise participation of the j^{th} mode and f_{ij} is the normalized modal displacement of the i^{th} mass in the j^{th} mode. The value of c_j is easily determined from the orthogonality of the principal modes and the theorem of virtual work. Since the inertia forces perform the work in a free vibration, the work done by the maximum inertia forces of the k^{th} mode when passing through the displacements u_i is

$$W_k = (M_1\omega_k^2 f_{1k})(u_1) + (M_2\omega_k^2 f_{2k})u_2 + \dots = \omega_k^2 \sum_{i \text{ masses}} M_i f_{ik} u_i \quad \dots(4)$$

But since u_i is given by equation (3), the work of the inertia forces in the k^{th} mode can also be expressed in terms of c_j, and ω_k, the circular frequency of vibration of the k^{th} mode.

$$W_k = \omega_k^2 \sum_j \left\{ c_j \sum_i M_i f_{ik} f_{ij} \right\} \quad \dots\dots\dots\dots\dots (5)$$

But from the orthogonality condition for the principal modes

$$\sum_i M_i f_{ik} f_{ij} = 0 \quad \text{if } j \neq k \quad \dots\dots\dots\dots\dots (6)$$

we find that only the k^{th} mode does any work, i.e.

$$W_k = \omega_k^2 c_k \sum_i M_i f_{ik}^2 \quad \dots\dots\dots\dots\dots (7)$$

Equating the work done from equations (4) and (7)

$$c_k = \frac{\sum_i M_i f_{ik} u_i}{\sum_i M_i f_{ik}^2} \quad \dots\dots\dots\dots\dots (8)$$

No apparent advantage has been obtained by these transformations, since c_k depends on u_i itself. Thus the modes still remain essentially coupled. We now observe that the coupling between modes was caused by the definition of u_i as given by equation (3). A more judicious assumption for u_i will indeed uncouple the modes if we introduce a modified value of c_k. The modified value of c_k is suggested by equation (8).

If we assume that

$$u_i = x_i - x_o = \sum_j c_j f_{ij} \varphi_j \quad \dots\dots\dots\dots\dots (9)$$

where the k^{th} value of c_j is given as

$$c_k = \frac{\sum_i M_i f_{ik}(\text{constant})}{\sum_i M_i f_{ik}^2} \quad \dots\dots\dots\dots\dots (10)$$

it will be shown that φ_j is a "normal" coordinate that uncouples the modes. For simplicity we choose the constant in equation (10) to be unity.

Since a forced vibration may occur such that *only the* k^{th} *mode is* excited, φ_k may be determined by substituting

$$(u_i)_k = c_k \, f_{ik} \varphi_k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

into equation (1) and setting $\delta_i = 0$. Then equation (1) is multiplied by the constant f_{ik}, and evaluated for each of the n masses of the system. If these n equations are then added, the forced vibration in the k^{th} mode is governed by the relation

$$\ddot{x}_o \sum_i M_i \, f_{ik} + c_k \, \ddot{\varphi}_k \sum_i M_i \, f_{ik}^2 + c_k \varphi_k \, [\ldots\ldots] = 0 \ldots\ldots\ldots \quad (12)$$

where the bracketed term is a long expression involving all the spring stiffnesses and the normalized mode shapes. Replacing c_k by its value from equation (10) reduces equation (12) to the form

$$\ddot{\varphi}_k + \frac{[\ldots\ldots\ldots]}{\sum_i M_i \, f_{ik}^2} \varphi_k = - \ddot{x}_o \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

It can be demonstrated by direct evaluation that the coefficient of φ_k in equation (13) is the square of the circular natural frequency of the k^{th} mode. Thus we have

$$\ddot{\varphi}_k + \omega_k^2 \varphi_k = - \ddot{x}_o \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

Equation (14) is simply the forced vibration equation for an undamped single degree of freedom system subjected to the base disturbance $\ddot{x}_o$. Thus by a proper choice of c_k (equation 10), the modes have been uncoupled, and the timewise response of each mode may now be computed by solving equation (14).

6. DAMPED MODAL ANALYSIS

As was mentioned in section 4, it is possible for damped modes to exist if the proper modal damping coefficients are chosen. These coefficients must also be a function of the mass to which the damping applies in order to keep the masses in phase in the mode. Thus the technique used is to assume that damped modes exist and solve for the coefficients necessary to keep the modal displacement ratios constant.

