ENERGY CONSUMPTION BY STRUCTURES IN STRONG-MOTION EARTHQUAKES

by

G. V. Berg* and S. S. Thomaides**

SYNOPSIS

The effect of energy dissipation upon structural response to earthquake is considered. A spectral study of the energy relations for an elasto-plastic system responding to strong ground motion, made by high-speed digital computer, is described. The results indicate that yielding does not increase the total energy input to the system, and that maximum drift may not be adversely affected by yielding as long as the yield level is maintained above a reasonable threshold.

INTRODUCTION

Engineers have long recognized that energy dissipation is a key factor in explaining the observed behavior of structures in strong-motion earthquakes. The earthquake feeds energy into the structure; and to survive, the structure must, without undue damage, consume all the energy imparted to it. Part of the energy is stored temporarily in the structure as strain energy and kinetic energy. Ultimately all the energy must be dissipated by internal friction and inelastic deformation in both the structural and nonstructural elements.

Linear response spectra show that if structures responded elastically to strong-motion earthquakes, they would, even though heavily damped, be required to withstand forces far above those stipulated in seismic building codes.[1] The reason they are able to survive is that both the main frame and the nonstructural components remain active when strained far beyond their

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lastic limits, continue to offer resistance to motion, and thus consume the nergy of the earthquake.

If a structure were capable of storing as strain energy the maximum energy input from an earthquake, it would certainly survive regardless of its issipative properties. On the other hand if a structure were able to dissipate energy as fast as the earthquake delivered it, without showing severe amage, it would survive regardless of its strain-energy capacity. Neither f these conditions is reasonable as a basis for aseismic design; indeed, the atter is impossible to achieve. Instead, a reasonable balance must be sought o that the strain-energy capacity and dissipative capacity, working together, an accommodate the energy delivered by the earthquake.

Taking this view of the problem, one can immediately make an observation bout present building codes; and the significant observation is not what is resent in the codes but what is lacking. The strain-energy capacity appears n the codes in the form of seismic coefficients, but the dissipative characteristics of the structure are not reflected at all. One cannot say that nergy dissipation is ignored completely, for it appears tacitly in the form of reduced seismic coefficients. But many existing codes place the same strain-energy requirements on the structure whether it is damped to one percent or thirty percent of critical damping. Also, the ductility of the structure is not accounted for. The danger of brittle fracture is circumvented by detailed specifications, but existing codes give no reward for a large reserve of plastic strength and they levy no penalty for a lack of it. The traditional building of a decade or two ago, with its steel frame and masonry walls and partitions, had a fairly large energy-absorbing capacity. In contemporary architecture, light-weight metal panels and glass are more the rule than the exception. It is quite apparent that structures of today may differ widely in their dissipative properties, and it seems only logical that seismic codes should somehow take this into account. The question how best to do so may not be completely answerable in the present state of the art.

If one adheres to the assumption that energy is dissipated only by viscous damping, one can use response spectrum techniques[1,2] to appraise the effect of damping upon the response of the structure. For other than linear viscous damped systems, the effects of damping and energy dissipation are more difficult to assess. If only the velocity terms in the equations of motion are nonlinear, one can sometimes bring an equivalent viscous damping into the picture.[3,4] However, an equivalent viscous damping is not very meaningful when significant nonlinearities are present in the restoring terms.[4]

There does appear to be good reason to give attention to systems which are strongly nonlinear. A real structure subjected to an earthquake strong enough to challenge its integrity significantly will respond quite differently from the ideal linear oscillator. In the early part of the response, the nonstructural elements may provide the dominant part of the restoring force. As the amplitude increases, the nonstructural components yield or crack, throwing to the main frame the burden of providing the restoring forces. Still, the nonstructural components continue to resist motion and dissipate energy, acting perhaps somewhat in the nature of Coulomb frictional forces. Finally, if the amplitude gets great enough, even the main frame yields and draws on its reserve plastic strength.[5]

ENERGY STUDIES

Energy relations provide a common basis for appraising the dynamic response of systems of widely differing nonlinear characteristics. To explore some of the basic questions concerning energy components for structures responding to earthquakes, a research project was initiated at The University of Michigan about a year ago under the sponsorship of the U. S. National Science Foundation. The objective of the research was to study the energy relations for elastic and elasto-plastic systems subjected to earthquake forces to learn how the energy components are influenced by variations in the structural parameters. The elasto-plastic system was chosen for study because of its simplicity. To avoid an excess of detail, the studies were restricted to an ideal single-degree-of-freedom elasto-plastic system with velocity damping, as shown in Fig. 1. The same system also serves as the linear viscous damped system when the yield strength is set high enough. The problem lends itself well to a spectral type of analysis using a high-speed digital computer.

