ANALOGUE COMPUTER TECHNIQUES IN ASEISMIC DESIGN

by

G. N. Bycroft*

Aseismic design is handicapped by a lack of statistical knowledge of earthquakes and the difficulty of analyzing the response of a complex frame to these irregular forcing functions. Structures subject to earthquake motion present a complex picture of linear and non-linear vibration. An explicit solution in terms of the participation factors of the modes and spectrum of the earthquake is useful when considering linear vibrations of the structure but the introduction of non-linearities such as friction and plastic yielding make this approach untenable. The general complexity of both linear and non-linear cases has led to the simulation of structures by various mechanical and electrical models. These models are highly idealized and, in general, simulate only the major motions of the structure such as shearing, bending, and rocking of the structure on its base. Mechanical modelling(1) produces satisfactory results but suffers from complexity and lack of versatility. Further, it is difficult to apply to a mechanical model, which is necessarily not small, a prescribed forcing function as complicated as that of an earthquake. On the other hand this method has the advantage that the structure may be more faithfully modelled than by other means. Electrical models of various types(2, 3, 4, 5, 6) have been made and have the advantage that their parameters may be readily changed over large ranges and measurements recorded by convenient instruments.

(a) Earthquake Representation

The obvious and direct method of earthquake representation is to reproduce the actual accelerograms of a number of large earthquakes, for example those given by Housner et al(4). The usual technique is to scan the accelerograms with some form of optical-photocell arrangement, thus obtaining an electrical equivalent of the original disturbance. This may be used as a source of excitation for an electrical or mechanical simulation of a building structure. As the relevant frequencies of the original disturbance are too low for suitable electrical operation it is necessary to decrease the time scale of the disturbance by a large factor and increase the characteristic frequencies of the simulated structure by a corresponding factor. Depending on the system used, it may be convenient to study a family of structures by varying either of these factors.

---

* Research Officer, Division of Building Research, National Research Council, Ottawa, Canada

Any one earthquake is characterized by its particular shape and also its "size" and duration. Earthquakes of the same energy content but different configurations may have completely different effects on different structures. As each earthquake bears little relation to its predecessors one is faced with the question of deciding how to represent future earthquakes. If enough records of strong motion earthquakes were available for each locality the situation could be given a sound statistical basis. Unfortunately, this is not the case. Housner(7,8) first conceived the idea of representing strong motion earthquakes by a random assemblage of acceleration impulses. This was a most important step in the conception of the true nature of a large earthquake. But the complete description of a random process is very difficult, if not impossible, and requires far more statistical knowledge of strong motion earthquakes than is available. If one regards earthquake accelograms of equal "size" as being finite bursts of a stationary gaussian random process then it is necessary to determine the power spectral density of such a source in order to make it correspond to what is known of the response spectra of typical earthquakes. In a recent paper(9) the author used an analogue computer to construct the conventional damped velocity spectra of bursts of electrical noise having a power spectral density that was constant over the relevant frequency range, i.e. "band-limited white noise". Figure 1 shows the mean of the maximum velocities occurring in a damped oscillator during successive bursts of acceleration of duration twenty-five seconds and spectral density 0.78 ft^2/sec^4/cycle, for various periods and damping coefficients, $\mathcal{R}$.

Also shown are Housner's(10) averaged earthquake spectra with the scale chosen to correspond to that of El Centro 1940. The correspondence is reasonable. As far as mean maximum responses are concerned bursts of random noise appear to be a suitable representation of typical large earthquakes. On this basis the "size" of the earthquake is determined by the spectral density and duration. Thus successive bursts of equal duration give equal earthquakes although they differ somewhat in configuration and hence in their effect on any particular structure. The objective here is to provide a representation of a particular "size" of earthquake, viz El Centro 1940. The distribution of relative "sizes", in any one locality, is another question. Depending on the band width of the noise used this representation permits higher peak accelerations corresponding to higher frequencies than is observed on accelograms. Where the band width should be limited could only be determined by seismograph instrumentation having a higher frequency response together with a comparison of corresponding spectral analyses.

As "white noise" generators are readily available this representation is particularly suitable for the excitation of electrical models.