As in section 5 it is assumed that

$$u_i = x_i - x_o = \sum_j c_j \, f_{ij} \, \varphi_j \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

where c_j and f_{ij} have the same definitions as previously chosen for the undamped modes, but φ_j is now a damped function of time.

Repeating the technique of section (5), the equilibrium relation for the k^{th} mode reduces to:

$$\ddot{\varphi}_k + \dot{\varphi}_k \frac{\sum_i \delta_{ik} f_{ik}^2}{\sum_i M_i f_{ik}^2} + \omega_k^2 \varphi_k = -\ddot{x}_o - \dot{x}_o \frac{\sum_i \delta_{ik} f_{ik}}{\sum_i M_i f_{ik}} \quad \ldots\ldots\ldots (16)$$

Also it may be shown that a single degree of freedom with undamped frequency ω_k and a viscous damping coefficient δ_k when subjected to the base disturbance $\ddot{x}_o$, satisfies the relation

$$\ddot{\varphi}_k + \frac{\delta_k}{M} \dot{\varphi}_k + \omega_k^2 \varphi_k = -\ddot{x}_o - \frac{\delta_k}{M} \dot{x}_o \quad \ldots\ldots\ldots\ldots\ldots (17)$$

Equation (16) reduces to the form of equation (17) if δ_{ik} is assumed to be a function of M_i. Thus the selection is made that

$$\delta_{ij} = 2\beta_j M_i \omega_j \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Substitution of δ_{ij} from equation (18) into equation (16), reduces equation (16) to:

$$\ddot{\varphi}_k + 2\beta_k \omega_k \dot{\varphi}_k + \omega_k^2 \varphi_k = -\ddot{x}_o - 2\beta_k \omega_k \dot{x}_o \quad \ldots\ldots\ldots\ldots\ldots (19)$$

Comparing equation (19) to equation (17) it is observed that φ_k is the response of a damped oscillator to the disturbance $\ddot{x}_o$ if

$$\delta_k = 2\beta_k M\omega_k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Since equation (20) satisfies equation (18), the modal damping as defined in equation (18) is correct as long as φ is interpreted as the response of a damped oscillator. The only restriction on the solution is that β_j be small for all modes. This restriction is necessary due to the fact that for large amounts of damping, the damping forces may be larger than the inertia and spring forces and the modal vibration will become aperiodic. If this happens the modal analysis no longer is valid.

7. NUMERICAL INTEGRATION

An alternate solution to equation (1) may be obtained by simultaneous numerical integration of the equations of motion for all the masses for specific ground motions. This integration has been performed on the University of Illinois digital computer, ILLIAC, using the iterative method developed by Newmark[7]. A time interval of seven milli-seconds was used in the integration thus assuring stability and providing rapid convergence of the numerical procedure for all structures analyzed. The ground motion records (see section 8) were integrated by the Trapezoidal Rule using a seven milli-second time interval, or a partial time interval where necessary to include all peaks of the accelerogram. The results of these integrations are summarized in Reference 8.

8. MODAL ANALYSIS COMPARED TO NUMERICAL INTEGRATION

The two methods were compared by direct application to specific cases. Three undamped multi-story structures were subjected to earthquake ground motion and the response computed by each method.

The shear beam type structures treated were a four, eight, and sixteen story building characterized by a linear variation of story mass and column stiffness. They are shown symbolically in Fig. 2.

The input consisted of 12 U.S.C.G.S. earthquake acceleration records reduced to digital form compatible with standard ILLIAC input routines. The base lines of these records were shifted slightly to make the residual ground velocity zero at the end of the record. Useful information regarding the integration of these records is presented in Table 1. In some cases the maximum ground displacement reached unreasonably large values, and the usefulness of the records was questioned. However additional minor adjustments reduces these maximum displacements considerably while still insuring that the terminal velocity is zero. Recent studies at the University of Illinois have shown that these minor adjustments have a relatively small effect on the response of the structures considered herein. The responses have been computed for those ground motion records which were not adjusted to reduce the maximum ground displacement.