The restoring element in the system of Fig. 1 is an elasto-plastic spring obeying the relations

$$\dot{Q} = k\,\dot{x} \quad \text{if} \quad |Q| < Q_y \quad \text{or} \quad Q\,\dot{x} \leq 0$$

$$\dot{Q} = 0 \quad \text{if} \quad |Q| = Q_y \quad \text{and} \quad Q\,\dot{x} > 0$$

in which Q is the spring force,

Q_y is the yield strength of the spring, and

k is the spring constant in the elastic range.

These are the usual elasto-plastic relations, with hysteresis, as shown in Fig. 2. The equation of motion for the system is

$$m(\ddot{x} + \ddot{y}) + c\,\dot{x} + Q = 0$$

in which c is a viscous damping coefficient, and the ground acceleration y is a known function of time. To reduce this to a unit mass basis, let

$$q = Q/m \quad ,$$

$q_y \;=\; Q_y/m \;=$ yield level

$\omega \;=\; \sqrt{\frac{k}{m}} \;=$ undamped natural frequency in the elastic range, and

$\beta \;=\; \frac{c}{2m\omega} \;=$ fraction of critical damping in the elastic range.

The equation of motion then becomes

$$\ddot{x} + 2\beta\omega\dot{x} + q = -\ddot{y}(t)$$

where

$$\dot{q} = 0 \quad \text{if} \quad |q| = q_y \quad \text{and} \quad q\,\dot{x} > 0$$

$$\dot{q} = \omega^2\dot{x} \quad \text{if} \quad |q| < q_y \quad \text{or} \quad q\,\dot{x} \leq 0$$

The yield level q_y is the lateral acceleration which would just bring the restoring force to the yield strength.

The force per unit mass exerted on the structure by the foundation is $-(q + 2\beta\omega\dot{x})$, and the rate at which energy is being delivered to the system is

$$\dot{E} = -(q + 2\beta\omega\dot{x})\,\dot{y}$$

The force per unit mass exerted on the damper is $2\beta\omega\dot{x}$, and the rate at which energy is being dissipated by the damper is

$$\dot{L} = 2\beta\omega\dot{x}^2$$

The kinetic energy per unit mass is

$$T = \frac{1}{2}(\dot{x} + \dot{y})^2$$

The energy of deformation may be resolved into elastic strain energy, which is recovered when the system returns to the equilibrium position, and energy dissipated in plastic deformation. The recoverable strain energy per unit mass is

$$U = \frac{q^2}{2\omega^2} \quad ,$$

and the rate of energy dissipation in plastic deformation is

$$\dot{D} = q\dot{x} \quad \text{if} \quad |q| = q_y \quad \text{and} \quad q\dot{x} > 0$$

$$\dot{D} = 0 \quad \text{if} \quad |q| < q_y \quad \text{or} \quad q\dot{x} \leq 0$$

The equation of motion and the equations for E and L were rewritten for computer solution as the following system of first-order differential equations:

$$\dot{z} = \ddot{y}(t) \qquad (z = \text{ground velocity})$$

$$\dot{v} = -\left[\ddot{y}(t) + q + 2\beta\omega v\right] \qquad (v = \text{system velocity relative to ground})$$

$$\dot{x} = v$$

$$\dot{E} = -(q + 2\beta\omega v)\, z$$

$$\dot{L} = 2\beta\omega v^2$$

The equations were solved by a Runge-Kutta third-order procedure.[6]

The plastic energy dissipation, D, and the restoring force, q, were obtained directly from difference relations rather than through differential equations. The reason is that both $\dot{q}$ and $\dot{D}$ are discontinuous whenever the system enters the plastic range. In a difference formulation the discontinuities cause no difficulty. Let x_n, q_n, and D_n denote the values of x, q, and D at the beginning of a small interval of time; let x_{n+1}, q_{n+1}, and D_{n+1} be the values at the end of the interval; and let Δx, Δq, and ΔD be the changes during the interval. Assuming that $\dot{x}$ does not change sign during the interval, the equations for $\dot{q}$ and $\dot{D}$ lead to the following relations:

If $(q_n + \omega^2 \Delta x) \geq q_y$, then $q_{n+1} = q_y$ and $\Delta D = q_{n+1}\left(\Delta x - \dfrac{\Delta q}{\omega^2}\right)$

or If $(q_n + \omega^2 \Delta x) \leq -q_y$, then $q_{n+1} = -q_y$ and $\Delta D = q_{n+1}\left(\Delta x - \dfrac{\Delta q}{\omega^2}\right)$

or If $|q_n + \omega^2 \Delta x| < q_y$, then $q_{n+1} = q_n + \omega^2 \Delta x$ and $\Delta D = 0$

The assumption of monotonic $\dot{x}$ during the interval requires the interval to be small compared with the natural period of the system. This is required in any event, to insure accuracy in the numerical solution of the differential equations, so the use of a difference formulation for Δq and ΔD imposes no additional restrictions.

In solving the system of equations, the computer evaluates all the variables at small intervals of time and records the maximum magnitude of each variable.