(b) Simulation of Structures by Passive Networks

The construction of electrical circuits obeying the same laws as those of a mechanical system is a technique used often in engineering analysis and also used on many occasions in aseismic design of structures. Within certain limitations electric networks have reciprocals and there is the possibility of two analogous networks for the one mechanical system. The mechanical equations are the summation of forces and these may have their analogy in the summation of voltages in one circuit or the summation of loop currents in the reciprocal circuit. In the one, force is represented by voltage and in the other by currents.

As an example of the voltage-force analogy one may consider a damped single degree of freedom oscillator having an idealized elasto-plastic stiffness characteristic. This is a fundamental unit of a structure vibrating into the plastic region. The restoring force is assumed proportional to displacement up to the yield point and then stays constant until the velocity changes sign. The force displacement characteristic now decreases along a line parallel to the initial linear portion until the reverse yield point is reached.

Let

m = mass of the oscillator,

k = the linear stiffness of the oscillator,

$\omega = \left(k/m\right)^{1/2}$ = angular frequency of the system,

b = viscous damping coefficient,

x = displacement of the mass relative to the base,

x_0 = displacement at which the system yields,

$k x_0 = F$ = yield force,

$\alpha(t)$ = ground acceleration,

then the equation of motion of the oscillator is

$$m\{\ddot{x} + \alpha(t)\} + b\dot{x} = -k(x - Y), \quad (x - Y) \leq x_0 \qquad (1)$$
$$= -F \operatorname{sgn} \dot{x}, \quad (x - Y) \geq x_0$$

where Y is the algebraic sum of all yield displacements up to the time in question.

Consider the circuit shown in Fig. 2. The condenser diode branch behaves as a condenser as long as the applied voltage is less than the biasing voltage E. If an attempt is made to raise the voltage above the biasing voltage current flows through one or other of the diodes. A loop analysis of the circuit gives,

$$L(\ddot{q}+\ddot{Q})+R\dot{q} = -\frac{(q-e)}{C}, \quad \frac{(q-e)}{C} \leq E$$
$$= -E\,\mathrm{sgn}\,\dot{q}, \quad \frac{(q-e)}{C} \geq E \qquad (2)$$

where q and Q are the charges corresponding to the loop currents i, I and e is the algebraic sum of the charges which have flowed in the diode branch of the circuit up to the time considered. Equations (1) and (2) are analogous and if L is made proportional to m, R to b, $1/C$ to k, E to F and $\ddot{Q}$ to $\alpha(t)$ then the charge q will be proportional to the displacement x. The requirement that $\ddot{Q}$ be proportional to $\alpha(t)$ is that the current input I be proportional to the velocity of the ground motion.

(c) Analog Computer Simulation

Another method of simulating the mechanical equations is to assemble a circuit of integrators and summers in such a fashion that the voltage relations are analogous to the relations of some particular variable in the mechanical system. The advantage of an analogue computer is that the necessary components are already organized in a fashion which enables any particular circuit to be readily assembled. Further, non-linear components, scanning devices, noise generators, and plotting facilities are generally provided. The main disadvantage is that one is limited, in general, to one dimension in space. This technique will be illustrated by a rather special problem in aseismic design.

(d) Nuclear Pile Subject to Earthquake Motion

The design of nuclear power stations in seismic areas introduces some rather special problems to earthquake engineering. One of these problems concerns a graphite moderated nuclear pile subject to earthquake ground motion.

Basically, the situation is that of a cylindrical stack of graphite blocks with control rods inserted vertically. The special feature is that the relative horizontal movements of the blocks during an earthquake must not be permitted to become large enough to jam the control rods. It has been suggested that the blocks be restrained by an encircling metal cylinder. It is of interest therefore to estimate the size of these relative displacements for various thicknesses of this cylinder when subjected to a large earthquake of the "size" of El Centro 1940.

As the height and diameter of the cylinder are approximately equal it is assumed that only horizontal shear of the surrounding cylinder is significant. Lengthening and shortening of the cylinder vertically resulting in bending is neglected. It is further assumed that the blocks are stiff enough horizontally to prevent any appreciable change in the circular shape of horizontal sections of the pile. Layers of the blocks now move together horizontally as a single flat disk. The idealized mechanical system is thus a pile of circular disks resting on top of each other. Between any two disks there is a friction force obeying the simple friction laws and also a horizontal spring force proportional to the relative displacements of these two disks. The top is assumed unrestrained and the bottom of the cylinder is rigidly fixed to the ground. The case where the top is also fixed is immediately deducible.