RESULTS OF MODAL ANALYSIS

The modal analysis technique is useful as an approximate procedure in predicting upper bounds on the maximum response of a structure, but holds no particular advantage over the numerical integration technique if an exact answer is desired. Thus it is desirable to use the modal analysis procedure only to compute the maximum modal responses rather than a complete time history of response, and superpose the modal maxima to obtain an upper bound on the true response. The maximum modal responses are proportional to the maximum value of φ for the particular excitation considered. For the recorded earthquakes these displacement maxima are available in the form of published spectra. Thus to predict the response of a particular structure to an earthquake of given intensity, the investigator need only compute the shear modes and frequencies (as an approximation, these may be computed by numerical techniques), extract the corresponding values of φ from published displacement spectra and superpose as many modes as deemed necessary.

The above procedure was followed in this study, but the modes and frequencies, and the spectral response φ were computed exactly for each mode by special computer programs.

The modal responses were superposed in two ways:

(a) The modes were all assumed to reach their maximum values simultaneously. Thus the absolute values of all modal shears were summed to provide an absolute maximum upper bound on the response.

(b) The most probable response for a uniform structure is given as the square root of the sum of the squares of the modal maxima. This value is also computed even though the structures tested are not uniform.

In Tables 2, 3, 4, and 5 typical results of this investigation are presented. For a more complete summary of data, the reader is referred to Reference 8. Table 2 presents results for the four story building. Table 3 presents the results for the eight story building, and Tables 4 and 5 present the results for the 16 story building. The column labeled "exact" presents results of numerical integration.

The results show that in all cases, the true answer computed by direct numerical integration is less than the upper bound. These data also indicate that for the short building, the maximum shear agrees best with the absolute maximum of the modal shears; for the intermediate height building the maximum shears range about in the middle between the root mean square value and the upper bound value; and for the tall building the maximum computed shears do not vary greatly from the root mean square value.

These results are quite reasonable and support the assumptions often made in earthquake resistant design that for short structures it is possible that all modes may act in phase, while for tall buildings subjected to random shock the fundamental modes dominate the response and thus it is reasonable to use the root mean square value.

9. RESULTS OF THE NUMERICAL INTEGRATION

Although results of the damped modal analysis are not available, the significant results of the numerical integration of equation (1) with absolute viscous damping included are presented.

This integration was carried out for all 12 earthquakes for the 3 structures previously mentioned. Absolute viscous damping was included, but the damping was assumed to be constant for all masses. The damping was defined to be:

$$\delta = \beta\delta_{cr} = 2\beta \, M_{av} \, \omega_1 \quad \text{.................... (21)}$$

where the damping coefficient, β, was chosen to be 0, 0.02, 0.10 and 0.15. M_{av} represents the average mass of the structure, while ω_1 is the fundamental circular frequency. This arbitrary definition of damping assumes the fundamental mode dominates the vibration. This assumption is not unrealistic since the higher modes damp out faster than the fundamental.

The important results are summarized here:

(a) For all values of damping, the shear distribution over the height of the structure is essentially parabolic. Figure 3 shows the distribution for the undamped response. This result agrees with earlier findings reported by Tung and Newmark[9].

(b) 2% damping reduces column shears to about 60% of their undamped maxima, while 10% damping reduces the response to about 30% of the undamped value. The results for 15% damping are not materially different from the 10% results.

(c) The maximum base shear does not appear to be generally a function of the maximum ground acceleration or the maximum ground velocity. Theoretically, the most probable value for the maximum base shear is given as:

$$V_{max} = \sqrt{\sum_j \left[(V_j)_{max}\right]^2} \quad \dots\dots\dots\dots\dots\dots (22)$$

where the maximum base shear in the j^{th} mode is

$$(V_j)_{max} = K_1 c_j f_{1j} \; (\varphi_j)_{max}$$

$$\therefore V_{max} = K_1 \sqrt{\sum_j \left[c_j f_{1j} \; (\varphi_j)_{max}\right]^2} \quad \dots\dots\dots\dots (23)$$

An average spectrum has been prepared by averaging all 12 earthquake spectra and is shown in Fig. 4. It is noted that it is a smooth curve in the range of frequencies tested. For these ranges of ω, the equation of the curve is roughly:

$$\frac{\varphi_j}{\varphi_1} = 1.2\, e^{-0.1\omega_j} \quad \dots\dots\dots\dots\dots\dots\dots\dots (24)$$