The input functions $\ddot{y}(t)$ consisted of punched card accelerograms[7] of the two horizontal components of four strong-motion earthquakes:

El Centro, California	December 30, 1934
El Centro, California	May 18, 1940
Olympia, Washington	April 13, 1949
Taft, California	July 21, 1952

The functions $\ddot{y}(t)$ were piecewise linear approximations of the accelerograms. Because a strong-motion accelerogram is very nearly a broken line diagram, especially during the strongest part of the ground motion, the error in the piecewise linear approximation is probably not large compared with the error in the accelerogram itself. The time-acceleration coordinates of the intersection points of successive line segments were put on punched cards. An arbitrary zero axis was chosen for the accelerogram, and the coordinates were scaled from the record using an arbitrary scale of measurement. The resulting data were then processed by machine to introduce appropriate scale factors for time and acceleration. The initial ground velocity, initial ground displacement, and the adjusted location of the zero axis were chosen to minimize the mean square displacement; i.e., to make

$$\int_0^L y^2 dt$$

a minimum, where y is the ground displacement found by integration, and L is the length of the punched card accelerogram.

This method of preparing the input data turned out to be quite satisfactory. A punched card accelerogram is compact, consisting of about 100 to 150 cards, each containing the coordinates of four points. Identification numbers and sequence numbers on the cards were checked by the machine during the computation to guarantee that no cards were missing or out of sequence.

RESULTS

The first computational phase of the project was the evaluation of linear response spectra. This was considered to be essential because it provided a basis for comparing the digitally computed data with previously published results obtained by electrical analog techniques.[1] The digital response spectra compared quite favorably with the analog spectra, differences being on the order of ten or fifteen percent. Also, the ground-velocity and ground-displacement curves obtained by integrating the punched card accelerograms were found to be in reasonable agreement with previously published results. This confirmed that the digital accelerograms are reasonably faithful representations of the records from which they were taken.

Elasto-plastic analyses have thus far been made for only the N-S component of the El Centro, California, earthquake of May 18, 1940, and the N69°W component of the Taft, California, earthquake of July 21, 1952. Analyses of other components are planned, but have not yet been accomplished. The most significant results are the energy input curves and the maximum displacement curves shown in the figures.

Figures 3 and 4 show spectral plots of energy input per unit mass versus period for damping ranging from zero to twenty percent of critical and for yield levels ranging from fully elastic (infinite yield level) down to one percent of gravity.

The following observations are pertinent:

The elastic undamped energy curve is a very jagged curve for which small changes in period may be accompanied by large changes in total energy input. The presence of dissipative forces in the system, either in the form of viscous damping or inelastic deformation, has a marked smoothing effect on the energy curves.

In the elastic case an increase in the fraction of critical damping does not cause any decided change in the average value of the energy curve. However, at low values of yield level an increase in damping is accompanied by an increase in total energy input.

On the average a decrease in yield level is accompanied by a <u>decrease</u> in total energy input at all levels of damping. An exception to this occurs at the low end of the period scale. In this region, for periods of about 0.4 second or less, the energy curves drop off as the period approaches zero; and the curves for high yield level drop more rapidly than the curves for the low yield level.

Figures 5 and 6 show maximum displacement plotted against natural period for various fractions of critical damping and various yield levels. The significant observation here is that a decrease in yield level is <u>not</u> accompanied by an increase in maximum displacement until the yield level gets below .06g. Even with the yield level as low as .03g, the maximum displacement is not severely affected. However, when the yield level drops to .01g, there is a marked increase in the maximum displacement.

ACCURACY

In this, as in any problem involving the response of a nonlinear dynamic system, one might justifiably question whether inaccuracies have crept into the analysis to destroy the validity of the results, or whether the problem is so ill-conditioned that small differences in the input data might become greatly magnified to produce large differences in the response.

The errors involved in the digital computation are small. Preliminary checks, using elementary input functions for which exact results were known, disclosed accuracy to four or more significant decimal digits. Each time an earthquake analysis was run, the energy balance, i.e., the difference between the energy input and the sum of the energy components, was recorded at the end of the solution. The balance should, of course, be zero. Its deviation from zero served as a check on the errors generated in the solution. It was found to be possible to shrink this energy balance nearly to zero simply by decreasing the size of the time interval used in the numerical integration process. This is not altogether desirable because it increases the time required to obtain the solution. For the most part the computations were controlled to keep this energy balance below five percent of the maximum energy input. In one or two cases it ran as high as seven percent, which was considered to be well within the range of tolerable error.

Concerning the second point, the nonlinearities of the elasto-plastic system could conceivably cause some magnification of the input errors. If this magnification were extreme, it would destroy the worth of the results. Intuitively, one feels that the nearly random nature of the input would preclude any extreme magnification. As a rough check, several solutions were run twice, first, using the accelerogram without modification, and then using the accelerogram with all acceleration readings magnified ten percent. This magnification in acceleration would cause a 21% increase in the energy input to a linear system. For the elasto-plastic system the increase in energy input ranged from 17% to 22%. As a second check, several cases were run with the accelerogram modified so that the odd-numbered ordinates were increased .01g and the even-numbered ordinates were decreased .01g. The effect of this was to superimpose a small noise on the earthquake signal. The change in output caused by this disturbance was almost immeasurably small. In view of these results, it seems that the problem is not ill-conditioned.