For convenience on the computer a system of five disks and springs was set up. The disks are of equal height and weight and the connecting points of the shear springs are at the mid-points of their heights. The spring stiffnesses are all k except at the base where the stiffness is $2k$. If λ is the coefficient of friction, y_n is the displacement of the n^{th} disk relative to the base and $W(t)$ is the representative "white" ground acceleration of t_0 seconds duration, then

$$m\ddot{y}_1 = m\,W(t) - 2ky_1 + (y_2 - y_1)k - 5mg\lambda \,\mathrm{sgn}\,\dot{y}_1 + 4mg\lambda\,\mathrm{sgn}(\dot{y}_2 - \dot{y}_1). \tag{3}$$

for the bottom disk,

$$m\ddot{y}_n = m\,W(t) + k(y_{n+1} - y_n) - k(y_n - y_{n-1}) + (5-n)mg\lambda\,\mathrm{sgn}(\dot{y}_{n+1} - \dot{y}_n) - (6-n)mg\lambda\,\mathrm{sgn}(\dot{y}_n - \dot{y}_{n-1}) \tag{4}$$

for the middle disks, and,

$$m\ddot{y}_5 = m\,W(t) - k(y_5 - y_4) - mg\lambda\,\mathrm{sgn}(\dot{y}_5 - \dot{y}_4) \tag{5}$$

for the top disk.

For the lower harmonics,

$(\dot{y}_n - \dot{y}_{n-1})$ and $(\dot{y}_{n+1} - \dot{y}_n)$ will always have same sign and we may simplify Equation (4) to,

$$\ddot{y}_n - \omega^2(y_{n+1} - 2y_n + y_{n-1}) + g\lambda\,\mathrm{sgn}(\dot{y}_n - \dot{y}_{n-1}) = W(t) \tag{6}$$

where $\omega^2 = k/m$, i.e., the angular frequency of one of the masses on one of the springs.

These equations were suitably scaled and simulated on the computer. Figure 3 shows the circuit for the n^{th} cell. Y_n refers to the scaled computer variable corresponding to y_n. The circuit is shown in the normal diagrammatical fashion where circles are potentiometers, triangles are summers, and double-based triangles are integrators. The relative displacements $(Y_n - Y_{n-1})$ were recorded simultaneously on a recorder. For each value of ω and λ, ten runs of 25 seconds duration were made; the mean of the maximum relative displacements of each of these ten runs is shown in Figs. 4, 5. These figures show the actual relative displacement in inches of $(y_n - y_{n-1})$. It must be remembered that y_1 is at a point one-tenth of the height of the pile from the base but that the other points differ by a distance one-fifth of the pile height. This is because y_n is the displacement at the mid-point of the n^{th} disk. There will be many more than five blocks in an actual pile height but a simple interpolation of these results should give a reasonable figure. Figure 6 shows the largest displacement of the top relative to the base that could occur if all the blocks moved in the same direction. This is merely the summation of all the maxima of the relative displacements.

The results show the trends that are to be expected. Naturally, increasing the stiffness of the system decreases the displacements as does a higher coefficient of friction. In a few cases one hundred runs were made in order to find the statistical distribution of the results. The dispersion was found to be comparatively small and in the many runs made the largest of the maxima was never greater than twice the mean.

The value of ω in terms of the pile dimensions may be simply derived.

Let

r = radius of the cylinder (ft)

h = height of the cylinder (ft)

ρ = density of the graphite (lb/cu ft)

G = modulus of rigidity of the cylinder (lb/sq ft)

t = thickness of cylinder (ft)

It may be shown that for five cells,

$$\omega^2 = \frac{50 g G t}{h^2 \rho r} \tag{7}$$

The shear stress S in the cylinder is given by

$$S = \frac{5G(y_n - y_{n-1})}{h} \tag{8}$$

If we assume as rough dimensions

$$G = 12^2 . 10^7, \quad h = 40, \quad r = 30,$$
$$\rho = 140$$

then, $$\omega = 540\, t^{1/2} \qquad (9)$$

As t ranges from 0.01 to 0.25, ω ranges approximately from 50 to 250.

Because of the nature of the non-linearity in this example it will be more sensitive to higher frequency components in the noise than would a linear situation. As a wide band of noise was used the resulting displacements must be qualified accordingly.