Thus equation (23) becomes

$$V_{max} = 1.2 K_1 \varphi_1 \sqrt{\sum_j \left[c_j^2 \; f_{1j}^2 \; e^{-0.2\omega_j}\right]} \quad \dots\dots\dots\dots (25)$$

All terms in equation (25) are constants for a particular structure except φ_1. Thus the maximum base shear is proportional to φ_1. For the undamped motion φ_1 is the solution of equation (14) which is known to be

$$\varphi_1(\tau) = -\frac{1}{\omega_1}\int_0^{\tau} \ddot{x}_o \sin \omega_1(\tau - t)dt \quad \text{.............} \quad (26)$$

The maximum values of φ_1 have been tabulated in Reference 8 for each of the three structures treated for all 12 earthquakes. It was found that in 17 cases φ_1 appeared to increase roughly in proportion to the increase of maximum ground acceleration, while in the 19 other cases φ_1, increased roughly proportional to the maximum ground velocity. Thus no general relation between φ_1, and earthquake intensity seems to exist, and so no specific correlation between maximum base shear and earthquake intensity was found.

(d) The problem of fatigue may have a very important bearing on the failure of multi-storied structures. For an eleven second earthquake (accelerogram 17) the undamped four story building had 39 reversals of shear in the base column. These intermediate maxima occurred approximately once for every one-fourth of a second of earthquake duration. Five times the intermediate maximum shear approached within 10 percent of the absolute maximum value. Fewer shear reversals occurred in the upper columns.

10. CONCLUDING REMARKS

It has been shown that shear beam type structures may be analyzed by modal analysis if restricted values of absolute viscous damping are assumed. Upper bounds and probable values for the maximum column shears may be computed with the assistance of published displacement spectra for recorded earthquakes. If the principal shear modes are computed by numerical techniques, the method enables solutions to be obtained for elaborate systems without setting up and solving the fundamental equations of motion.

11. BIBLIOGRAPHY

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12. NOMENCLATURE

c_j = Modal participation constant for the j^{th} mode.

f_{ij} = Normalized modal displacement of the i^{th} mass in the j^{th} mode.

i = Subscript labeling mass or displacement. $i = 1, 2, \ldots n$.

j = Subscript labeling mode. $j = 1, 2, \ldots, k, \ldots n$.

k = Specific value of j.

K_i = Shear stiffness of i^{th} spring.

M_i = i^{th} mass.

n = Number of degrees of freedom of system. This is also the number of masses and principal modes.

u_i = Displacement of i^{th} mass relative to base of structure.

x_i, $\dot{x}_i$, $\ddot{x}_i$ = Displacement, velocity and acceleration of the i^{th} mass relative to a datum fixed in space.

x_o, $\dot{x}_o$, $\ddot{x}_o$ = Displacement, velocity and acceleration of the ground (base of structure) relative to a fixed datum.

V_i = Shear in i^{th} column.

V_{ij} = Shear in i^{th} column in j^{th} mode.

W_k = Work done by the inertia forces of the k^{th} mode during a free vibration.

β = Coefficient defining percent of critical absolute viscous damping in the structure.

β_j = Coefficient defining percent of critical damping in the j^{th} mode.

δ_i = Viscous damping constant for the i^{th} mass.

δ_{ij} = Viscous damping constant for the i^{th} mass in the j^{th} mode.

τ = Time variable of integration.

φ_j = Timewise dynamic amplification of j^{th} mode.

ω_j = Undamped circular frequency (Rad./sec.) of j^{th} principal mode.