CONCLUSIONS

It would be presumptuous to draw any broad conclusions or design recommendations from the data presented in this paper. The elasto-plastic analyses were made for only two earthquake components. The authors will hazard the opinion that other components will produce results agreeing with these in the over-all picture, although differing in the details. But this is yet unconfirmed. Also, one must remember that the ideal elasto-plastic system, while it is a convenient mathematical model and a step closer to reality than the linear viscous-damped system, still does not describe the behavior of a real structure very accurately.

Despite these limitations, the results suggest the possibility of employing a design procedure that would take into account both the elastic and inelastic properties of a structure. This is essentially the same as a method first proposed by Housner[8] and the foregoing results lend strength to Housner's views. The design philosophy is that a structure should be able to withstand an earthquake of moderate intensity with no more than superficial damage, and should also be able to survive a very intense earthquake without catastrophic damage. This dual objective can be attained by requiring that the structure remain in the elastic range in its response to a moderate earthquake, and also that it be able to consume the energy of a very intense earthquake by calling on its reserve strength beyond the yield deformation.

For either the moderate or intense earthquake, the amount of energy input involved can be estimated by spectrum techniques. Because yielding tends to reduce the total energy input to the system, one can get a conservative estimate of the energy input from linear response spectrum curves or through the use of spectrum intensity concepts, rather than resorting to the more complex elasto-plastic analyses described above. Linear response spectrum curves and spectrum intensities have been published for most of the strong-motion earthquakes of record.[1,8]

A simple structure might be designed as follows:

1. Select velocity-response-spectrum values S_{vm} to represent the moderate earthquake, and S_{vs} to represent the strong earthquake. Because the S_v curves tend to be essentially horizontal lines, the natural period of the structure would not be critical.

2. Design the structure elastically to withstand the moderate earthquake without exceeding the design stress, i.e., so that

$$\frac{1}{2} Q_d X_d \geq \frac{W}{2g} S_{vm}^2$$

where

- W = effective weight of structure,
- S_{vm} = velocity-response-spectrum value for moderate earthquake, taken at a fraction of critical damping appropriate to the type of structure,
- Q_d = restoring force at design stress, and
- X_d = lateral displacement at design stress.

3. To avoid excessive drift in the strong earthquake, impose the requirement that

$$Q_y \geq .05W$$

where

Q_y = the restoring force at yield stress.

4. Check the design plastically for the intense earthquake, according to the criterion

$$Q_y X_y + \frac{1}{2} Q_d X_d \geq \frac{W}{2g} S_{vs}^2$$

where

S_{vs} = the velocity response spectrum value for the strong earthquake, taken at the same fraction of critical damping as for S_{vm}, and
X_y = permissible total yield displacement.

The procedure as stated here could be applied only to simple structures that have essentially one degree of freedom. Even so, there are questions that call for further study. The parameter X_y, for example, is the permissible total yield displacement, i.e., the sum of the magnitudes of all displacements in the plastic range, in both directions. Yield could ordinarily be expected to occur in both directions. Present knowledge of structural materials is for the most part restricted to one-directional loading and unloading, or to oscillatory loading at low stresses. Relatively few tests have been carried far beyond the yield displacement, and almost none has explored reversed loading cycles after exceeding the yield displacement. Attempts to assign numerical values to X_y would necessarily involve the use of unproved assumptions.

The extension of these design concepts to multi-story structures is another topic for further investigation. On the basis of preliminary studies, the authors believe that the energy-input curves for elasto-plastic multistory structures will fall below the corresponding curves for elastic structures, much the same as in the single-degree-of-freedom systems. It then appears that a similar dual-purpose design might be appropriate for this case as well. The elastic portion of the design could be executed for a multistory structure with no difficulty, but the limit design portion is less clear. One cannot readily predict where inelastic deformation will occur. It might occur in many locations in the structure, or it might be concentrated at one or two locations. In exceptional circumstances it might even be desirable to control the location of yielding by purposely building a weak link in the structure—a structural fuse, so to speak. But this appears to have little merit as a general practice.

Much additional research needs to be done before these concepts can be applied effectively to multi-story or complex structures. The analyses presented herein are only a short step down a long road.