BIBLIOGRAPHY

(1) Jacobsen, Lydik S. and Robert S. Ayre. "Experimentally Determined Dynamic Shears in a Sixteen-story Model", Bull. Seism. Soc. Am., 28: 269-311, 1938.

(2) Housner, G. W. and G. D. McCann. "The Analysis of Strong Motion Earthquake Records with the Electric Analog Computer", Bull. Seism. Soc. Am., 39: 47-56 (1949).

(3) Murphy, M.J., G.N. Bycroft and L.W. Harrison. "Electrical Analog for Earthquake Shear Stresses in a Multi-Story Building", First World Conference on Earthquake Engineering, Berkeley, California, June 1956, Paper No. 9.

(4) Housner, G. W., R.R. Martel, and J.L. Alford. "Spectrum Analysis of Strong-Motion Earthquakes", Bull. Seism. Soc. Am., 43, p. 97-119, 1953.

(5) Merritt, R. G. and G. W. Housner. "Effect of Foundation Compliance on Earthquake Stresses in Multistory Buildings", Bull. Seism. Soc. Am., 44: 551-569 (1954).

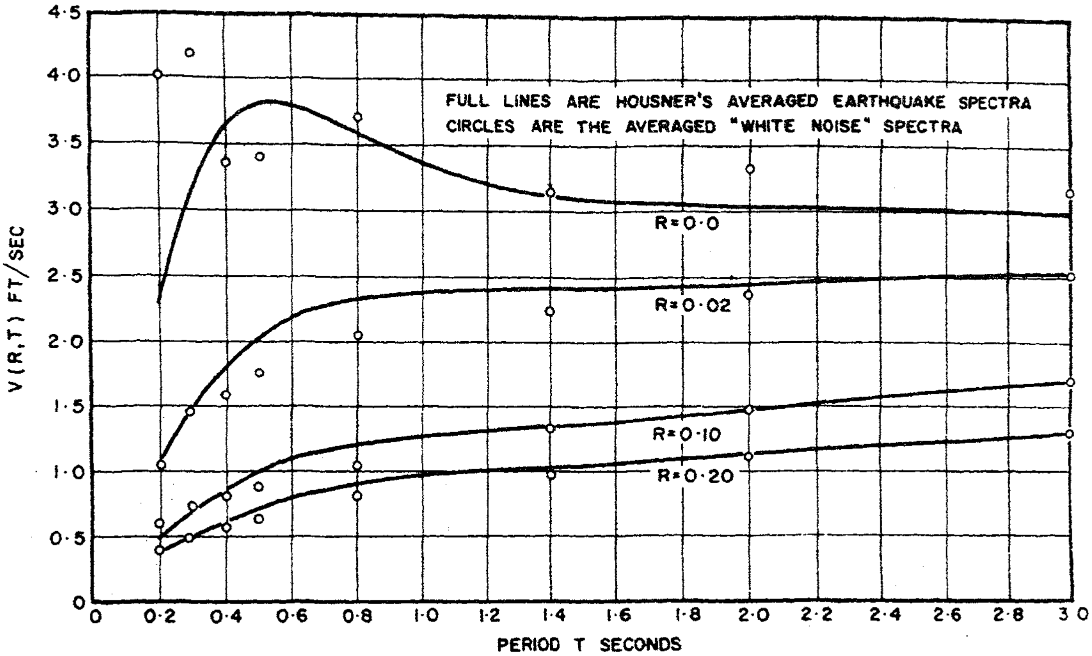
(6) Bycroft, G.N., M.J. Murphy and K.J. Brown. "Electrical Analog for Earthquake Yield Spectra", Engineering Mechanics Division Proc. A.S.C.E., 85: 43-64 (1959).

(7) Housner, G. W. "Characteristics of Strong Motion Earthquakes", Bull. Seism. Soc. Am., 37: 19-31 (1947).

(8) Housner, G. W. "Properties of Strong Motion Earthquakes", Bull. Seism. Soc. Am., 45, p. 197-218, 1955.

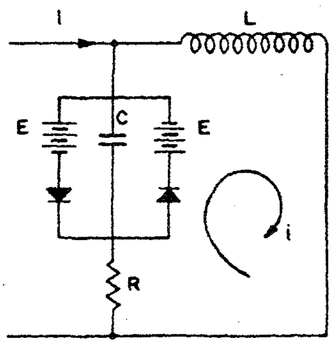
(9) Bycroft, G.N. "White Noise Representation of Earthquakes", to be published shortly in the Engineering Mechanics Division, Proc. A.S.C.E.