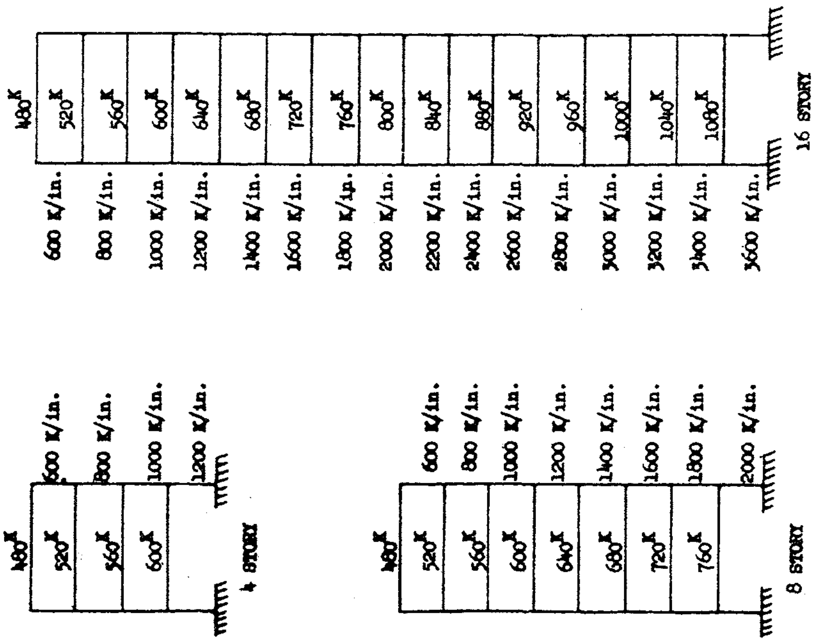

FIGURE 2

BUILDING MODELS USED IN ANALYSIS

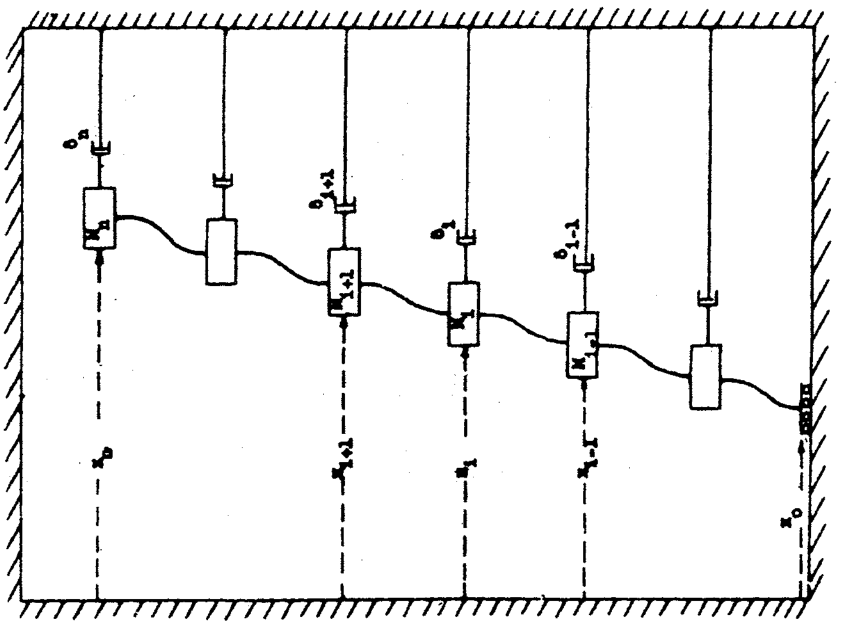

FIGURE 1

MECHANICAL ANALOGY APPROXIMATING AN ABSOLUTELY DAMPED SHEAR-BEAM

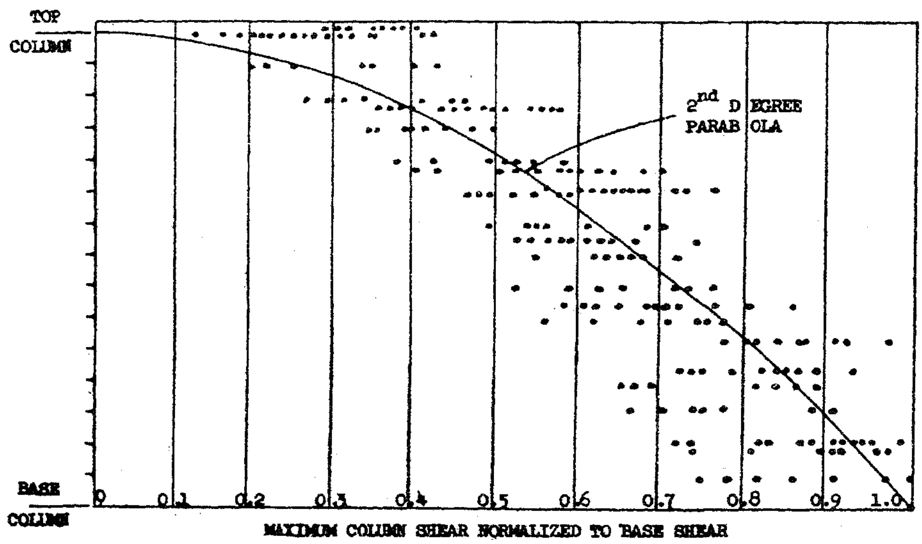

FIGURE 3

RELATIVE MAXIMUM SHEAR DISTRIBUTION OVER HEIGHT OF BUILDING

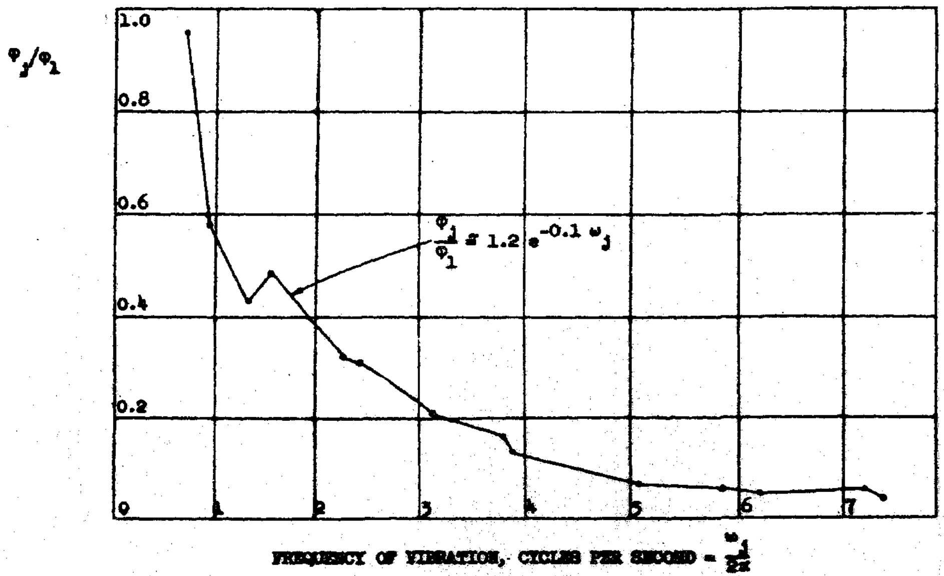

FIGURE 4

RESPONSE SPECTRUM - AVERAGE OF ALL ACCELEROGRAMS

NO.	LOCATION	DATE	BEARING	DURATION (SEC.)	MAX.ACCEL. (IN./SEC.²)	MAX.VEL. (IN./SEC.)	MAX.DISPL* (IN.)	M.MERCALLI INTENSITY
2	L.A. Subway Term.	Oct. 2, 1933	N 39° E	21.448	18.194	2.940	11.345	5.75
3	L.A. Subway Term.	Oct. 2, 1933	N 51° W	21.140	42.711	3.814	34.960	5.75
4	L.A. Subway Term.	Mar. 10,1933	N 39° E	26.859	12.725	5.426	31.900	6.50
5	L.A. Subway Term.	Mar. 10,1933	N 51° W	25.081	8.988	3.745	30.698	6.50
6	El Centro, Calif.	May 18, 1940	E-W	27.671	88.025	17.847	38.672	7.50
	El Centro, Calif.	May 18, 1940	N-S	29.386	123.798	17.438	175.837	7.50
9	El Centro, Calif.	Dec. 30,1934	N-S	25.291	95.004	10.595	78.424	6.00
10	El Centro, Calif.	Dec. 30,1934	E-W	25.326	67.124	13.410	76.470	6.00
15	Vernon, Calif.	Mar. 10,1933	E-W	42.469	75.158	9.488	16.365	7.50
16	Vernon, Calif.	Mar. 10,1933	N-S	40.047	52.240	9.826	78.783	7.50
17	Vernon, Calif.	Oct. 2, 1933	S 82° E	10.962	45.688	3.832	4.232	6.00
19	Vernon, Calif.	Oct. 2, 1933	N 08° E	9.478	34.825	2.140	1.009	6.00