ACKNOWLEDGMENT

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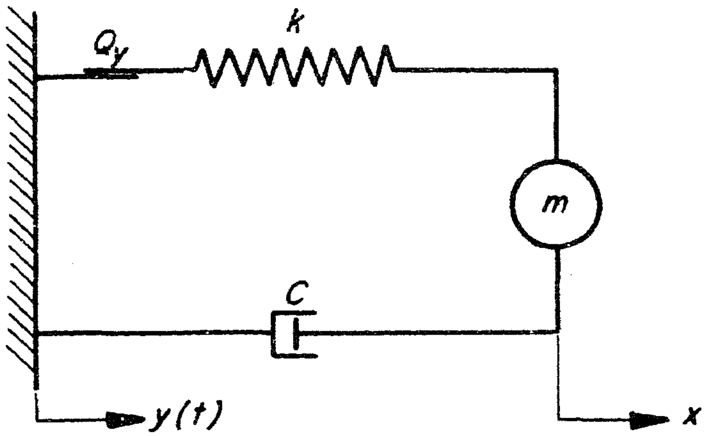

Fig. 1. Elasto-plastic system.

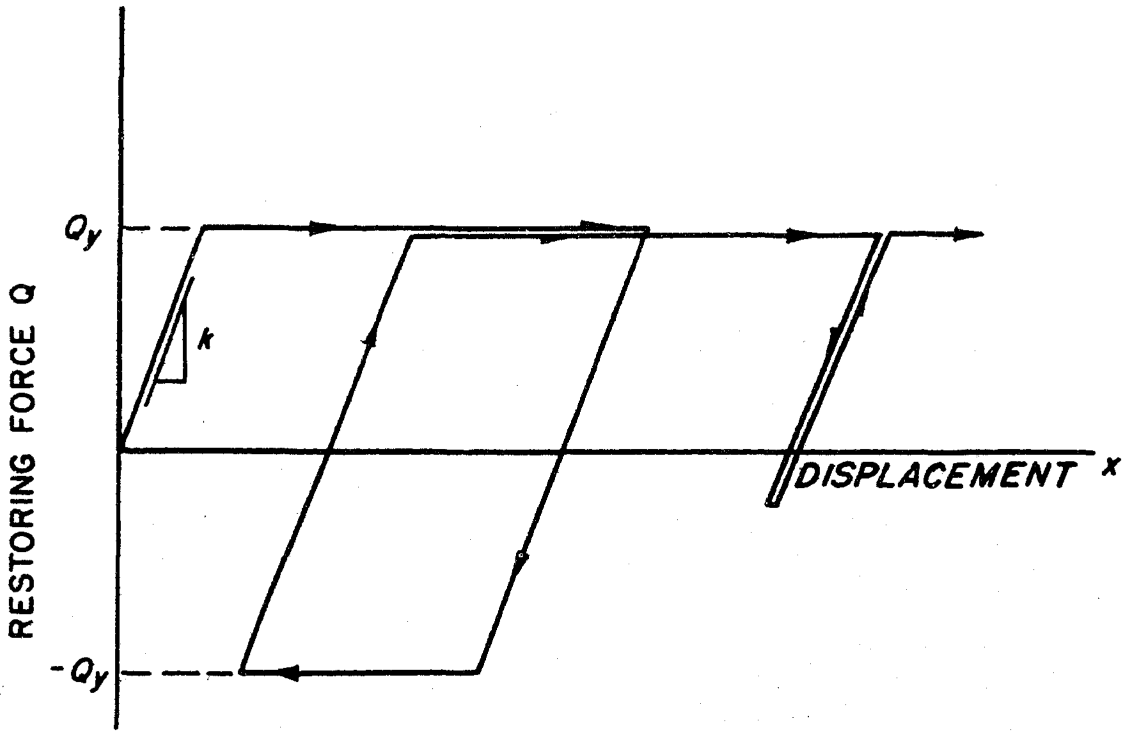

Fig. 2. Force-displacement relations.