(10) Housner, G.W. "Limit Design of Structures", First World Conference on Earthquake Engineering, Berkeley, California, June 1956.



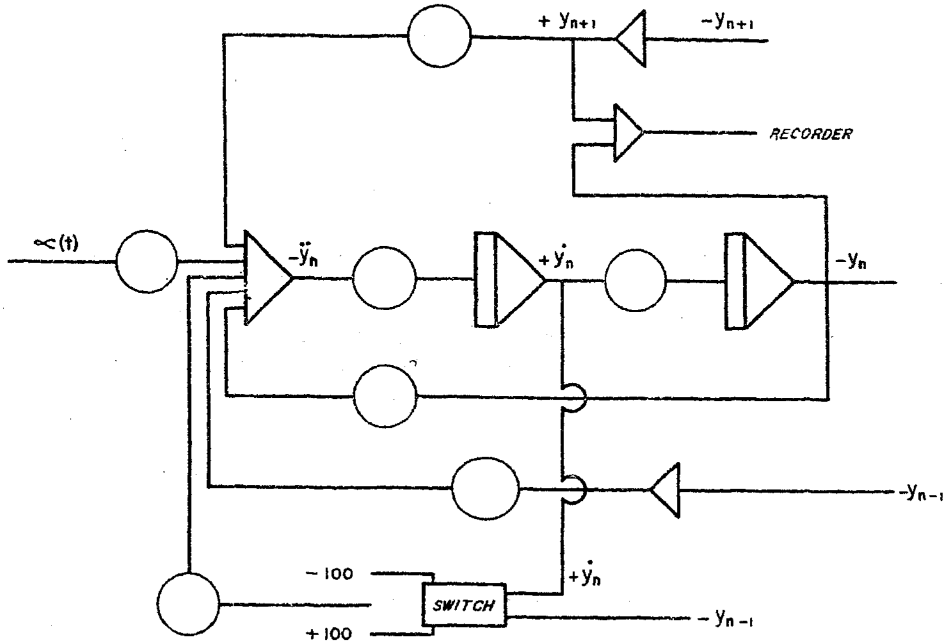
EARTHQUAKE AND NOISE SPECTRA

FIGURE 1



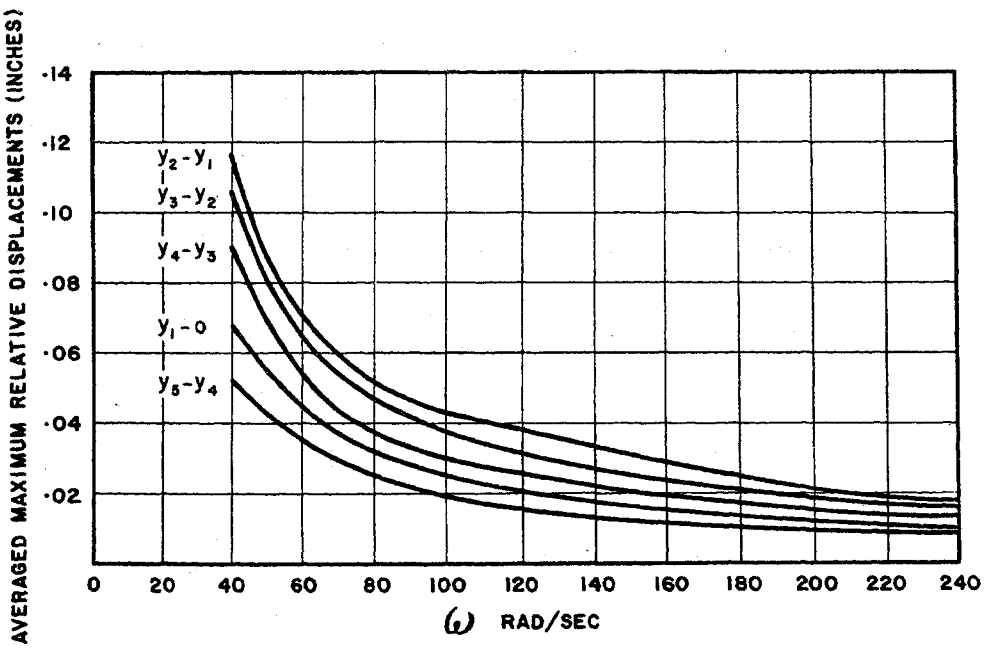
ELECTRICAL ANALOGUE OF YIELDING OSCILLATOR