*Uncorrected Values

TABLE 1

CHARACTERISTICS OF ACCELEROGRAMS

	EXACT RESPONSE	Σ MODAL RESPONSES	$\sqrt{\Sigma(\text{MODAL RESPONSES})^2}$
ACCELEROGRAM 2			
4th Floor	134.92	148.10	101.93
3rd Floor	218.65	233.34	183.19
3nd Floor	267.90	285.08	243.27
1st Floor	305.66	334.42	276.28
ACCELEROGRAM 7			
4th Floor	1417.39	1557.17	972.52
3rd Floor	2003.26	2167.14	1556.16
2nd Floor	2333.54	2410.69	2008.85
1st Floor	2912.58	3143.57	2348.94
ACCELEROGRAM 17			
4th Floor	331.17	368.39	239.22
3rd Floor	446.89	500.59	358.68
2nd Floor	518.69	541.97	454.07
1st Floor	662.53	734.10	542.72

TABLE 2

TYPICAL RESPONSES OF UNDAMPED 4 STORY BUILDING
MAXIMUM SHEAR GIVEN IN KIPS

	EXACT RESPONSE	Σ MODAL RESPONSES	$\sqrt{\Sigma(\text{MODAL RESPONSES})^2}$
ACCELEROGRAM 3			
8th Floor	127.43	157.99	85.34
7th Floor	195.89	232.79	146.39
6th Floor	228.35	279.44	195.20
5th Floor	283.34	327.82	240.64
4th Floor	312.82	344.52	280.56
3rd Floor	332.52	387.45	314.49
2nd Floor	381.76	436.39	344.45
1st Floor	445.15	519.64	365.23
ACCELEROGRAM 9			
8th Floor	703.32	885.89	458.95
7th Floor	955.65	1154.38	673.21
6th Floor	1109.03	1290.19	834.70
5th Floor	1349.70	1590.60	1057.89
4th Floor	1472.89	1706.68	1228.53
3rd Floor	1577.77	1780.77	1330.02
2nd Floor	1850.70	1948.20	1456.06
1st Floor	2159.88	2506.91	1590.99

TABLE 3

TYPICAL RESPONSES OF UNDAMPED 8 STORY BUILDING
MAXIMUM SHEAR GIVEN IN KIPS

ACCELEROGRAM 4	EXACT RESPONSE	Σ MODAL RESPONSES	$\sqrt{\Sigma(\text{MODAL RESPONSES})^2}$
16th Floor	140.29	244.17	109.97
15th Floor	260.71	388.72	207.23
14th Floor	357.86	509.03	301.36
13th Floor	424.71	610.27	391.42
12th Floor	512.26	702.21	480.01
11th Floor	605.53	791.97	565.91
10th Floor	713.30	892.36	646.85
9th Floor	800.95	944.30	724.68
8th Floor	876.02	996.07	798.43
7th Floor	908.31	1025.25	866.92
6th Floor	985.12	1131.46	928.03
5th Floor	1039.15	1206.27	984.18
4th Floor	1103.82	1278.92	1033.47
3rd Floor	1142.59	1358.76	1071.32
2nd Floor	1184.98	1439.13	1101.33
1st Floor	1217.23	1547.00	1117.77

TABLE 4

TYPICAL RESPONSES OF UNDAMPED 16 STORY BUILDING
MAXIMUM SHEAR GIVEN IN KIPS

ACCELEROGRAM 5	EXACT RESPONSE	Σ MODAL RESPONSES	$\sqrt{\Sigma(\text{MODAL RESPONSES})^2}$
16th Floor	126.70	183.37	128.01
15th Floor	223.91	309.23	179.55
14th Floor	300.25	408.41	262.16
13th Floor	372.72	487.07	343.33
12th Floor	454.04	561.67	425.27
11th Floor	528.01	650.92	506.22
10th Floor	599.27	750.07	586.87
9th Floor	688.16	814.23	662.44
8th Floor	780.69	882.72	731.25
7th Floor	841.85	928.97	795.48
6th Floor	872.35	994.13	849.03
5th Floor	904.28	1031.46	897.36
4th Floor	951.17	1083.25	939.37
3rd Floor	1002.72	1155.61	971.29
2nd Floor	1056.96	1235.37	998.93
1st Floor	1097.92	1305.93	1011.13

TABLE 5

TYPICAL RESPONSES OF UNDAMPED 16 STORY BUILDING
MAXIMUM SHEAR GIVEN IN KIPS