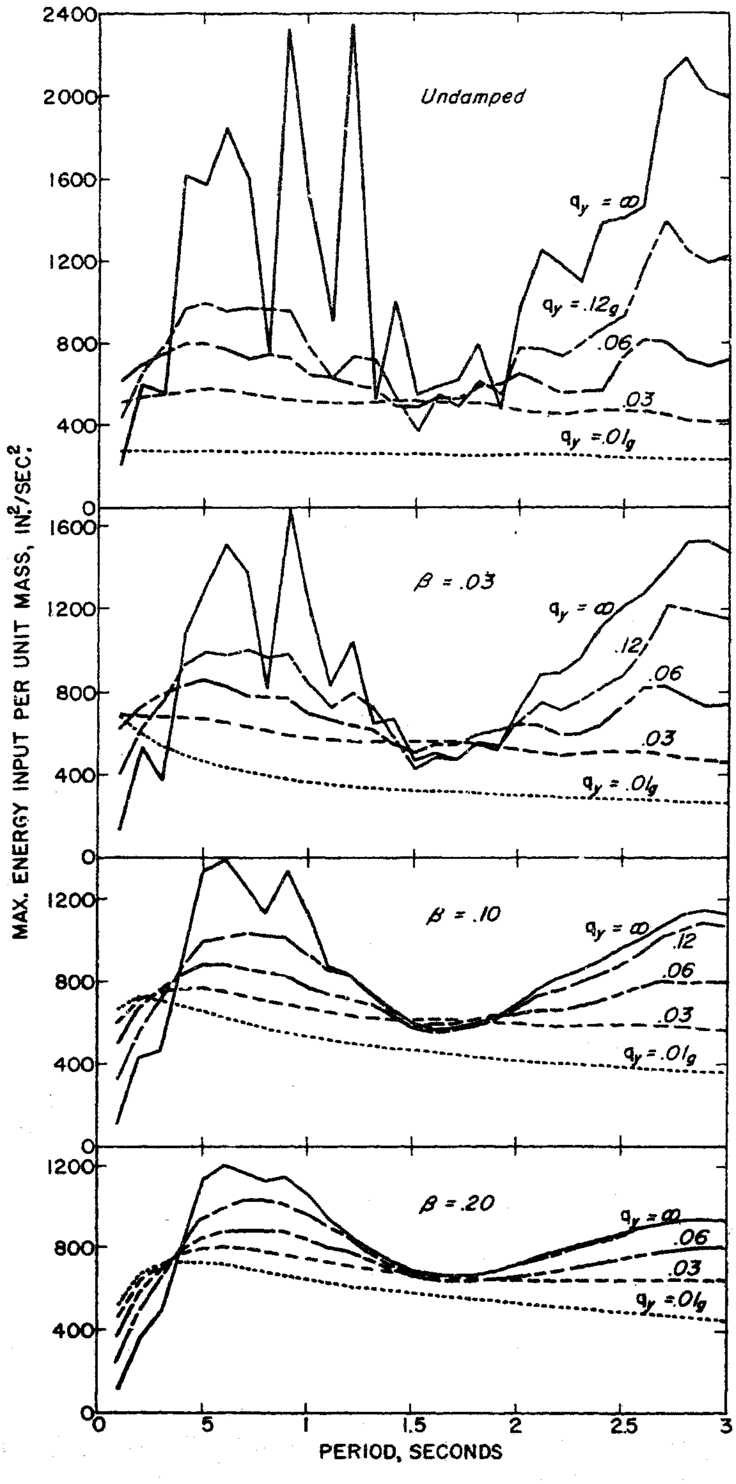

Fig. 3. Energy input spectrum (N-S component, El Centro, California, earthquake of May 18, 1940).

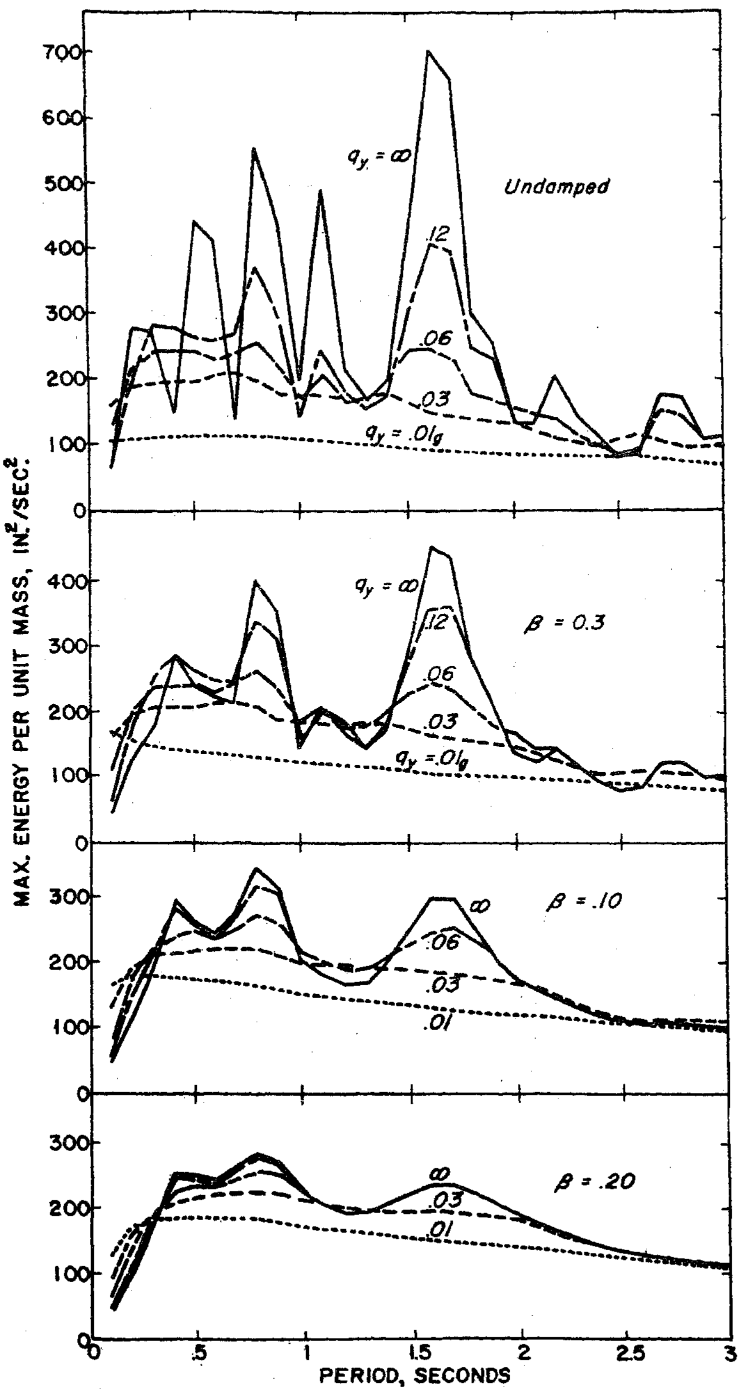

Fig. 4. Energy input spectrum (N69°W component, Taft, California, earthquake of July 21, 1952).