FIGURE 2



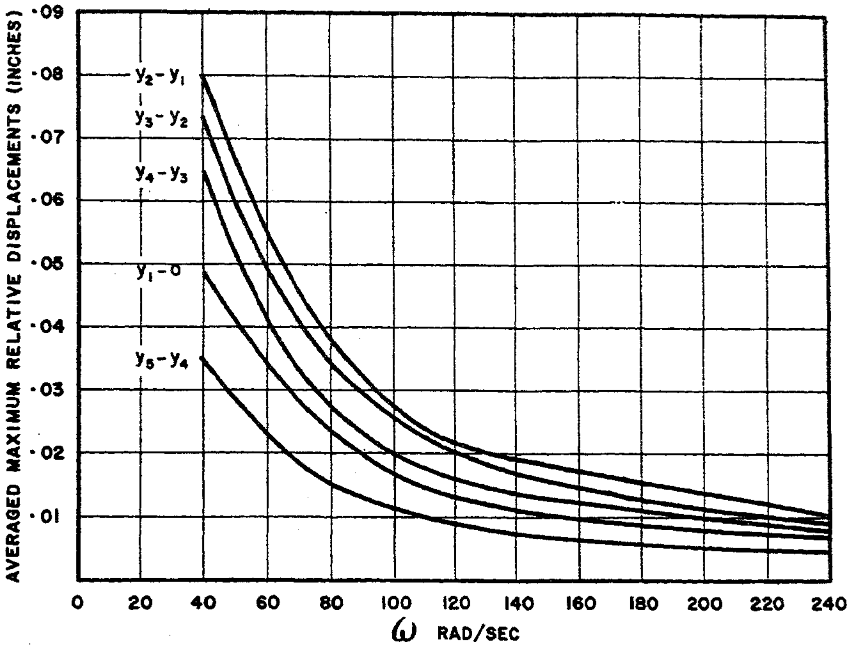
COMPUTER DIAGRAM FOR ANALOGUE CELL N

FIGURE 3



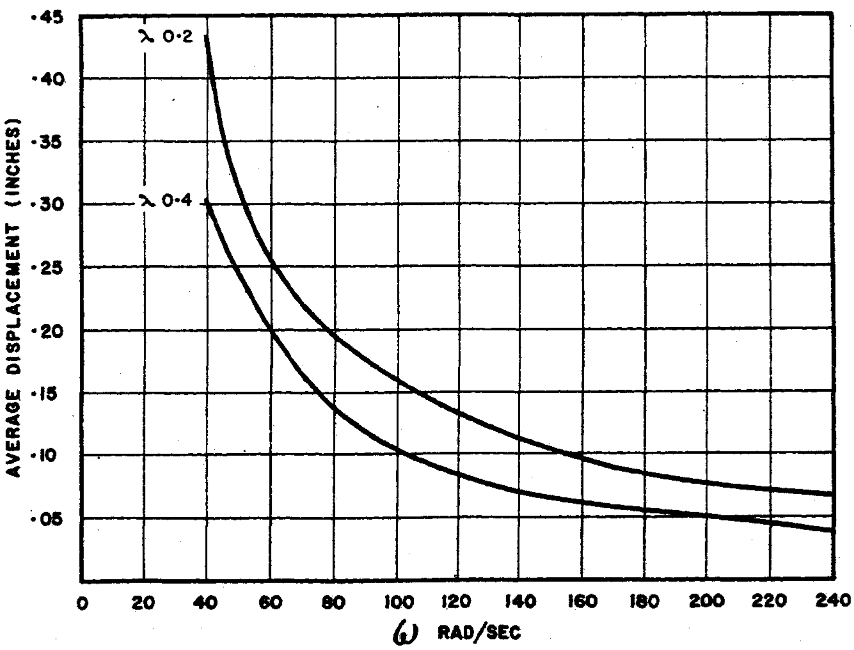
RELATIVE DISPLACEMENTS FOR λ= 0·2

FIGURE 4



RELATIVE DISPLACEMENTS FOR λ= 0·4

FIGURE 5



DISPLACEMENT OF TOP RELATIVE TO THE BASE

FIGURE 6