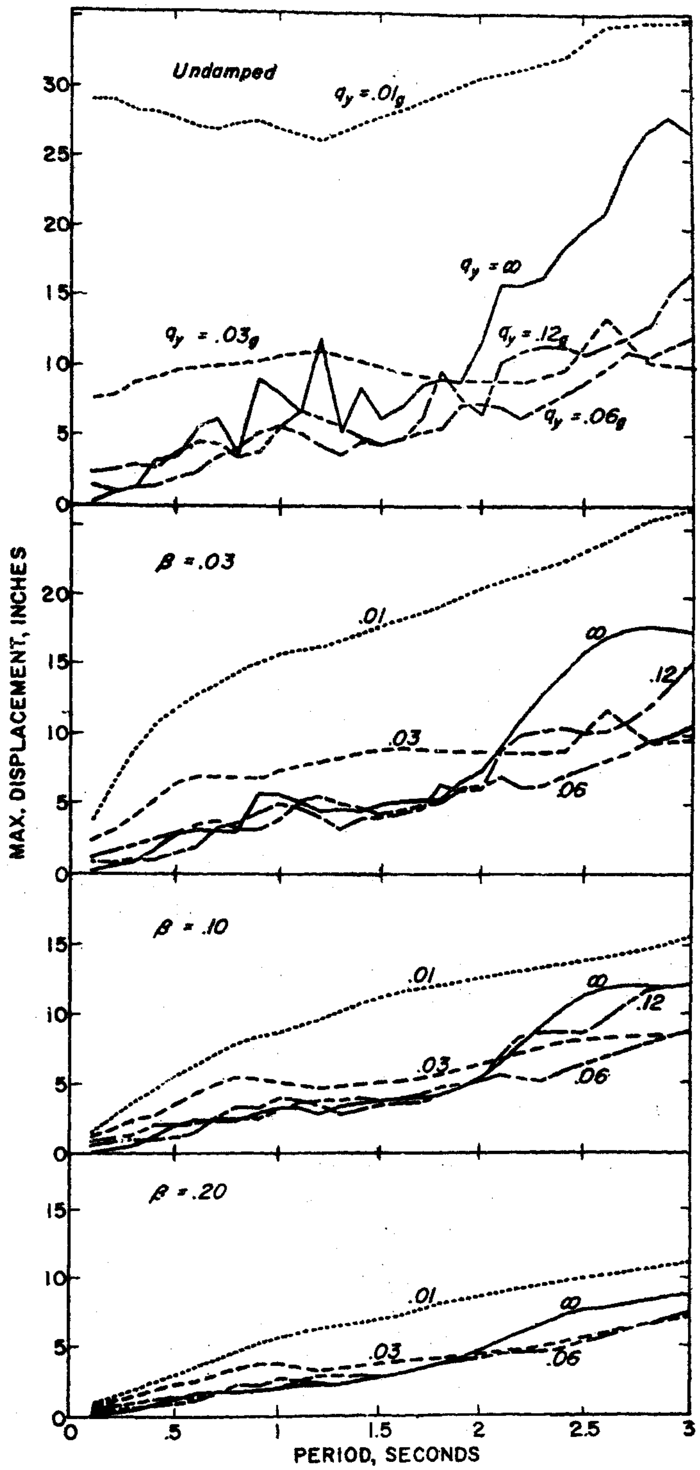

Fig. 5. Displacement spectrum (N-S component, El Centro, California, earthquake of May 18, 1940).

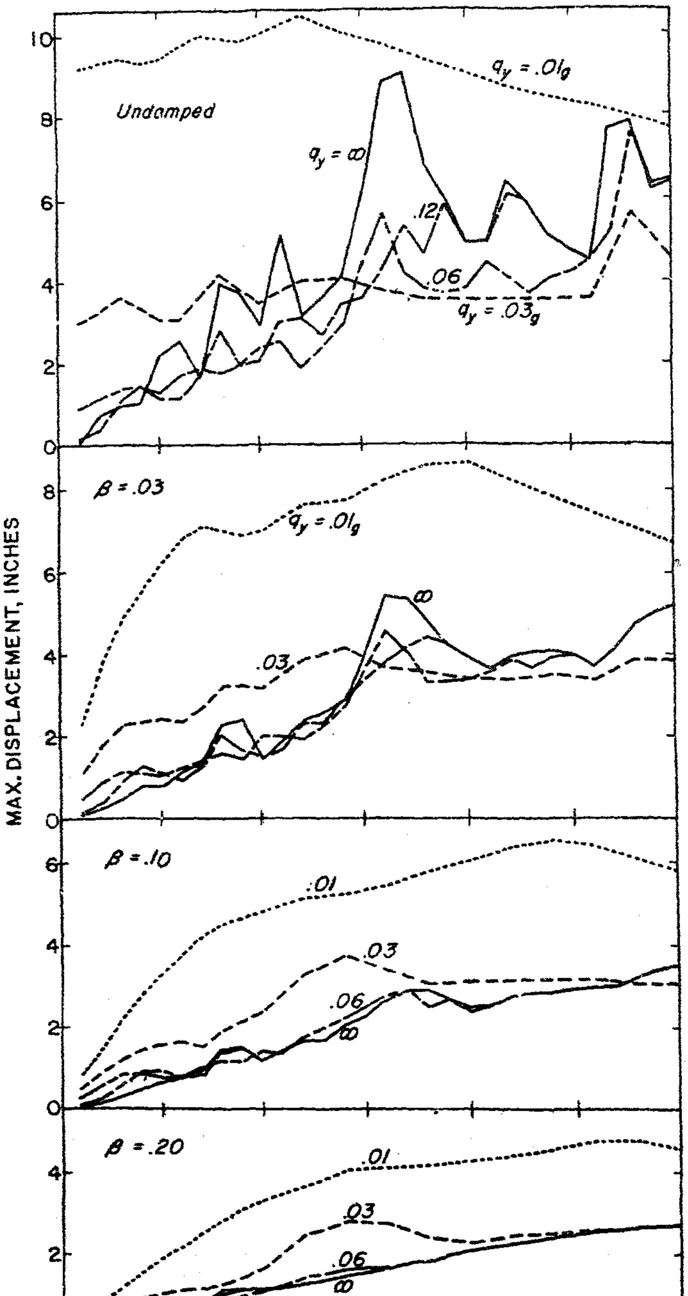

Fig. 6. Displacement spectrum (N69°W component, Taft, California, earthquake of July 21, 1952).

DISCUSSION

T. Kobori, Kyoto University, Japan:

1. This report is based on the hysteretic energy dissipation with linear viscous damping. Why did you adopt higher value of fraction of critical damping in the elastic range up to 0.10? Because I think that the linear damping is one of the equivalent to the nonlinear (bilinear) damping.

2. Why did you use linear velocity spectrum Svs for the check of structural design in plastic domain?

G. V. Berg:

1. The higher values of viscous damping were included in the studies for completeness, even though they may be unrealistic in an actual physical system. The same might be said for the very low values of yield level.

   With regard to the "equivalent viscous damping" for an elasto-plastic system, one must have either a periodic response or small nonlinearity to get a meaningful equivalence. For transient response involving large nonlinearities no reasonable equivalence can be established.

   This has been discussed by Professor Jacobsen in reference 4.

2. The linear response spectrum Sv was suggested because it is readily available for most strong-motion earthquakes of record. See reference 1.
   Inelastic deformation tends to reduce the total energy input, so the use of the linear response spectrum should give a conservative design.

ELASTIC RESPONSE OF MULTI-STORY SHEAR BEAM TYPE STRUCTURES SUBJECTED TO STRONG GROUND MOTION

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1. ABSTRACT

The elastic response of a multiple degree of freedom system vibrating as a base excited shear-beam is analyzed by superposition of normal modes. Absolute viscous damping is included in the theoretical treatment, both for modal analysis and for direct numerical integration of the governing equations of motion. The two methods of analysis are used to compute the undamped response of three multi-story structures to 12 recorded earthquakes. The results of the two methods are compared and conclusions drawn about the use of modal analysis to compute the maximum response of structures to time-varying ground displacements.

2. INTRODUCTION

The total effect of an earthquake on a structure with an infinite number of degrees of freedom is complex. To obtain a useful solution it is necessary to make simplifying assumptions about the structure and the energy input to the structure.

If the ground motion is essentially horizontal, and the structure is of rigid frame construction it is not unreasonable to treat the system as a compound oscillator with rigid masses connected by shear springs. With these assumptions the response of the structure can be analyzed mathematically.

Other authors have made modal analyses of undamped structures treating them as cantilever rods vibrating either in flexure[1,2], or in shear. The question as to whether the structure will vibrate in shear, flexure or a combination of both is not easy to predict, although some relative measurement of this is determined by the rigidity of the structure[3].

It is the intent of this paper to develop the modal analysis of damped structures vibrating in shear modes only. Absolute viscous damping is considered. The modal analysis of a two degree of freedom shear beam with intermass (relative) damping has been developed independently of this investigation.[4] The results are generally of the same nature as those presented herein.

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